

Department of Economics
The Ohio State University
Midterm Answers–Econ 805

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February 7, 2008

1. (35 points)

Consider the following economy with two goods, two firms, and one consumer. The consumer's utility function is given by

$$u(x^1, x^2) = A \log(x^1) + \log(x^2),$$

where A is a strictly positive parameter. The consumer owns both firms and has the initial endowment vector, $\omega = (1, 1)$. Firm 1 produces good 1 with good 2 as an input (that is, $y_1^1 \geq 0$ and $y_1^2 \leq 0$), and has a production function or boundary of the production set given by

$$y_1^1 = -\frac{1}{2}y_1^2.$$

Firm 2 produces good 2 with good 1 as an input (that is, $y_2^2 \geq 0$ and $y_2^1 \leq 0$), and has a production function or boundary of the production set given by

$$y_2^2 = -\frac{1}{2}y_2^1.$$

- (a) (10 points) Define a competitive equilibrium for this economy.
(b) (20 points) Normalize the price of good 2 to be one, so the price vector is given by $(p, 1)$. Compute the competitive equilibrium price and allocation, as a function of the parameter A . (Hint: Depending on A , it is possible that one or both firms do not produce.)
(c) (5 points) For what values of A is the competitive equilibrium allocation Pareto optimal?

Answer:

(a) Note: because of strict monotonicity, we can write budget constraints and market clearing as equalities. A CE is a price vector (p^1, p^2) and an allocation, $(x^1, x^2, y_1^1, y_1^2, y_2^1, y_2^2)$, such that

(i) (x^1, x^2) solves

$$\begin{aligned} & \max A \log(x^1) + \log(x^2) \\ & \text{subject to} \\ p^1 x^1 + p^2 x^2 &= p^1 + p^2 + \pi_1 + \pi_2 \\ x &\geq 0, \end{aligned}$$

(ii) (y_1^1, y_1^2) solves

$$\begin{aligned} & \max p^1 y_1^1 + p^2 y_1^2 \\ & \text{subject to} \\ y_1^1 &= -\frac{1}{2} y_1^2 \\ y_1^1 &\geq 0, y_1^2 \leq 0, \end{aligned}$$

(iii) (y_2^1, y_2^2) solves

$$\begin{aligned} & \max p^1 y_2^1 + p^2 y_2^2 \\ & \text{subject to} \\ y_2^2 &= -\frac{1}{2} y_2^1 \\ y_2^1 &\leq 0, y_2^2 \geq 0, \end{aligned}$$

(iv) markets clear

$$\begin{aligned} x^1 &= 1 + y_1^1 + y_2^1 \\ x^2 &= 1 + y_1^2 + y_2^2. \end{aligned}$$

(b) First we solve the consumer's problem. Because technologies are CRS, equilibrium profits will be zero. Demand is characterized by the f.o.c.,

$$\begin{aligned} \frac{Ax^2}{x^1} &= p \\ px^1 + x^2 &= p + 1. \end{aligned}$$

Solving for the demand functions, we have

$$\begin{aligned} x^1 &= \frac{A(p+1)}{(A+1)p} \\ x^2 &= \frac{p+1}{A+1}. \end{aligned}$$

There are three types of equilibria, depending on A . If firm 1 produces, its profit max problem implies $p = 2$, so firm 2 must not be producing. From the demands, we have

$$\begin{aligned} x^1 &= \frac{3A}{2(A+1)} \\ x^2 &= \frac{3}{A+1}. \end{aligned}$$

From market clearing and the fact that firm 2 does not produce, we have

$$\begin{aligned} y_1^1 &= \frac{A-2}{2(A+1)} \\ y_1^2 &= \frac{2-A}{(A+1)} \\ y_2^1 &= y_2^2 = 0. \end{aligned}$$

This type of equilibrium occurs whenever firm 1's output is positive, which occurs if and only if $A > 2$ holds.

If firm 2 produces, we have $p = \frac{1}{2}$, and firm 2 produces nothing. From the demands, we have

$$\begin{aligned}x^1 &= \frac{3A}{(A+1)} \\x^2 &= \frac{3}{2(A+1)}.\end{aligned}$$

Market clearing implies

$$\begin{aligned}y_2^2 &= \frac{1-2A}{2(A+1)} \\y_2^1 &= \frac{2A-1}{(A+1)} \\y_1^1 &= y_1^2 = 0.\end{aligned}$$

This type of equilibrium occurs whenever firm 2's output is positive, which occurs if and only if $A < \frac{1}{2}$ holds. Finally, if we have $\frac{1}{2} \leq A \leq 2$, neither firm produces, $y_1^1 = y_1^2 = y_2^1 = y_2^2 = 0$, and the equilibrium consumption is the endowment, $x^1 = x^2 = 1$. The equilibrium price is found by solving the marginal rate of substitution equation, $p = A$.

(c) The equilibrium allocation is Pareto optimal for all positive values of A , because we can apply the FFTWE.

2. (35 points)

The following economy has 2 consumers, 2 states of nature, and one commodity per state. The probability of state 1 is $\frac{1}{3}$, and the probability of state 2 is $\frac{2}{3}$. Before observing the state, the two consumers trade contingent commodities. For $i = 1, 2$, consumer i is a von Neumann-Morgenstern expected utility maximizer, with Bernoulli utility function $u_i(x_i) = \log(x_i)$. The endowment vector of consumer 1 is $\omega_1 = (2, 0)$, and the endowment vector of consumer 2 is $\omega_2 = (0, 1)$.

(a) (10 points) Define a competitive equilibrium for this economy, with complete contingent commodity markets.

(b) (20 points) Calculate the competitive equilibrium price vector and allocation.

(c) (5 points) Given that each consumer's expected endowment is $\frac{2}{3}$, how do you explain the difference in expected equilibrium consumption in part (b)?

Answer:

(a) Note: because of strict monotonicity, we can write budget constraints and market clearing as equalities. A CE is a price vector (p^1, p^2) and an allocation, $(x_1^1, x_1^2, x_2^1, x_2^2)$, such that

(i) (x_1^1, x_1^2) solves

$$\begin{aligned} & \max \frac{1}{3} \log(x_1^1) + \frac{2}{3} \log(x_1^2) \\ & \text{subject to} \\ & p^1 x_1^1 + p^2 x_1^2 = 2p^1 \\ & x_1 \geq 0, \end{aligned}$$

(ii) (x_2^1, x_2^2) solves

$$\begin{aligned} & \max \frac{1}{3} \log(x_2^1) + \frac{2}{3} \log(x_2^2) \\ & \text{subject to} \\ & p^1 x_2^1 + p^2 x_2^2 = p^2 \\ & x_2 \geq 0, \end{aligned}$$

and (iii) markets clear

$$\begin{aligned} x_1^1 + x_2^1 &= 2 \\ x_1^2 + x_2^2 &= 1. \end{aligned}$$

(b) Normalize prices to be $(p, 1)$. First we solve consumer 1's problem. Demand is characterized by the f.o.c.,

$$\begin{aligned} \frac{\frac{1}{3}x_1^2}{\frac{2}{3}x_1^1} &= p \\ px_1^1 + x_1^2 &= 2p. \end{aligned}$$

Solving for the demand functions, we have

$$\begin{aligned} x_1^1 &= \frac{2}{3} \\ x_1^2 &= \frac{4p}{3}. \end{aligned}$$

For consumer 2, demand is characterized by the f.o.c.,

$$\begin{aligned} \frac{\frac{1}{3}x_2^2}{\frac{2}{3}x_2^1} &= p \\ px_2^1 + x_2^2 &= 1. \end{aligned}$$

Solving for the demand functions, we have

$$\begin{aligned} x_2^1 &= \frac{1}{3p} \\ x_2^2 &= \frac{2}{3}. \end{aligned}$$

Market clearing for good 2 can be solved for the equilibrium price

$$\begin{aligned}\frac{4p}{3} + \frac{2}{3} &= 1 \\ p &= \frac{1}{4}.\end{aligned}$$

Substituting the equilibrium price into the demand functions yields the equilibrium allocation,

$$\begin{aligned}x_1 &= \left(\frac{2}{3}, \frac{1}{3}\right) \\ x_2 &= \left(\frac{4}{3}, \frac{2}{3}\right).\end{aligned}$$

(c) Even though each consumer has the same expected endowment, $\frac{2}{3}$, consumer 2 receives twice as much consumption as consumer 1. The reason is that consumer 2 receives her endowment in state 2, where consumption is scarce, and consumer 1 receives his endowment in state 1, where consumption is plentiful. Therefore, the relative price of consumption in state 2 is even greater than the probability ratio, so consumer 2 is richer than consumer 1.

3. (30 points)

Consider the following version of the Akerlof lemons model. There are two types of goods, numeraire or money consumption, M , and cars. The quality of car i is denoted by x_i , and is uniformly distributed between $\frac{1}{4}$ and 1,

$$x_i \sim U\left(\frac{1}{4}, 1\right),$$

independently of the quality of the other cars. Type 1 traders (potential sellers) have the utility function over consumption of money and car qualities (of the n cars she owns) given by

$$u_1 = M + \sum_{i=1}^n x_i.$$

Type 2 traders (potential buyers) have the utility function over consumption of money and car qualities (of the n cars he owns) given by

$$u_1 = M + \sum_{i=1}^n \frac{3}{2}x_i.$$

Assume that type 2 traders are endowed with a large amount of money, so that they can afford to buy as many cars as they want. Note: the only difference between this model and the Akerlof model presented in class is that quality is now uniform between $\frac{1}{4}$ and 1.

Question: What will be the equilibrium price, and what will be the average quality of cars sold?

Answer:

If the equilibrium price is p , then every seller whose quality is below p will sell her car. Therefore, the distribution of car qualities on the market is uniform over the range $[\frac{1}{4}, p]$. Thus, the expected quality of cars on the market, given price p , is

$$\mu(p) = \frac{\frac{1}{4} + p}{2} = \frac{1}{8} + \frac{p}{2}.$$

Buyers' net gain in utility from consuming a car when the price is p is given by

$$\frac{3}{2}\mu(p) - p.$$

Thus, there will be excess demand for cars whenever the previous expression is positive, and excess supply of cars when the previous expression is negative. In equilibrium, we must have

$$\begin{aligned}\frac{3}{2}\mu(p) &= p, \\ \frac{3}{2}\left[\frac{1}{8} + \frac{p}{2}\right] &= p, \\ p &= \frac{3}{4}.\end{aligned}$$

At the equilibrium price of $\frac{3}{4}$, the expected quality is $\frac{1}{2}$.