Oligopoly

**Cournot Oligopoly**

Firm 1 \( \max_{Y_1} \Pi_1(Y_1, Y_2) = p(Y)Y_1 - c_1(Y_1) \)

Firm 2 \( \max_{Y_2} \Pi_2(Y_1, Y_2) = p(Y)Y_2 - c_2(Y_2) \)

where \( Y_1 + Y_2 = Y \)

\[ \text{F.A.C.} \quad p(Y) + p'(Y)Y_i = c_i'(Y_i) \quad i = 1, 2 \]

reaction function is determined implicitly \( Y_1 = f_1(Y_2) \)

**EX.** \( X(p) = \frac{80}{p} \)

for output \( Y \) to be demanded, \( p(Y) = \frac{80}{Y} \)

\( c_1'(Y_1) = 20 = c_2'(Y_2) \)
\( f.o.c. \quad 80 - Y_2 - 2Y_1 = 0 \)

\[
Y_1 = 30 - \frac{Y_2}{2} \quad \text{and} \quad Y_2 = 30 - \frac{Y_1}{2}
\]

\( Y_1 = Y_2 = 20 \)

\( Y = 40 \quad \text{and} \quad P = 40 \)

\[
\begin{align*}
\text{Competitive:} & \quad p = MC \\
\text{Monopoly:} & \quad MR = MC \\
\text{At} & \quad p = 20, Y = 60 \quad \text{and} \quad p = 50, Y = 30 \\
& \quad \pi_1 = \pi_2 = 400
\end{align*}
\]
More generally, if there are $m$ firms, then $c_i'(y_i) = c$ for all $i$.

For any $y$, $p(y) + p'(y) \frac{y}{m} = c$ by symmetry.

$p(y) \left[ 1 + \frac{p'(y) y}{m p(y)} \right] = c$

$p(y) \left[ 1 + \frac{1}{m \zeta} \right] = c$ where $\zeta = \frac{dY}{dp} \frac{p}{Y}$

Assume demand elasticity is bounded above zero.

$\lim_{n \to \infty} p(y) = c$

**Diagram:**

- Linear demand:
- $E$ bounded above zero
- Zero elasticity

In general, as long as demand curve is never vertical.
**Bertrand Model**

firms choose $p_1, p_2$

low-priced firm serves the entire market at that price

\[ d_1(p_1, p_2) = \begin{cases} D(p_1) & \text{if } p_1 < p_2 \\ \frac{D(p_1)}{2} & \text{if } p_1 = p_2 \\ 0 & \text{if } p_1 > p_2 \end{cases} \]

\[ \Pi_i(p_1, p_2) = p_i d_i(p_1, p_2) - c_i d_i(p_1, p_2) \]

suppose $c_2 \geq c_1$ w.l.o.g.

\underline{Nash Equilibrium}

suppose $p_1 > c_2$, Firm 2 could undercut firm 1's price and increase profits if $p_2 < p_1$; Firm 1 could choose...
Nash equil. \( c_1 = c_2 = c \)

\( p_1 = p_2 = c \)

Cutting price loses money; raising price does not help.

This is the competitive equilibrium.

This is the only Nash equil.:

Some:

\( p_i < c \): one or both firms lose money.

\( p_1 \gtrapprox c, \ p_2 \gtrapprox c, \ p_1 \neq p_2 \): high-price should undercut.

\( p_1 > c, \ p_2 = p_1 \): either firm should undercut.

w.l.o.g.: \( p_2 = c, \ p_1 > c \): firm 2 should raise \( p_2 \).
Nash equil? $c_2 > c_1$

$p_1 = c_2$

$p_2 > p_1$

Firm 1 serves the market at firm 2's marginal cost

If $p_2 = p_1$, then firm 1 is only getting half the market. Profits are higher by slightly undercutting.

If $p_2 > p_1$ (and $p_2$ is below the monopoly price) then firm 1 can raise the price, while staying below $p_2$.

What we want is $p_1 = c_2$, $p_2 = c_2 + 1$
Variations on Bertrand:

1. finite grid (pennies)
2. U-shaped cost curves
3. specify price and maximum quantity
4. simultaneously produce \( Y_i \) and name price \( P_i \)
5. heterogeneous products
6. uncertainty

→ Stackelberg model

Firm 1 is the quantity "leader"; firm 2 is the "follower".

\[
\begin{align*}
X(p) &= 80 - p \\
p(Q) &= 80 - Q \\
Q_1, Q_2 &\text{ chosen} \\
C'(Q_1) &= C'(Q_2) = 20
\end{align*}
\]
f.o.c. for firm 2 (reacting to $q_1$):

$$q_2 = 30 - \frac{q_1}{2}$$

$$\Pi_1 (q_1, q_2(q_1)) =$$

$$\left[ 80 - q_1 - \left(30 - \frac{q_1}{2}\right) \right] q_1 - 20 q_1$$

$$\left[ 50 - \frac{q_1}{2} q_1 \right] q_1 - 20 q_1$$

$$\max_{q_1} \Pi_1 : \quad 50 - \frac{q_1}{2} = 20$$

$$q_1 = 30, \quad q_2 = 15$$

$$p = 35$$

$$\Pi_1 = 450 \quad \Pi_2 = 225$$

Notice: (30, 15) is not a N.E. — firm 1 would choose a different output if the play is truly simultaneous.

Firm 2 must actually observe $q_1$

Revised Oligopoly