The Ohio State University
Department of Economics
Econ 808–Problem Set #2
Due Tuesday, April 23

Spring 2002
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1. Consider the following economy with one physical commodity per state of nature and three consumers, each of whom seek to maximize expected utility. For i = 1,2, consumer i is risk averse, with utility of certain consumption given by $u_i(x_i) = \log(x_i)$. For i = 1,2, consumer i is endowed with 1 unit of consumption when she does not have an accident, 0 units of consumption when she has an accident.

Consumer 1 is a "low risk" consumer, with a probability of an accident equal to 1/3. Consumer 2 is a "high risk" consumer, with a probability of an accident equal to $\frac{1}{2}$. Consumer 1 having an accident and consumer 2 having an accident are independent events.

Consumer 3 is risk neutral, with utility of certain consumption given by $u_3(x_3) = x_3$, and has an endowment of 2 units of consumption in all states of nature. For parts (i) and (ii), assume that consumer 3 knows that consumer 1 is low risk and that consumer 2 is high risk, so information is symmetric.

(i) Define a competitive equilibrium for the economy with complete state-contingent commodity markets. Specify how many states of nature there are and the probability of each state.

(ii) Calculate the competitive equilibrium price vector and allocation for the economy with complete state-contingent commodity markets.

For parts (iii) and (iv), suppose that consumers 1 and 2 are in a Rothschild-Stiglitz world. That is, instead of consumer 3, there are many risk-neutral firms who cannot observe which consumer is low risk and which consumer is high risk. Firms compete by offering contracts, specifying consumption a policyholder receives when she has an accident and consumption she receives when she does not have an accident.

(iii) Find the pooling contract providing full insurance (consumption is independent of whether the policyholder has an accident) and yielding zero expected profits when both consumers accept the contract.

(iv) Show that the pooling contract of part (iii), call it α, is not an equilibrium. That is, find another contract, β, that would be chosen by the low risk consumer, would not be chosen by the high risk consumer, and yields expected profits for the firm offering the contract. Be as explicit as you can.

2. Consider the following version of Spence's signaling model. There are two types of workers, where type 1 workers have productivity A, and type 2 workers have productivity 2A. Type 1 workers make up a fraction q_1 of the population. There are two possible signals that workers can choose from, which are costly but do not enhance productivity.

A worker's cost of choosing y>0 units of signal Y is y for type 1 and y/2 for type 2. That is, we have $c_1(y) = y$ and $c_2(y) = y/2$. A worker's cost of choosing z>0 units of signal Z is z^2 (z squared) for type 1 and z for type 2. That is, we have $c_1(z) = z^2$ and $c_2(z) = z$.

As in the Spence model, firms are risk neutral and workers' utility is their wage minus their signal cost. The timing is that workers first choose their signal, followed by firms choosing a wage schedule.

(a) Assume that Y and Z are mutually exclusive activities, so that workers must choose either activity Y or activity Z, but not both. As a function of the nonnegative parameter, A, find the best separating equilibrium for the type 2 workers. Specify the full equilibrium, including "beliefs," as carefully as you can.

(b) Show whether or not there would be an even better separating equilibrium if workers could choose both activities. [where $c_1(y, z) = y + z^2$ *and* $c_2(y, z) = \frac{y}{2} + z$.*]*

3. In the Ackerlof lemons model with asymmetric information, suppose that the 3/2 in the utility function of type 2 consumers is replaced with 5/2. Solve for the equilibrium price, and determine whether the equilibrium is efficient.