Department of Economics
The Ohio State University
Midterm Exam Questions and Answers–Econ 805

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1. (30 points)

Consider the following pure exchange economy with two goods and n consumers. All utility functions satisfy strict monotonicity, strict quasi-concavity, and continuity. Endowment vectors are strictly interior. Also, the aggregate endowment is given by

$$\sum_{i=1}^{n} \omega_i = (20, 80),$$

and the aggregate demand function for good 1, as a function of the price vector $p = (p^1, p^2)$, is given by

$$\sum_{i=1}^{n} x_i^1(p) = \frac{(20p^1 + 80p^2)p^2}{p^1(p^1 + p^2)}.$$

(a) Calculate the competitive equilibrium price ratio (p^1/p^2).
(b) Calculate the aggregate demand function for good 2.

Answer:

(a) We have an equilibrium price ratio if and only if supply equals demand for good 1,

$$\frac{(20p^1 + 80p^2)p^2}{p^1(p^1 + p^2)} = 20.$$

Cross-multiplying, we have

$$\begin{aligned} 20(p^1)^2 + 20p^1p^2 &= 20p^1p^2 + 80(p^2)^2 \\ 20(p^1)^2 &= 80(p^2)^2 \\ \frac{p^1}{p^2} &= 2. \end{aligned}$$

(b) By Walras' Law, we have

$$\begin{aligned} p^1\left[\frac{(20p^1 + 80p^2)p^2}{p^1(p^1 + p^2)}\right] + p^2\sum_{i=1}^{n} x_i^2(p) &= 20p^1 + 80p^2. \\ \sum_{i=1}^{n} x_i^2(p) &= 20\frac{p^1}{p^2} + 80 - \frac{p^1}{p^2}\left[\frac{(20p^1 + 80p^2)p^2}{p^1(p^1 + p^2)}\right], \end{aligned}$$

which can be simplified to

$$\sum_{i=1}^{n} x_i^2(p) = \frac{(20p^1 + 80p^2)p^1}{p^2(p^1 + p^2)}.$$

2. (20 points)

A pure exchange economy has n consumers and k goods. Endowments are strictly interior and all utility functions satisfy strict monotonicity, strict quasi-concavity, and continuity. Also, suppose that the economy has two competitive equilibria, (p^*, x^*) and (p^{**}, x^{**}), where $x^* \neq x^{**}$. For the following statement, either prove it or find a counterexample. (If you use a theorem from class as part of your argument, you do not have to prove the theorem again here.)

Statement: There exists at least one consumer i such that $u_i(x_i^*) > u_i(x_i^{**})$.

Answer:

The statement is true. Suppose that the statement is not true. Then for all consumers i, we have

$$u_i(x_i^*) \leq u_i(x_i^{**}).$$

If the inequality is strict for some i, then x^{**} Pareto dominates x^*, which contradicts the fact that x^* is a C.E. allocation, and therefore Pareto optimal by the FFTWE. The remaining case is that, for all consumers i, we have

$$u_i(x_i^*) = u_i(x_i^{**}).$$

Since $x^* \neq x^{**}$, there must be at least one consumer i such that $x_i^* \neq x_i^{**}$. Consider the allocation

$$\widetilde{x} \equiv \frac{x^* + x^{**}}{2}.$$

It follows from the fact that x^* and x^{**} are feasible that $\widetilde{x}$ is feasible. Also strict quasi-concavity implies that

$$\begin{aligned} u_i(\widetilde{x}_i) &> u_i(x_i^*) \quad \text{and} \\ u_h(\widetilde{x}_h) &\geq u_h(x_h^*) \quad \text{for all } h \end{aligned}$$

contradicting the fact that x^* is Pareto optimal.

3. (30 points)

The following economy has 2 consumers, 2 states of nature, and one commodity per state. The probability of state 1 is $\frac{1}{3}$, and the probability of state 2 is $\frac{2}{3}$. The endowment vector of consumer 1 is $\omega_1 = (\omega_1(1), \omega_1(2)) = (3, 1)$, and the endowment vector of consumer 2 is $\omega_2 = (\omega_2(1), \omega_2(2)) = (1, 3)$. For $i = 1, 2$, consumer i is a von Neumann-Morgenstern expected utility maximizer, with strictly monotonic, differentiable, and strictly concave Bernoulli utility function $u_i(x_i)$. In other words, consumer i's expected utility function is

$$\frac{1}{3}u_i(x_i(1)) + \frac{2}{3}u_i(x_i(2)).$$

Before observing the state, the two consumers trade two securities. Security 1 pays 2 units of account in state 1 and 1 unit of account in state 2. Security 2 pays 1 unit of account in state 1 and 2 units of account in state 2. After the securities market, the state of nature is observed, securities are redeemed, and consumption is traded on a spot market. Denote the security holdings as b_i^1 and b_i^2 and denote security prices as q^1 and q^2.

(a) (15 points) Define a competitive equilibrium for this economy.

(b) (15 points) Calculate the competitive equilibrium consumptions, $x_i(s)$ for $i = 1, 2$ and $s = 1, 2$. You do not have to compute the entire equilibrium. HINT: This market structure is equivalent to complete markets, and there is no aggregate uncertainty.

Answer:

(a) A C.E. is a vector of securities prices and spot prices, $(q^1, q^2, p(1), p(2))$, securities holdings, $(b_1^1, b_1^2, b_2^1, b_2^2)$, and consumptions, $(x_1(1), x_1(2), x_2(1), x_2(2))$, such that

(i) consumer 1's securities holdings and consumption solve (equalities because of strict monotonicity)

$$\begin{aligned} & \max \frac{1}{3}u_1(x_1(1)) + \frac{2}{3}u_1(x_1(2)) \\ & \text{subject to} \\ q^1 b_1^1 + q^2 b_1^2 &= 0 \\ p(1)x_1(1) &= 3p(1) + 2b_1^1 + b_1^2 \\ p(2)x_1(2) &= p(1) + b_1^1 + 2b_1^2 \\ x_1 &\geq 0, \end{aligned}$$

(ii) consumer 2's securities holdings and consumption solve (equalities because of strict monotonicity)

$$\begin{aligned}
& \max \frac{1}{3}u_2(x_2(1)) + \frac{2}{3}u_2(x_2(2)) \\
& \text{subject to} \\
q^1 b_2^1 + q^2 b_2^2 &= 0 \\
p(1)x_2(1) &= p(1) + 2b_2^1 + b_2^2 \\
p(2)x_2(2) &= 3p(1) + b_2^1 + 2b_2^2 \\
x_2 &\geq 0,
\end{aligned}$$

(iii) markets clear:

$$\begin{aligned}
b_1^1 + b_2^1 &= 0 \\
b_1^2 + b_2^2 &= 0 \\
x_1(1) + x_2(1) &= 4 \\
x_1(2) + x_2(2) &= 4.
\end{aligned}$$

(b) The short cut to solving for the consumptions is to use the hint that the market structure is equivalent to complete markets, so that the consumption will be the same as under the contingent commodity market structure. Also, since there is no aggregate uncertainty, the functional form of the utility function does not matter; each consumer will consume the same amount in the two states of nature and the relative price ratio equals the probability ratio, $\frac{1}{2}$. That is, there is complete insurance at fair odds, so consumption in each state equals the expected value of endowment consumption. Thus, we have

$$\begin{aligned}
x_1(1) &= x_1(2) = \frac{1}{3}(3) + \frac{2}{3}(1) = \frac{5}{3}, \\
x_2(1) &= x_2(2) = \frac{1}{3}(1) + \frac{2}{3}(3) = \frac{7}{3}.
\end{aligned}$$

4. (20 points)

Consider the standard Rothschild-Stiglitz competitive screeening model, with a proportion λ of high risk drivers with accident probability p^H, and a proportion $1-\lambda$ of low risk drivers with accident probability p^L. Consumers have the same concave Bernoulli utility function and seek to maximize expected utility. Now suppose that there is a government or social planner **whose sole objective is to maximize the expected utility of the low risk drivers**. The planner can offer whatever contracts it wants, and can prevent other firms from offering contracts. However, the planner cannot observe a driver's type, and is constrained so that any contract it offers makes non-negative expected profits.

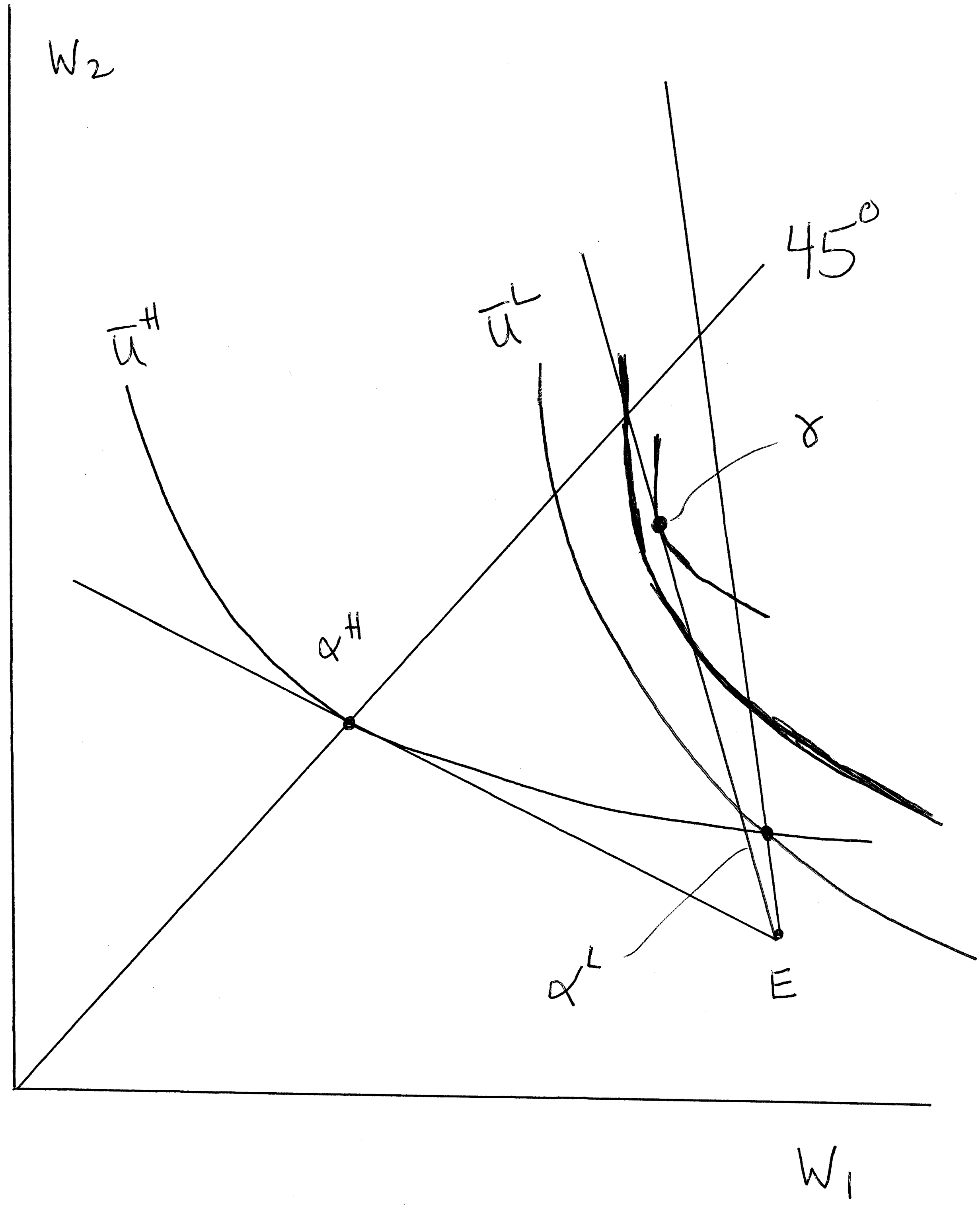
Using a carefully drawn, labeled, and neat diagram, show that when λ is sufficiently small, the planner will offer a single (pooling) contract, and indicate the optimal pooling contract on the diagram. Explain whether or not such an optimal pooling contract is on the 45 degree line. You will probably want your diagram to take up almost the full page so that I will be able to read it.

Answer:

See the diagram on the next page. The choice will be between the candidate separating equilibrium and the best pooling contract for the low risk drivers. If λ is small enough so that the pooled fair-odds line intersects low-risk indifference curve through their contract in the candidate separating equilibrium, α^L, then a pooling contract is superior. The optimal pooling contract will be on the pooled fair-odds line and tangent to an indifference curve for the low-risk drivers. This contract will be below the 45 degree line. To see this, notice that on the 45 degree line, the low-risk indifference curve has slope

$$-\frac{1-p^L}{p^L},$$

which is steeper than the slope of the pooled fair odds line. Therefore, the tangency point is below the 45 degree line.



W2
W1
45°
$\bar{u}^H$
$\bar{u}^L$
α^H
α^L
γ
E