

Department of Economics  
The Ohio State University  
Midterm Questions and Answers—Econ 805

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**Directions:** *Answer all questions, carefully label all diagrams, and show all work.*

**1. (30 points)**

Consider the following “quality disclosure” game, in which a seller owns an object that has no value to her, but is valuable to a potential buyer. The value to the buyer is either \$10, \$20, or \$30, each with probability one third. The seller observes the quality of the good (i.e., the value to the buyer), but the buyer does not observe the quality.

The game starts with nature informing the seller about the quality of the good. Next, the seller decides whether or not to disclose the true quality to the buyer. That is, the buyer either observes the disclosed quality (which is known to be the truth), or else observes that quality was not disclosed. Finally, the buyer decides whether or not to buy the good at a price of \$15. If the buyer does not buy, both players receive a payoff of zero. If the buyer buys, the seller receives a payoff of \$15 and the buyer receives a payoff equal to  $q - 15$ , where  $q$  denotes the quality of the good.

- (a) *Draw and label the game tree. Be neat!*
- (b) *This game has several weak perfect Bayesian equilibria. Construct a WPBE in which the seller **always** discloses its quality. (Remember to specify beliefs as well as strategies.)*
- (c) *Construct a WPBE in which the seller **never** discloses its quality. (Remember to specify beliefs as well as strategies.)*

**Answer:**

For part (a), refer to the handout.

We will construct a WPBE in which the seller always discloses its quality. The seller’s strategy is (D,D,D), meaning that the seller discloses when  $q = 10$ , discloses when  $q = 20$ , and discloses when  $q = 30$ . The buyer’s strategy is (NB,B,B,NB), meaning that the buyer does not buy when the seller discloses  $q = 10$ , the buyer buys when the seller discloses  $q = 20$ , the buyer buys when the seller discloses  $q = 30$ , and the buyer does not buy when the seller does not disclose. At all singleton information sets, the conditional probability of being at that node is one. At the buyer’s information set following no disclosure, the beliefs assign probability one to  $q = 10$ , probability zero to  $q = 20$ , and

probability zero to  $q = 30$ . To see that this is a WPBE, notice first that Bayes' rule cannot be applied, so the beliefs are consistent. Given these beliefs, not buying is sequentially rational after the seller does not disclose. When the seller discloses, it is sequentially rational for the buyer to buy only when the quality is at least 20. Finally, it is rational for the seller to disclose a quality of 20 or 30, because the buyer will buy. It is also rational for the seller to disclose a quality of 10, because the buyer will not buy whether the seller discloses or not.

We will construct a WPBE in which the seller never discloses its quality. The seller's strategy is (ND,ND,ND), meaning that the seller does not disclose when  $q = 10$ , does not disclose when  $q = 20$ , and does not disclose when  $q = 30$ . The buyer's strategy is (NB,B,B,B), meaning that the buyer does not buy when the seller discloses  $q = 10$ , the buyer buys when the seller discloses  $q = 20$ , the buyer buys when the seller discloses  $q = 30$ , and the buyer buys when the seller does not disclose. At all singleton information sets, the conditional probability of being at that node is one. At the buyer's information set following no disclosure, the beliefs assign probability one third to  $q = 10$ , probability one third to  $q = 20$ , and probability one third to  $q = 30$ . To see that this is a WPBE, notice first that Bayes' rule determines the buyer's beliefs when quality is not disclosed, so the beliefs are consistent. Given these beliefs, buying is sequentially rational after the seller does not disclose, because the expected payoff of the buyer is 5. When the seller discloses, it is sequentially rational for the buyer to buy only when the quality is at least 20. Finally, it is rational for the seller not to disclose, because the buyer will buy.

## 2. (30 points)

Two firms are engaging in Cournot (quantity) competition. The market demand curve is given by

$$D(p) = 2300 - 10p,$$

where  $p$  denotes the market price and  $D(p)$  denotes the total quantity demanded at that price. Letting  $y_i$  denote the output quantity of firm  $i$  (for  $i = 1, 2$ ), the firms have the cost functions

$$\begin{aligned} c_1(y_1) &= \frac{(y_1)^2}{5} \quad \text{and} \\ c_2(y_2) &= \frac{(y_2)^2}{10}. \end{aligned}$$

*Find the Cournot-Nash equilibrium strategies, and find the equilibrium price.*

**Answer:**

First, we solve for the inverse demand, as a function of the two quantities

$$p = 230 - \frac{y_1}{10} - \frac{y_2}{10}. \quad (1)$$

From (1), we can write firm 1's profit function as

$$\pi_1 = \left(230 - \frac{y_1}{10} - \frac{y_2}{10}\right) y_1 - \frac{(y_1)^2}{5}. \quad (2)$$

To find firm 1's reaction function, differentiate (2) with respect to  $y_1$ , set the expression equal to zero, and solve for  $y_1$ , yielding

$$y_1 = \frac{2300 - y_2}{6}. \quad (3)$$

Going through the same steps for firm 2, its profit function is

$$\pi_2 = \left(230 - \frac{y_1}{10} - \frac{y_2}{10}\right) y_2 - \frac{(y_2)^2}{10},$$

and its reaction function is

$$y_2 = \frac{2300 - y_1}{4}. \quad (4)$$

To find the equilibrium strategies, simultaneously solve (3) and (4), by substituting (4) into (3), yielding

$$y_1 = \frac{2300 - \frac{2300 - y_1}{4}}{6}.$$

Solving, we have

$$\begin{aligned} y_1 &= 300, \\ y_2 &= 500, \\ p &= 150. \end{aligned}$$

### 3. (40 points)

Consider a market that is initially served by a single, monopolistic firm with constant average and marginal costs equal to  $c$ . There is one consumer, with a utility function over consumption of the produced good,  $x$ , and consumption of the numeraire good,  $M$ , given by

$$u(x, M) = 2\sqrt{x} + M.$$

Also, assume that the consumer has a zero endowment of good  $x$ , and a large enough endowment of the numeraire good so that we do not have to worry about nonnegativity constraints on numeraire consumption. For the questions below, your answer will depend on the parameter,  $c$ .

(a) Assuming that the monopolist sets a price (of good  $x$  in terms of the numeraire) denoted by  $p$ , and allows the consumer to purchase as much as he/she wants to purchase at that price, what will be the profit maximizing monopoly price and what will be the monopoly profit?

(b) Suppose that the monopolist can choose the number of units of output,  $x$ , in its “package,” and can choose a price for its package,  $P$ . The consumer decides whether to purchase the package or to purchase nothing. What values of  $(x, P)$  will maximize the monopolist’s profits, and what will be the profit level?

(c) Now suppose that this industry is characterized by long run perfect competition, with free entry of firms who have constant average and marginal cost equal to  $c$ . What would be the equilibrium price and quantity demanded?

**Answer:**

(a) This is a standard monopoly problem, but first we need to compute the demand function. Letting  $\omega$  denote the consumer’s numeraire endowment, the utility maximization problem is

$$\begin{aligned} & \max_{x, M} 2\sqrt{x} + M \\ & \text{subject to} \\ px + M &= \omega \end{aligned}$$

The first order conditions are the budget equation and the marginal rate of substitution condition, which gives us the inverse demand function:

$$x^{-1/2} = p. \tag{5}$$

From (5), we can write the monopolist’s profit maximization problem as follows.

$$\max(\pi) = x^{-1/2}x - cx. \tag{6}$$

Differentiating (6), setting the expression equal to zero, and solving for  $x$ , we have

$$x = \frac{1}{4c^2}. \tag{7}$$

Substituting (7) into (5) and (6), we have the monopoly price and profit,

$$\begin{aligned} p &= 2c, \\ \pi &= \frac{1}{4c}. \end{aligned}$$

(b) Here the consumer is not solving the above utility maximization problem, but rather must make a take it or leave it decision to purchase the package or not. In other words, the monopoly is able to price discriminate. The outcome will be the efficient quantity but no surplus for the consumer. Letting  $x$  be the

amount of output in the package and  $P$  be the price of the entire package (not the price per unit as in part a), the monopolist's problem is

$$\begin{aligned} & \max_{x,P} P - cx \\ & \text{subject to} \\ 2\sqrt{x} + (\omega - P) &= \omega. \end{aligned} \tag{8}$$

The left side of (8) is the utility of the consumer, since  $x$  is the consumption of the good and  $(\omega - P)$  is the numeraire consumption after paying for the package. This utility must be at least as high as the utility of refusing to purchase,  $\omega$ , and the two utilities will be equal at the profit maximum. Substituting the constraint into the objective, we have an equivalent unconstrained problem,

$$\max_x 2\sqrt{x} - cx \tag{9}$$

Differentiating (9) with respect to  $x$ , setting the expression equal to zero, and solving for  $x$ , we have

$$x = \frac{1}{c^2}. \tag{10}$$

Substituting (10) into (8), we have

$$P = \frac{2}{c}, \tag{11}$$

and substituting (10) and (11) into (9), we have

$$\pi = \frac{1}{c}.$$

Notice that profits are four times greater in part (b) than in part (a).

(c) Under perfect competition, the price equals minimum long run average cost, which in this case is simply  $c$ . The quantity supplied will equal the quantity demanded, which is determined by the consumer's demand function, calculated in part (a). Substituting  $p = c$  into (5), and solving for  $x$ , we have

$$x = \frac{1}{c^2}.$$

Not surprisingly, we have the efficient quantity derived in part (b).