Oligopoly

Cournot Oligopoly

Firm 1 $\quad \max_{Y_1} \quad \Pi_1(Y_1, Y_2) = p(Y)Y_1 - c_1(Y_1)$

Firm 2 $\quad \max_{Y_2} \quad \Pi_2(Y_1, Y_2) = p(Y)Y_2 - c_2(Y_2)$

where $Y_1 + Y_2 = Y$

F.O.C. $\quad p(Y) + p'(Y)Y_i = c_i'(Y_i) \quad i = 1, 2$

reaction function is determined implicitly

$Y_1 = f_1(Y_2)$

ex. $\quad X(p) = 80 - p$

for output Y to be demanded, $p(Y) = 80 - Y$

$c_1'(Y_1) = 20 = c_2'(Y_2)$

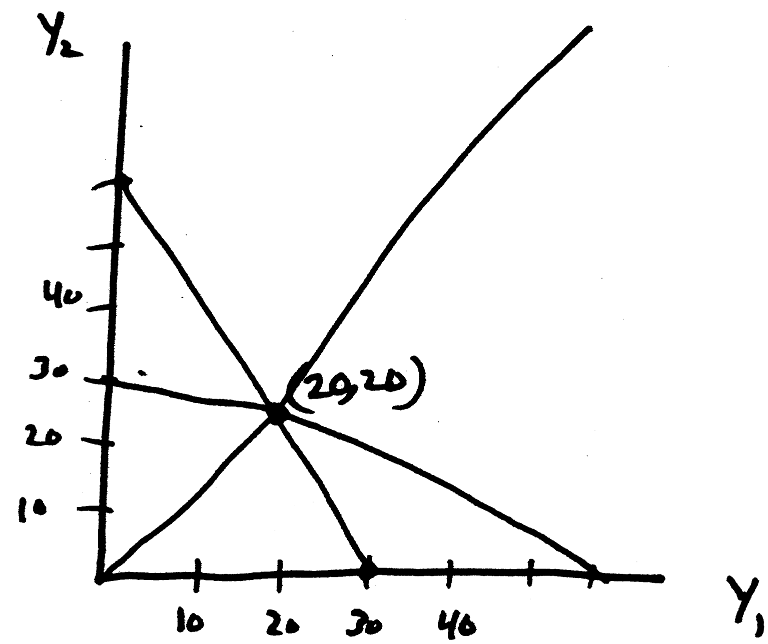
$$p(Y) = 80 - Y$$

f.o.c. $80 - Y_2 - 2Y_1 = 20$

$Y_1 = 30 - \frac{Y_2}{2}$ $Y_2 = 30 - \frac{Y_1}{2}$

$Y_1 = Y_2 = 20$

$Y = 40$
$p = 40$



Competitive : $p = MC$ $p = 20$, $Y = 60$
Monopoly : $MR = MC$ $p = 50$ $Y = 30$ $\pi^m/2 = 450$

$$\pi_1 = \pi_2 = 400$$

More Generally

m firms, $c_i'(Y_i) = c$ for all i

f.o.c. $P(Y) + P'(Y)\frac{Y}{m} = c$ by symmetry

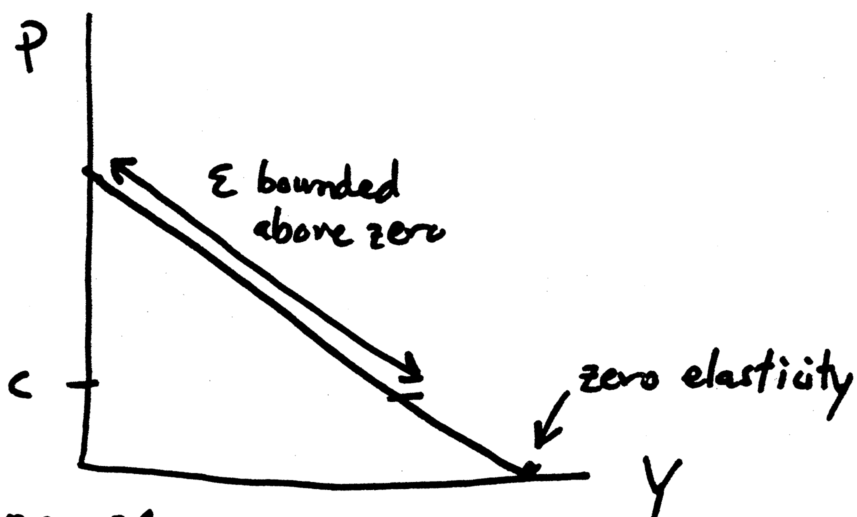
$$P(Y)\left[1 + \frac{P'(Y)\,Y}{m\,P(Y)}\right] = c$$

$$P(Y)\left[1 + \frac{1}{m\varepsilon}\right] = c \qquad \text{where } \varepsilon \equiv \frac{dY}{dP}\frac{P}{Y}$$

assume demand elasticity is bounded above zero

$$\lim_{m \to \infty} P(Y) = c$$

linear demand:



In general, as long as demand curve is never vertical

Bertrand Model

firms choose p_1, p_2
low-priced firm serves the entire market at that pri

$$d_1(p_1,p_2) = \begin{cases} D(p_1) & \text{if } p_1 < p_2 \\ D(p_1)/2 & \text{if } p_1 = p_2 \\ 0 & \text{if } p_1 > p_2 \end{cases}$$

$$\pi_i(p_1,p_2) = p_i\, d_i(p_1,p_2) - c_i\, d_i(p_1,p_2)$$

suppose $c_2 \geq c_1$ w.o.l.o.g.

~~Nash equilibrium~~

~~suppose $p_1 > c_2$. Firm 2 could undercut firm 1's price and increase profits if $p_2 \geq p_1$; firm 1 could choose~~

Nash equil. $c_1 = c_2 = c$

$p_1 = p_2 = c$

Cutting price loses money ; raising price does not help

This is the competitive equilibrium

This is the only Nash equil. :

some $p_i < c$: one or both firms lose money

$p_1 \geqslant c$, $p_2 \geqslant c$, $p_1 \neq p_2$: high-price should undercut

$p_1 > c$, $p_2 = p_1$: either firm should undercut

w.o.l.o.g. $p_2 = c$, $p_1 > c$: firm 2 should raise p_2

Nash equil ? $c_2 > c_1$

$p_1 = c_2$

$p_2 > p_1$

Firm 1 serves the market at firm 2's marginal cost

If $p_2 = p_1$, then firm 1 is only getting half the market. Profits are higher by slightly undercutting.

If $p_2 > p_1$ (and c_2 is below the monopoly pri[ce]) then firm 1 can raise the price, while staying below p_2.

$\boxed{\text{What we want is} \quad p_1 = c_2, \quad p_2 \le c_2 + 1¢}$

Variations on Bertrand:

1. finite grid (pennies)
2. U-shaped cost curves
3. specify price and maximum quantity
4. simultaneously produce Y_i and name price P_i
5. heterogeneous products
6. uncertainty

→ **Stackelberg model**

Firm 1 is the quantity "leader"; firm 2 is the "follower".

ex. $X(p) = 80 - p$ $\quad\quad$ $p(Q) = 80 - Q$

q_1, q_2 chosen $\quad\quad$ $c'(q_1) = c'(q_2) = 20$

f.o.c. for firm 2 (reacting to q_1):

$$q_2 = 30 - \frac{q_1}{2}$$

$$\Pi_1(q_1, q_2(q_1)) =$$

$$\left[80 - q_1 - \left(30 - \frac{q_1}{2}\right)\right] q_1 - 20 q_1$$

$$\left[50 - \frac{1}{2} q_1\right] q_1 - 20 q_1$$

$\max\limits_{q_1} \Pi_1$: $50 - q_1 = 20$

$q_1 = 30$, $q_2 = 15$ $p = 35$

$\Pi_1 = 450$ $\Pi_2 = 225$

Notice: $(30, 15)$ is not a N.E. to the game with simultaneous play — firm 1 would choose a different output if the play is truly simultaneous.

$\boxed{\text{Repeated Oligopoly}}$

Firm 2 must actually observe q_1