Monopoly

Reasons for monopoly: (1) patent, government protection, (2) ownership of a key input, (3) natural monopoly, because LRAC falls until it crosses the market demand curve.

$$\max \pi = p(y)y - C(y)$$
yields the first order condition

$$p(y) + yp'(y) - C'(y) = 0.$$  

The first two terms comprise marginal revenue. When a monopolist produces more output, it takes into account the fact that the price it can charge is lower.

We can think of the monopolist as choosing $p$ or as choosing $y$.

second order conditions:

$$p''(y)y + 2p'(y) - C''(y) < 0$$
Monopoly

The monopoly quantity satisfies $MR(y) = C''(y)$. The monopolist charges the price the market will pay for that quantity.
Comparative Statics with constant Marginal Costs, $c$

F.o.c. \[ p'(y) y + p(y) - c = 0 \]

\( \text{effect of } c \text{ on } y \text{ and } p : \)

1. \[ p'(y) \frac{dy}{dc} + y p''(y) \frac{dy}{dc} + p'(y) \frac{dy}{dc} - 1 = 0 \]

\[ \frac{dy}{dc} \left[ y p''(y) + 2p'(y) \right] = 1 \]

\( \quad \text{negative from F.o.c.} \)

\( \therefore \frac{dy}{dc} < 0 \)

2. \[ \frac{dp}{dc} = \frac{dp}{dy} \frac{dy}{dc} \quad \therefore \frac{dp}{dc} > 0 \]

\( \text{neg. slope of demand} \)
Price Discrimination

First Degree: Charge everyone their marginal willingness to pay for each unit.

Consumer $i$ receives a take it or leave it offer of $x_i$ units at total payment of $T_i$. To extract all surplus, set

$$T_i = u_i(x_i).$$

Therefore,

$$\max_{x_1, \ldots, x_n} \sum_{i=1}^{n} u_i(x_i) - C(\sum_{i=1}^{n} x_i)$$

The first order condition for $x_i$ is

$$u'_i(x_i) = C'(\sum_{i=1}^{n} x_i)$$

which equates marginal social benefit to marginal social cost, with no deadweight loss.
Problem: resale. What if a group of high-utility consumers refuses to purchase and buys from the low-utility consumers instead?

example of f.d.p.d.: college tuition and financial aid.
Second Degree: nonlinear pricing applied equally to everyone.

example: two-part tariffs. Suppose a typical cell phone customer is willing to pay $2 for each of 10 "essential" calls per month, and has a demand curve for nonessential calls given by \( D(p) = 50(1 - p) \). If the cost of providing a call is \( c \), the monopolist should induce the efficient quantity by charging \( p = c \), and extract the surplus from both types of calls with a monthly fixed fee of \( 25(1 - c)^2 + 20 - 10c \).

Note that with one type of consumer this is essentially a first degree price discrimination problem. With two types, the optimal plan will extract all the surplus from the lowest valuation consumers, and leave the highest valuation consumers with some surplus. This would be a monopoly screening problem, which you will see next quarter.
Third Degree: charge different consumers different constant prices.

examples: student discounts, domestic and foreign markets, coupons

Letting the inverse demand curves in the two markets be $p_1(y_1)$ and $p_2(y_2)$, the monopolist solves

$$\max_{y_1, y_2} p_1(y_1)y_1 + p_2(y_2)y_2 - C(y_1 + y_2)$$

First order conditions:

$$MR_1 = C''(y_1 + y_2) = MR_2$$