

The Ohio State University
Department of Economics
Econ 808–Problem Set #3 Answers

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1. (a) Differentiating the Lagrangean expression with respect to $w(\pi)$, NOT with respect to π , we have:

$$-f(\pi | e) + \gamma \frac{f(\pi | e)}{w(\pi) - g(e)} = 0.$$

Simplifying, we have

$$\gamma = w(\pi) - g(e) \text{ for all } \pi.$$

That is, the wage is a constant function of output. The other first-order condition is the individual rationality constraint, holding as an equality.

(b) Letting w_e^* denote the (constant) wage corresponding to effort e , the individual rationality constraint simplifies to

$$\begin{aligned} \log(w_e^* - g(e)) &= 0, \text{ or} \\ w_e^* - g(e) &= 1. \end{aligned}$$

Thus, we have $w_L^* = \frac{1}{4} + 1 = \frac{5}{4}$, and $w_H^* = \frac{1}{3} + 1 = \frac{4}{3}$.

(c) Since wages are independent of π , the principal's objective can be simplified to

$$\int_0^1 \pi f(\pi | e) d\pi - w_e^*. \quad (1)$$

If high effort is specified, equation (1) becomes

$$\int_0^1 \pi d\pi - \frac{4}{3} = -\frac{5}{6}.$$

If low effort is specified, equation (1) becomes

$$\int_0^1 \pi 2(1 - \pi) d\pi - \frac{5}{4} = -\frac{11}{12}.$$

Since the principal's objective is higher (less negative) with high effort than low effort, the optimal contract is to specify high effort and a wage offer of $\frac{4}{3}$.

2. (a) The full-information first-best contract specifies a wage profile that does not depend on π , in order to provide full insurance. Therefore, it is not

incentive compatible to provide high effort, because the agent would be incurring a higher disutility of effort without affecting the distribution of wages.

(b) Let us solve for the optimal contract that implements high effort when effort is hidden. Using the log specification, and letting the multipliers on the individual rationality and incentive compatibility constraints be γ and μ , the first-order condition is

$$w(\pi) = \gamma + \mu \left(1 - \frac{2(1-\pi)}{1} \right) = \gamma + \mu(2\pi - 1).$$

This is a linear contract of the form $w(\pi) = a + b\pi$, where we have $a = \gamma - \mu$ and $b = 2\mu$. Since the incentive compatibility constraint binds, $\mu > 0$ holds, so higher profits yield higher wages.

(c) To find the parameters a and b , use the equations for the two constraints. The binding individual rationality constraint is

$$\int_0^1 \log(a + b\pi) d\pi = G. \quad (2)$$

The binding incentive compatibility constraint says that the agent's utility from choosing high effort (which is zero according to individual rationality constraint (2)) must equal the agent's utility from choosing low effort. Therefore, incentive compatibility can be written as

$$\int_0^1 \log(a + b\pi) 2(1 - \pi) d\pi = 0. \quad (3)$$

To determine whether to implement high effort or low effort, compare the principal's payoff in both cases. If high effort is implemented, the principal's payoff is

$$\int_0^1 (\pi - a - b\pi) d\pi = \frac{1-b}{2} - a.$$

If low effort is implemented, the wage profile is constant, determined by individual rationality:

$$\begin{aligned} \log(w) - g(e_L) &= \bar{u}, \text{ or} \\ w &= 1. \end{aligned}$$

Therefore, the principal's payoff would be

$$\int_0^1 \pi 2(1 - \pi) d\pi - 1 = -\frac{2}{3}.$$

High effort is preferred if and only if we have

$$\frac{1-b}{2} - a > -\frac{2}{3}.$$

3. Suppose the optimal contract with observable effort is (e^*, w^*) , where e^* is either e_L or e_H . The effort, e^* , always yields profit π^* (high profit if effort is high, or low profit if effort is low). In other words, since π is deterministic, we can infer the effort that was chosen.

When effort is not directly observable, simply set the following wage profile:

$$\begin{aligned}w(\pi) &= w^* && \text{if } \pi = \pi^* \\w(\pi) &= -\infty && \text{if } \pi \neq \pi^*.\end{aligned}$$

Clearly, the agent will accept the contract and choose the effort, e^* . Wage payments, principal's payoff, and agent's payoff are at the first-best levels. The punishment for profits other than π^* does not have to be so strong, as long as the agent does not want to choose e_L when $e^* = e_H$.