

Department of Economics
The Ohio State University
Midterm Answers–Econ 805

Prof. Peck

1. (30 points)

Consider the following game with two players, a plaintiff and a defendant engaged in a civil law suit. Nature moves first, selecting which side would win if the case goes to trial. The probability that the plaintiff would win a trial is $\frac{1}{3}$, and the probability that the defendant would win a trial is $\frac{2}{3}$. The plaintiff observes whether or not he would win, but the defendant does not. If the plaintiff wins a trial, his payoff is 3 and the defendant's payoff is -4 . If the plaintiff loses a trial, his payoff is -1 and the defendant's payoff is 0.

Now suppose that the plaintiff, after observing who would win a trial, is forced to propose a settlement, either a low settlement, $m = 1$, or a high settlement, $m = 2$. The defendant, after observing the settlement offer but not nature's move, must either accept the offer or refuse. If a settlement offer of m is accepted, the plaintiff's payoff is m and the defendant's payoff is $-m$. If the settlement offer is not accepted, the case goes to trial.

(a) (15 points) Give the extensive form representation of this game, by drawing the game tree and labeling things properly. Try to be neat! (If I can't read it, I will not assume that it is right.)

(b) (15 points) Find a weak perfect Bayesian equilibrium (WPBE) for this game. Remember to fully specify the strategies and the defendant's beliefs. Explain why the conditions for a WPBE are satisfied.

Answer: (a) See the diagram.

(b) The WPBE strategies are the following. The plaintiff always offers a settlement, $m = 2$, whether he observes that he would win at trial or he observes that he would lose at trial. The defendant rejects all settlement offers, $m = 2$ or $m = 1$. For the information set in which the settlement offer is $m = 2$, the defendant's beliefs are: $\mu(Pwins) = \frac{1}{3}$ and $\mu(Dwins) = \frac{2}{3}$. For the information set in which the settlement offer is $m = 1$, the defendant's beliefs are: $\mu(Pwins) = 0$ and $\mu(Dwins) = 1$.

Let us check that the conditions for a WPBE are met. Given the strategies, the plaintiff's settlement offer satisfies sequential rationality. Whatever settlement offer he makes is rejected, so changing the settlement offer does not change his payoff, no matter who would win at trial. Given the strategies and the beliefs, the defendant's rejection is sequentially rational. When the settlement offer is $m = 2$, the expected payoff from going to trial is $-\frac{4}{3}$, which is better than receiving -2 . When the settlement offer is $m = 1$, the expected payoff from going to trial is 0, which is better than receiving -1 .

Beliefs are consistent. When the settlement offer is $m = 2$, Bayes' rule requires the beliefs to be as specified. When the settlement offer is $m = 1$, Bayes' rule cannot be applied, so consistency is not violated. Furthermore,

it makes intuitive sense that a deviation to a lower settlement offer is a sign of weakness. Other beliefs are possible when $m = 1$, as long as $\mu(Dwins)$ is sufficiently high to induce the defendant to refuse the offer.

2. (40 points)

Rain Forest Nuts (RFN) is a monopolist that gathers nuts from tropical rain forests without damaging trees or interfering with wildlife. Its cost function is given by $c(y) = 4y$, where y is the total production of nuts. Some of RFN's potential customers read the magazine, *Endangered Species Quarterly*, and love to clip coupons. The demand curve from the first class of customers is given by

$$X_1(p, C) = 42 - (p - C),$$

where $X_1(p, C)$ represents the quantity demanded by the first class of customers, $p - C$ represents the actual price paid, p is the posted price, and C is the coupon value to be deducted from the posted price.

The rest of RFN's potential customers do not clip coupons or read magazines. The demand curve from the second class of customers is given by

$$X_2(p) = 32 - \frac{p}{2}.$$

(a) (15 points) If RFN is able to price discriminate by offering coupons in *Endangered Species Quarterly*, what is the profit maximizing posted price, p , and coupon value, C ?

(b) (15 points) If RFN cannot offer coupons or otherwise price discriminate, what posted price and total quantity will maximize its profits?

(c) (10 points) Assuming that the only way to price discriminate is to advertise a coupon in *Endangered Species Quarterly*, what is the most that RFN would be willing to pay for its coupon advertisement?

Answer: (a) This is a standard price discrimination problem. If the coupon is confusing you, an alternative way to solve the problem is to act as if you could charge a different price to the two classes of customers, p_1 and p_2 , and then realize that the list price is the price charged to class 2 and the coupon is the difference between the two prices: $p = p_2$ and $C = p_2 - p_1$. Below, we solve the problem directly.

Solving for the inverse demand as functions (actual price paid as a function of the quantity), we have

$$p - C = 42 - y_1 \text{ for class 1, and} \tag{1}$$

$$p = 64 - 2y_2 \text{ for class 2.} \tag{2}$$

RFN's profits are given by

$$\pi = (42 - y_1)y_1 + (64 - 2y_2)y_2 - 4(y_1 + y_2). \quad (3)$$

Differentiating (3) with respect to y_1 and y_2 , we have the first order conditions which equate marginal revenue and marginal cost for each customer class:

$$42 - 2y_1 = 4, \quad (4)$$

$$64 - 4y_2 = 4. \quad (5)$$

Solving (4) and (5), we have

$$y_1 = 19, \quad y_2 = 15. \quad (6)$$

Substituting (6) into (2), we have $p = 34$. Substituting (6) and $p = 34$ into (1), we have $C = 11$.

(b) Without the ability to offer coupons or otherwise price discriminate, we must charge everyone the same actual price. The total demand from both classes of customers (assuming each class demands a positive quantity) is given by

$$X(p) = (42 - p) + (32 - \frac{p}{2}) = 74 - \frac{3p}{2}. \quad (7)$$

Solving (7) for the inverse demand function, we have

$$\begin{aligned} \frac{3p}{2} &= 74 - y, \text{ or} \\ p &= \frac{148}{3} - \frac{2y}{3}. \end{aligned} \quad (8)$$

The profit function is

$$\pi = (\frac{148}{3} - \frac{2y}{3})y - 4y. \quad (9)$$

Differentiating (9) with respect to y we have the first order condition which equates marginal revenue and marginal cost:

$$\frac{148}{3} - \frac{4y}{3} = 4. \quad (10)$$

Solving (10) for y , we have $y = 34$. Substituting $y = 34$ into (8), we have $p = \frac{80}{3}$.

(c) The most that RFN would pay for the advertisement is the additional profits it can receive by price discriminating. From (3), profits from price discriminating are

$$\pi = (23)19 + (34)15 - 4(34) = 811.$$

From (9), profits without price discriminating are

$$\pi = \left(\frac{80}{3}\right)34 - 4(34) = 770.67.$$

Therefore, the most that RFN would pay is $811 - 770.67 = 40.33$.

3. (30 points)

In the following model of Cournot (quantity) competition, the market demand curve is given by

$$X(p) = 100 - 2p.$$

There are two firms. Firm 1 is known to have constant marginal cost of 10. Firm 2 observes its marginal cost before choosing its output, but firm 2's marginal cost is not observed by firm 1. Ex ante, firm 2's marginal cost is 0 with probability $\frac{1}{2}$, and firm 2's marginal cost is a positive constant, h , with probability $\frac{1}{2}$.

(a) (20 points) Assuming that both firms produce a positive quantity, $q_1 > 0$, $q_2^0 > 0$, and $q_2^h > 0$, solve for the Bayesian Nash equilibrium of this game.

(b) (10 points) For what values of h will firm 2 choose not to produce when its marginal cost is h ?

Answer: (a) The inverse demand curve, as a function of q_1 and q_2 , is given by

$$p = \frac{100 - q_1 - q_2}{2} \quad (11)$$

Firm 1 faces uncertainty about firm 2's cost, and therefore does not know which quantity firm 2 will choose. Using (11), we can express firm 1's expected profits as a function of q_1 , q_2^0 , and q_2^h

$$\pi_1 = \frac{1}{2} \left(\frac{100 - q_1 - q_2^0}{2} \right) q_1 + \frac{1}{2} \left(\frac{100 - q_1 - q_2^h}{2} \right) q_1 - 10q_1. \quad (12)$$

Differentiating (12) with respect to q_1 , equating to zero, and simplifying, we have the reaction function,

$$q_1 = 40 - \frac{q_2^0}{4} - \frac{q_2^h}{4}. \quad (13)$$

Firm 2 faces no uncertainty. When its costs are zero, its profit function is

$$\pi_2^0 = \left(\frac{100 - q_1 - q_2^0}{2} \right) q_2^0. \quad (14)$$

Differentiating (14) with respect to q_2^0 , equating to zero, and simplifying, we have the reaction function,

$$q_2^0 = \frac{100 - q_1}{2} \quad (15)$$

When firm 2's costs are h , its profit function is

$$\pi_2^h = \left(\frac{100 - q_1 - q_2^h}{2} \right) q_2^h - hq_2^h. \quad (16)$$

Differentiating (16) with respect to q_2^h , equating to zero, and simplifying, we have the reaction function,

$$q_2^h = \frac{100 - q_1 - 2h}{2}. \quad (17)$$

Now we can simultaneously solve (13), (15), and (17) for the equilibrium quantities. Substituting (15) and (17) into (13), we have

$$q_1 = 40 - \frac{100 - q_1}{4} - \frac{100 - q_1 - 2h}{4},$$

from which we solve for q_1 ,

$$q_1 = 20 + \frac{h}{3}. \quad (18)$$

Substituting (18) into (15) and (17), we have

$$q_2^0 = 40 - \frac{h}{6} \quad \text{and} \quad q_2^h = 40 - \frac{7h}{6}. \quad (19)$$

(b) The above calculation assumed that all quantities are nonnegative. The crucial condition is that q_2^h should be nonnegative. When firm 2's marginal cost is h , its quantity becomes zero when we have

$$40 - \frac{7h}{6} = 0. \quad (20)$$

Equation (19) tells us that $q_2^h = 0$ when we have $h = \frac{240}{7}$. Thus, firm 2 does not produce when its cost is high whenever $h \geq \frac{240}{7}$ holds.