The Ohio State University
Department of Economics
Econ 808–Problem Set #1
Due Thursday, April 11

Spring 2002
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(1) In the following economy, there are two consumers, two firms, and two goods (labor/leisure and food). For i = 1,2, consumer i is endowed with zero units of food and 1 unit of leisure, $\omega_i = (0, 1)$. Letting x_i denote consumer i's consumption of food and ℓ_i denote consumer i's consumption of leisure, the utility function is: $\log(x_i) + \log(\ell_i)$.

Let y_1 denote firm 1's output of food and L_1 denote firm 1's labor input (so that L_1 must be nonnegative). Then firm 1's production function, the frontier of its production set, is given by: $y_1 = AL_1$, where the parameter A is a positive real number. Firm 1 is owned by consumer 1.

Let y_2 denote firm 2's output of food and L_2 denote firm 2's labor input (so that L_2 must be nonnegative). Then firm 2's production function, the frontier of its production set, is given by: $y_2 = (L_2)^{1/2}$. Firm 2 is owned by consumer 2.

(a) Define a competitive equilibrium for this economy.

(b) Calculate the competitive equilibrium price vector and allocation, as a function of the parameter, A. Assume that we have an interior solution, where both firms produce output.

(c) For what values of the parameter, A, will we have a corner solution, where one of the firms produces zero output?

(2) In the following economy, there are 2 consumers, one good, x, and 2 states of nature, a and b. The probabilities of the states are A and B, where A + B = 1. The utility functions and endowments of the consumers are given by

$$\begin{aligned} V_1 &= A\log(x_1^a) + B\log(x_1^b) \\ (\omega_1^a, \omega_1^b) &= (2, 0) \\ V_2 &= Ax_2^a + Bx_2^b \\ (\omega_2^a, \omega_2^b) &= (1, 1) \end{aligned}$$

(a) Draw an Edgeworth box diagram, indicating the endowment point and the contract curve.

(b) Calculate the C.E. of this economy, as a function of the parameters, A and B. Be careful to remember the constraint that consumption must be nonnegative.

(c) Interpret the C.E. as an insurance market. In other words, who is offering the insurance, what is the amount of the premium, what is the amount of the claim in the event of an "accident," and how does the premium compare with the expected value of the claim?

(3) Consider the following exchange economy with two von Neumann-Morgenstern expected utility maximizers, two states of nature, $s = \alpha$ and $s = \beta$, and one commodity per state. We have

$$V_i = \sum_{s=\alpha,\beta} \pi_s u_i(x_i^s),$$

where V_i is consumer i's utility function, π_s is the probability of state s, and x_i^s is the consumption of consumer i in state s. Assume that each u_i is twice continuously differentiable, and that we have $u_i' > 0$ and $u_i'' < 0$. Also assume that the initial endowments, ω_i^s, are strictly positive for each consumer in each state.

(a) If the following statement is true, then carefully argue why, and if it is false, then present a counterexample: If the initial endowments of state-contingent commodities are Pareto optimal for $\pi_\alpha = \pi_\beta = \frac{1}{2}$, then the endowments are Pareto optimal for any other specification of the probabilities.

(b) Define a competitive equilibrium for this economy.

(c) Does this economy satisfy the assumptions required to apply the Second Welfare Theorem?