

The Ohio State University
 Department of Economics
 Econ 808–Problem Set #3
 due Thursday, May 2

Spring 2002
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Questions 1 and 2 relate to the principal-agent problem with hidden action presented in class and taken from Mas-Colell chapter 14B. Suppose the parameters are as follows:

$$\bar{u} = 0, \pi \in [0,1],$$

$$f(\pi|e_H) = 1 \text{ for all } \pi, f(\pi|e_L) = 2(1 - \pi),$$

There are two possible effort choices, $e = e_H$ or $e = e_L$, where $e_H > e_L$. The project yields a random payoff, which depends on the effort choice, e , given by the density function $f(\pi|e)$ over the support $[0,1]$.

1. Consider the **first-best** contract, in which effort is observable. Suppose the Bernoulli utility function is given by $v(w,e) = \log(w-g(e))$, where $g(e_L) = 1/4$ and $g(e_H) = 1/3$.

The contract specifies an effort level, $e = e_H$ or $e = e_L$, and a wage profile $w(\pi)$ to solve:

$$\max_{w(\pi)} \int_0^1 (\pi - w(\pi)) f(\pi|e) d\pi$$

subject to: $\int_0^1 \log(w(\pi) - g(e)) f(\pi|e) d\pi = 0.$

(a) Derive the first order conditions that characterize the optimal wage profile for effort level e . Hint: keep the density expressed as $f(\pi|e)$.

(b) Use the first-order-conditions to calculate the optimal wage profile for effort level e_L and for effort level e_H .

(c) Calculate the values of the principal's objective function for the optimal wage profiles for the two effort levels. What effort level will be specified in the (first-best) optimal contract with observable effort? (If the objective is negative, find the contract that yielding the least negative payoff. Profits could include a sunk cost that cannot be recovered by abandoning the project.)

2. Now suppose that the principal **cannot** observe effort. Suppose the Bernoulli utility function is given by $v(w,e) = \log(w) - g(e)$, where $g(e_L)=0$ and $g(e_H)=G$.

(a) If the principal wants to implement high effort, briefly explain whether or not the wage profile from the full-information first-best contract will be incentive compatible.

(b) Show that, with hidden effort, the optimal contract that implements e_H is of the form: $w(\pi) = a + b \pi$, where $b > 0$. In other words, the optimal compensation is increasing and linear in profits.

(c) Specify the two equations that must be solved to find the optimal values of a and b from question 2b. Do not try to solve them. Assuming you knew the optimal values of a and b , how would you determine whether the overall optimal contract is the one that implements e_L or the one that implements e_H ?

3. In the principal-agent problem with hidden action presented in class and taken from Mas-Colell chapter 14B, suppose that the distribution of profits, conditional on effort, is deterministic. That is, suppose that whenever the agent chooses effort e_L , profits are π_L , and that whenever the agent chooses effort e_H , profits are π_H , where $\pi_L < \pi_H$. Show that the optimal contract yields the same effort choice and payoffs to the principal and agent as the optimal contract with observable actions, even when the agent is risk averse.