When and Why not to Auction*

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Abstract

Standard auctions are known to be a revenue-maximizing way to sell an object under broad conditions when buyers are symmetric and have independent private valuations. We show that when buyers have interdependent valuations, auctions may lose their advantage, even if symmetry and independence of information are maintained. In particular, selling mechanisms that sometimes allow a buyer who does not have the highest valuation to win the object will in general increase all buyers' willingness to pay, possibly enough to offset the loss to the seller of not always selling to the buyer with the greatest willingness to pay.

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1. INTRODUCTION

Auctions hold a grand place among institutions of exchange. The universal appeal of auctions has its source in their transparency to the participants, their competitive benefits for the seller (if a sales auction) or buyer (if a procurement auction), and their allocative efficiency. In addition to having a long tradition of use and associated interest, auctions have received a recent surge of attention from government, industry, and academia, due in part to the progress of wireless technology and expanded demand, from many sources, for spectrum rights. Current debate on spectrum sales centers mostly on how best to design an auction that takes account of specific features of that market; that an auction ought to be employed at all is taken as virtually self-evident.

Theoretic justification for auctions is strong. A signature result of auction theory is that certain standard, recognizable auction rules, when formalized as games, can in fact be the revenue-maximizing way to sell an object even when a fully general and complex set of potential selling rules is available. The environment for which this result holds, viz., buyers are ex-ante identical, and have independently determined private valuations the distribution of which satisfies certain regularity conditions, represents the core case of auction theory. In this case, it is optimal for the seller to sell the object to the buyer with the highest valuation conditional on selling at all, an outcome that standard auction formats (first-price, second-price, English, Dutch) deliver as an equilibrium.

The suboptimality of these simple rules outside of the symmetric independent private values benchmark has been demonstrated most explicitly in the case in which buyers' signals (equivalent to valuations in the private values case) are not independent.[1] In two papers, Crémer and McLean [5],[6] show that by exploiting variations in buyers' beliefs about each other's information, something that standard auctions do not achieve, it is possible under some conditions for the seller to extract all expected surplus from buyers. McAfee and Reny [8] extend the results of Crémer and McLean from a finite to an infinite signal space. Although the mechanisms that achieve this surplus extraction are liable to be extremely complex and bear little resemblance to standard auctions or to other familiar selling procedures, they nevertheless maintain the property that it is optimal for the seller to sell to

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[1] The suboptimality of standard auctions when buyers are asymmetric is, in contrast, relatively transparent.

the buyer with the highest valuation, as doing so generates the maximum surplus to be extracted.

This paper investigates the optimality of standard auctions in a third environment, one that maintains the assumption of independent, identically distributed signals of the core model, but allows for interdependence in the buyers' valuations. This case has been studied previously and referred to as an environment of "affiliated values, independent signals." It is of independent interest because it isolates issues arising from the interdependence of buyer valuations from issues arising from the statistical interdependence of buyer information. Milgrom and Weber [9], for instance, derive many results on the revenue properties of auctions in a general affiliated values, affiliated signals model, but many of those results also hold for private but affiliated values, suggesting that the forces at work are purely statistical in nature, much as in the Crémer and McLean papers.

To examine the potential optimality of standard auctions in our setting, we employ the techniques of optimal mechanism design first exposited by Myerson [10] for the independent private values benchmark. Myerson's great contribution was to demonstrate that for independent signals, the equilibrium expected revenue from any selling mechanism can be expressed as the expectation of the virtual valuation of the buyer who wins the object. A buyer's virtual valuation is his true ex-post valuation less an "information rent" term, equal to the product of the marginal effect of his information on his ex-post valuation and the inverse hazard rate of his informational type. For private valuations, it will be the case that the buyer with the highest valuation also has the highest virtual valuation under fairly weak conditions on the hazard rate, because the first term in the virtual valuation is the true valuation. When such an alignment between virtual and true valuations holds, selling to the buyer with the highest virtual valuation, which maximizes expected revenue, is equivalent to selling to the buyer with the highest true valuation, which the standard auction rules achieve.

A formal interpretation of our question is whether the consistency between virtual and true ex-post valuations is as easily satisfied for interdependent valuations as it is for private valuations; whenever there is alignment between virtual and true valuations, standard auctions are optimal. Our answer is negative: for interdependent valuations, conditions ensuring the optimality of auctions are more difficult to satisfy, and hence there is an important array of circumstances under which selling mechanisms other than auctions ought to be considered. We demonstrate this in the body of the paper by showing that a particularly simple selling mechanism, the posted price rule, may outperform the best possible standard auction mechanism when values are interdependent. The posted price we construct is itself not necessarily optimal among all mechanisms, and hence weakly understates the loss of revenue from using the best auction rather than the best unconstrained mechanism.

The intuition for our results relates to the “winner's curse" notion that is typically used to provide insight into equilibrium bidding behavior in auctions where buyers have interdependent valuations. For any strategy that a given buyer might use in a selling game against a fixed profile of strategies for his competitors, it is possible to identify the set of types of other buyers against whom such a strategy will win the object. If valuations are private, this set has no effect on a buyer's willingness to pay for the object. If valuations are interdependent, then a buyer's willingness to pay depends explicitly on this set. For interdependent valuations, the property of auctions that the buyer with the highest signal wins means that buyers' willingnesses to pay are depressed, because any given strategy wins only against a selection of relatively low signals. A mechanism that permits a buyer of a certain signal to win sometimes against buyers with higher signals entails less adverse selection for that buyer, and hence raises the willingness to pay of each type. However, there is a tradeoff, in that for given willingnesses to pay, selling to the buyer with the highest willingness to pay tends to raise more revenue. Our results show that the first effect can sometimes dominate the second, making auctions less favorable than some mechanisms that permit buyers with lower valuations to win.

This paper is closely related to elements of one written by Bulow and Klemperer [3], and while we point out specific differences in the text, we feel it is important to provide an initial summary of the distinction between the papers. Bulow and Klemperer, much as we do, examine the relationship between expected selling price and interdependency of buyer valuations. They do so under a particular specification of asymmetric buyer valuations, which generally does not fall into the symmetric class that we consider. Furthermore, while they study auctions, they do not consider optimal auctions (the distinction lying in the possibility of not selling the object). Showing that even the best auctions perform worse than other mechanisms is a stronger result than the equivalent result for auctions that are not best. Thus, while some of the conclusions of the two papers are similar regarding the potential suboptimality of auctions when valuations are interdependent, the analyses are rather different.

One important implication of using a selling mechanism other than an auction, and hence sometimes selling to a buyer who does not have the maximum valuation, is that there is some efficiency loss. That the seller may value such a possibility demonstrates a tension between revenue and efficiency of a different sort than is found in the symmetric private values case. There, inefficiency can occur in the optimal auction because the seller effectively posts a reserve price, and hence sometimes fails to sell the object. This same incentive exists for interdependent buyer valuations, but the possibility of selling to the wrong (from a social standpoint) buyer represents a novel source of inefficiency in optimal mechanisms for symmetric buyers.

We also offer related results for environments in which a seller has more than one object to sell. Here, a standard auction in which all buyers compete directly with each other can still be used, as in, for instance, a "highest rejected bid" uniform-price auction. We show that for some interdependencies of valuations, the seller may be better off partitioning the buyers into subsets, and holding distinct auctions for different objects within each subset. Analogously to the posted-price mechanism for one object, this has the detrimental effect of reducing overall competition, but the beneficial effect of allowing some buyers to win against other buyers with higher signals, thereby raising buyers' equilibrium bids relative to a single auction in which each buyer competes against all others. Our results help identify conditions under which the latter effect can dominate the former, making segmented auctions desirable for the seller.

2. MODEL

The framework is a standard one, and can be found in, for instance, Bulow and Klemperer [2] for the case of independent signals. N risk-neutral buyers $\{1; 2; \ldots; N\}$ value an indivisible object owned by a seller, whose valuation for the object is normalized to 0. The buyers' valuations for the object depend on N signals $\{t_1; \ldots; t_N\}$. The signals are independent and identically distributed random variables, with signal t_n drawn from a continuous distribution $F(\cdot)$ on $[0; 1]$. Let t denote a typical vector of realized signals and t_{-n} denote the ordered vector of signals excluding t_n. Typical buyer n observes the realization of t_n but not of t_{-n}; the seller observes no signals. Buyer n's valuation of the object is determined by the real-valued function $v(t_n, t_{-n})$, whose range is $[0, 1]$. We assume that the function $v(\cdot, \ldots, \cdot)$ is continuous and weakly increasing in all of its arguments, with $v(0, 0, \ldots, 0) = 0$ and $v(1, 1, \ldots, 1) = 1$. We assume two forms of symmetry for $v(\cdot, \ldots, \cdot)$: the function is the same for all buyers n, and for any t_{-n} and any permutation of it $\bar{t}_{-n}$, $v(t_n, t_{-n}) = v(t_n, \bar{t}_{-n})$ for all n and t_n. We also assume that the buyer with the highest signal has the (weakly) highest ex-post valuation for the object: for any vector of signals $\{t_1, \ldots, t_N\}$, $n^* \in \operatorname{argmax}_n \{t_1, \ldots, t_N\}$ implies $n^* \in \operatorname{argmax}_n v(t_n, t_{-n})$.[2] This environment nests the popular case of symmetric buyers with independent private values.

A selling mechanism is a direct revelation mechanism characterized by an assignment function $\left(\pi_n(\hat{t})\right)_{n=1}^{N}$ and a payment function $\left(y_n(\hat{t})\right)_{n=1}^{N}$. $\pi_n(\hat{t})$ denotes the probability that buyer n is assigned the object as a function of a vector of announced signals $\hat{t}$; hence, we have $\pi_n(\hat{t}) \geq 0$ for all n and $\hat{t}$, and $\sum_n \pi_n(\hat{t}) \leq 1$ for all $\hat{t}$. $y_n(\hat{t})$ is the payment buyer n makes to the seller as a function of the announced signals.

We are interested in the revenue-raising properties of different mechanisms in this environment. To explore this, we review some familiar results that can be found in, for instance, Branco [1] and Bulow and Klemperer [2]. Let $V_n(t_n, \hat{t}_n)$ be the expected payoff to buyer n when his signal is t_n, he reports signal $\hat{t}_n$, and all other buyers report truthfully. Given the allocation and payment functions, we have

$$V_n(t_n, \hat{t}_n) = \int_{T_{-n}} \left(\pi_n(\hat{t}_n, t_{-n}) v(t_n, t_{-n}) - y_n(\hat{t}_n, t_{-n})\right) f(t_{-n}) dt_{-n},$$

where we have abused notation and written $f(t_{-n})$ for $f(t_1) f(t_2) \ldots f(t_{n-1}) f(t_{n+1}) \ldots f(t_N)$. Incentive compatibility implies that for any two signals t_n and t'_n, it is the case that $V_n(t_n, t_n) - V_n(t'_n, t_n) \geq V_n(t_n, t_n) - V_n(t'_n, t'_n) \geq V_n(t_n, t'_n) - V_n(t'_n, t'_n)$, or

$$\text{(IC)} \qquad \begin{aligned} &\int_{T_{-n}} \pi_n(t_n, t_{-n}) \left(v(t_n, t_{-n}) - v(t'_n, t_{-n})\right) f(t_{-n}) dt_{-n} \\ &\geq V_n(t_n, t_n) - V_n(t'_n, t'_n) \\ &\geq \int_{T_{-n}} \pi_n(t'_n, t_{-n}) \left(v(t_n, t_{-n}) - v(t'_n, t_{-n})\right) f(t_{-n}) dt_{-n}. \end{aligned}$$

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[2] Given our assumption of symmetry, this condition therefore implies the single-crossing condition of Maskin [7].

For any signal t, define $V_n^*(t) \equiv V_n(t,t)$ to be type t's expected payoff from revealing his signal truthfully, which will be his equilibrium payoff in an incentive compatible mechanism. Standard arguments establish that $V_n^*(t)$ is weakly increasing in t, so it is differentiable almost everywhere; (IC) therefore establishes that its derivative is necessarily equal to $\int_{T_{-n}} \pi_n(t,t_{-n})\frac{\partial}{\partial t}v(t,t_{-n})f(t_{-n})dt_{-n}$. If we impose the restriction that the allocation function $\pi_n(\cdot)$ must be Riemann integrable[3] in t_n for all n, then by the monotonicity of $v(t,t_{-n})$ in t, the derivative of $V_n^*(t)$ is Riemann integrable and we have

$$V_n^*(t) = V_n^*(0) + \int_0^t \int_{T_{-n}} \pi_n(z,t_{-n})\frac{\partial}{\partial z}v(z,t_{-n})f(t_{-n})dt_{-n}dz.$$

Thus, the expected payment of type t of buyer n equals

$$\int_{T_{-n}} \pi_n(t,t_{-n})v(t,t_{-n})f(t_{-n})dt_{-n} - V_n^*(0) - \int_0^t \int_{T_{-n}} \pi_n(z,t_{-n})\frac{\partial}{\partial z}v(z,t_{-n})f(t_{-n})dt_{-n}dz.$$

Differentiation by parts then establishes that the ex ante expected payment by buyer n is

$$\int_0^1 \int_{T_{-n}} \pi_n(t_n,t_{-n})\left[v(t_n,t_{-n}) - \frac{1-F(t_n)}{f(t_n)}\frac{\partial}{\partial t_n}v(t_n,t_{-n})\right]f(t_{-n})f(t_n)dt_{-n}dt_n - V_n^*(0).$$

The function $v(t_n,t_{-n}) - \frac{1-F(t_n)}{f(t_n)}\frac{\partial}{\partial t_n}v(t_n,t_{-n})$ has been called, variously, the virtual valuation of type t_n of buyer n, and the marginal revenue of type t_n of buyer n (as in, e.g., Bulow and Roberts [4] and Bulow and Klemperer [2]).

Characterization of the seller's expected revenue as the sum of the virtual valuations of the buyers, weighted by the probabilities of the assignment function, has allowed great insight into the problem of solving for the revenue-maximizing feasible selling mechanism. In particular, the seller would always like to assign the good with probability one to the buyer with the highest virtual valuation, conditional on that virtual valuation being nonnegative, if such an assignment function respects incentive compatibility. As referred to in the introduction, a sufficient condition for this to hold is that for any realized profile of signals $(t_1,\ldots,t_N)$, the

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[3] For some specifications of $v(\cdot,\ldots,\cdot)$, incentive compatibility implies that the derivative of $V_n^*(t)$ is increasing in t, in which case Riemann integrability is implied rather than imposed; this is true of the private-values case, for instance.

buyer with the highest realized signal also has the highest virtual valuation, in which case the seller sells to the buyer with the highest signal if she sells at all. This condition amounts to a joint restriction on the valuation function $v(\cdot;\ldots;\cdot)$ and the distribution function $F(\cdot)$.

As we have noted, a reason to focus on the case in which the seller optimally always sells to the buyer with the highest signal when she sells is that such a mechanism can be implemented via any of the standard auction rules (first-price, second-price, English, Dutch), with an appropriately chosen reserve price (starting price in an English auction, stopping price in a Dutch auction). Environments of symmetric buyers in which such a selling rule is suboptimal are often deemphasized as pathological, or at least unlikely. For example, in the private-values case, the virtual valuation of type t_n is $t_n - (1 - F(t_n))/f(t_n)$. If the inverse hazard rate $(1 - F(t_n))/f(t_n)$ is nonincreasing, then the virtual valuation is increasing in t_n, in which case the buyer with the highest valuation also has the highest virtual valuation, and auctions are optimal. Numerous papers in mechanism design impose a nonincreasing inverse hazard rate as an assumption, often noting that it is satisfied for many familiar distributions, such as the uniform and normal.

Our goal in this paper is to demonstrate that the suboptimality of auctions as selling mechanisms is perhaps not so unusual a case as has been regarded previously. In particular, certain features of the valuation function $v(\cdot;\ldots;\cdot)$ may tend to favor selling mechanisms that do not always award the object to the buyer with the highest signal, and hence cannot be implemented with a standard auction. This point is also made in Bulow and Klemperer [3], and we begin our motivation of the forces at work with an example adapted from one in their paper. Suppose that $v(t_n; t_{-n}) = \sum_{i=1}^{N} \frac{t_i}{N}$. This is a case of pure common values (all buyers have the same valuation ex post), with the common value equal to the average of the signals. Buyer n of type t_n has a virtual valuation equal to $\sum_{i=1}^{N} \frac{t_i}{N} - \frac{1}{N}(1 - F(t_n))/f(t_n)$. Note that different buyers' virtual valuations differ only via the inverse hazard rate. Thus, a nonincreasing inverse hazard rate is both sufficient and necessary for the buyer with the highest signal to have the highest virtual valuation in this case, whereas for the private values case it is merely sufficient. In particular, there are distributions for which an auction would be optimal under private values, but may not be optimal in this common-value specification. The reason is clear from the virtual valuation function, which is ex-post valuation less information rent: when buyer n's ex-post valuation is his signal t_n, it broadens the range of distributions for which his virtual valuation is the highest whenever his signal is the highest, relative to the common-value case in which all ex-post valuations are identical. We comment more on the significance of common values later.

While the above example goes some way to demonstrating that the optimality of auctions can become fragile when we depart from the private values case, it may or may not be convincing. Since nonincreasing inverse hazard rates are often assumed as a matter of course in the literature, it might be preferable to show that we need not violate that assumption to discover that auctions can be suboptimal when values are interdependent. We provide an example of this in the next section.

3. A CASE WHEN AUCTIONS ARE NOT BEST

For this section we consider some examples with two buyers; we discuss how they extend naturally to environments of more buyers at the end of the section. We impose some mild extra structure on the function $v(t_n; t_{-n})$: the function $v(x; x)$, defined on signals $x \in [0; 1]$, is assumed to be strictly increasing in x. Under this assumption, it can be specified without further loss of generality that for any two signals t and t', $v(t; t') \in [\min\{t; t'\}; \max\{t; t'\}]$, so that $v(x; x) = x$. We shall later refer to valuation functions satisfying this property as "permissible." This specification simplifies expressions for the seller's revenue under standard auctions.

To explore the possible optimality of standard auctions, we start by identifying the revenue-maximizing mechanism among those with the property that the seller sells to the buyer with the higher signal, or else does not sell at all. We shall call mechanisms satisfying this property "standard." Any standard mechanism can be uniquely characterized by a single signal t^*, defined as the highest type of buyer who wins the object with probability 0. The complete assignment function is: $\pi_1(\hat{t}_1; \hat{t}_2) = 1$ if $\hat{t}_1 > \hat{t}_2$ and $\geq t^*$, 0 else; $\pi_2(\hat{t}_1; \hat{t}_2) = 1$ if $\hat{t}_2 > \hat{t}_1$ and $\geq t^*$, 0 else.[4]

Under the revenue-maximizing standard mechanism, buyer type t^* must earn zero ex-

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[4] The assignment function in the zero-probability event that the buyers have identical signals above t^* can be specified arbitrarily.

pected profit. Thus, type t^*'s expected payment is

$$\bar{y}(t^*) = \int_0^{t^*} v(t^*,t)f(t)dt.$$

To express the expected payment of higher types, it is convenient to apply revenue equivalence, which holds when signals are independent: any standard mechanism can be implemented via a second-price auction with a reserve price, in which case the equilibrium bid by a given buyer of type t who bids is $v(t,t) = t$. Using this fact, the expected payment of type $\hat{t} > t^*$ is

$$\bar{y}(\hat{t}) = \int_{t^*}^{\hat{t}} tf(t)dt + \int_0^{t^*} v(t^*,t)f(t)dt.$$

Announced types below t^* pay zero. The seller's expected revenue from this mechanism is

$$2(1 - F(t^*))\int_0^{t^*} v(t^*,t)f(t)dt + 2\int_{t^*}^1 \int_{t^*}^z tf(t)dt f(z)dz.$$

The optimal standard mechanism is the standard mechanism characterized by the t^* that maximizes this expression.

We now consider a different mechanism that sometimes assigns the object to the buyer with the lower signal. The assignment function for this mechanism is as follows. Continue to define t^* as the cutoff signal in the optimal regular mechanism. If both buyers announce signals below t^*, neither is awarded the object and no payments are made. If one of the buyers announces a signal greater than or equal to t^* and the other a signal lower than t^*, the buyer with the higher announced signal wins the object and pays a price p. If both buyers announce signals greater than or equal to t^*, the buyer chooses one of them via a uniform 50-50 randomization and sells to that buyer at price p. This mechanism can be interpreted as a posted-price rule with rationing: each buyer decides whether he is willing to pay a fixed price p for the object, and the seller simply fills the order. Hereafter we refer to this mechanism as the posted-price mechanism.

The price p can be determined by requiring a zero expected payoff for type t^*, the lowest type of buyer willing to accept the offer. Type t^*'s expected gross return from participating is

$$\int_0^{t^*} v(t^*,t)f(t)dt + \frac{1}{2}\int_{t^*}^1 v(t^*,t)f(t)dt,$$

since the expected payment of type t^* (and every type greater than t^*) is $F(t^*)p + \frac{1-F(t^*)}{2}p$, we have

$$p = \frac{2\int_0^{t^*} v(t^*;t)f(t)dt + \int_{t^*}^1 v(t^*;t)f(t)dt}{1+F(t^*)}:$$

The seller's expected revenue from the posted-price mechanism is

$$2(1-F(t^*))\int_0^{t^*} v(t^*;t)f(t)dt + (1-F(t^*))\int_{t^*}^1 v(t^*;t)f(t)dt:$$

The difference between the seller's expected revenues under these two mechanisms is that revenue from the standard mechanism contains the term

$$2\int_{t^*}^1\int_{t^*}^z tf(t)dtf(z)dz;$$

while the revenue from the posted-price mechanism contains the term

$$(1-F(t^*))\int_{t^*}^1 v(t^*;t)f(t)dt:$$

We can illuminate the significance of these terms by dividing each by $(1-F(t^*))^2$. After this transformation, the former becomes

$$\frac{2\int_{t^*}^1\int_{t^*}^z tf(t)dtf(z)dz}{(1-F(t^*))^2};$$

$$\text{(A)} \qquad = \int_{t^*}^1 t\frac{2(1-F(t))f(t)}{(1-F(t^*))^2}dt$$

which is the expectation of the first[5] (smaller) order statistic of a sample of two from distribution $F(\cdot)$, conditional on its realization being greater than or equal to t^*. This term represents the benefit to the seller of having two buyers compete for the object: to increase his chance of winning the object, a buyer must offer a larger payment to the seller.

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[5] We adopt the convention used in statistics in which the Nth order statistic from a sample of N is the largest realization, and the first is the smallest.

Dividing the term in the posted-price mechanism by $(1-F(t^*))^2$ yields

$$\int_{t^*}^{1} v(t^*;t)\frac{f(t)}{(1-F(t^*))}\,dt; \tag{B}$$

the expectation of $v(t^*;t_{-n})$ conditional on t_{-n} being greater than or equal to t^*. This term represents the benefit to the seller of allowing a buyer to win when he has the lower signal: it is in these instances that a buyer most values winning, and the willingness to pay of a buyer who receives signal t^* goes up relative to the auction accordingly. In particular, the price p that just induces type t^* to participate in the posted-price mechanism is always larger than the reserve price in the standard mechanism when interpreted as an auction.

We now show that the posted-price mechanism can do better than the best mechanism that can be implemented via a standard auction, and examine for what properties of the environment this ranking will hold. To begin, consider the special case in which $v(t_n;t_{-n}) = \max\{t_n;t_{-n}\}$. Now (B) is simply the expectation of a random variable drawn from $F(\cdot)$ conditional on its realization being greater than or equal to t^*. This is clearly larger than the expectation of the lower of two random draws from $F(\cdot)$ conditional on both being greater than or equal to t^*. Thus, we have

Proposition 1: For $v(t_n;t_{-n}) = \max\{t_n;t_{-n}\}$, the posted-price mechanism yields more expected revenue than any standard mechanism for all possible specifications for $F(\cdot)$.

We emphasize that the posted-price mechanism against which we compare the optimal standard mechanism is not itself necessarily optimal, possibly even within the class of posted-price mechanisms. Thus, any difference in expected revenue between the specified posted-price mechanism and the optimal standard mechanism is a lower bound on the loss from using a standard mechanism rather than the true optimum.

Before examining some comparative statics on when one or the other mechanism performs better, we comment on why the posted-price mechanism can be superior. At a formal level, recall the virtual valuation $v(t_n;t_{-n}) - \frac{1-F(t_n)}{f(t_n)}\frac{\partial}{\partial t_n}v(t_n;t_{-n})$. When values are common and the derivative $\frac{\partial}{\partial t_n}v(t_n;t_{-n})$ is increasing in t_n, it is possible that the seller prefers to assign the object to buyers with lower signals. Because values are common there is no efficiency loss from doing so, and if $\frac{\partial}{\partial t_n}v(t_n;t_{-n})$ is increasing in t_n, then the higher types may claim the larger information rent, even if the inverse hazard rate is nonincreasing. In the example in which $v(t_n, t_{-n}) = \max\{t_n, t_{-n}\}$ examined above, since $\frac{\partial}{\partial t_n} v(t_n, t_{-n}) = 0$ if $t_n < t_{-n}$, it is in fact the case that the buyer with the lower signal nevertheless has the higher virtual valuation. Thus, the property of the posted-price mechanism that the buyer with the lower signal sometimes receives the object is beneficial to the seller.

On an intuitive level, in a standard auction, each buyer calculates his willingness to pay by conditioning on having a signal at least as high as his opponent's, since those are the only instances in which he wins the object in equilibrium. Thus, the specification of $v(t_n, t_{-n})$ for $t_{-n} > t_n$ has no bearing whatever on the choice of optimal standard mechanism nor on the performance of that mechanism. In particular, if $v(t_n, t_{-n})$ is generally large relative to $v(t_n, t_n)$ for $t_{-n} > t_n$ (of which $v(t_n, t_{-n}) = \max\{t_n, t_{-n}\}$ is the most extreme example), the seller is foregoing an opportunity to extract value from the buyers by removing this portion of the domain of the valuation function from consideration. This yields the following comparative static, which is immediate:

Proposition 2: For two permissible valuation functions $v(t_n, t_{-n})$ and $u(t_n, t_{-n})$, let $v(t_n, t_{-n}) \quad u(t_n, t_{-n})$ for all $t_n < t_{-n}$. Then if the posted-price mechanism yields a higher expected revenue than the optimal standard mechanism when the buyers have valuation function $v(\cdot, \cdot)$, it also yields a higher expected revenue for $u(\cdot, \cdot)$.

To attach intuitive significance to this feature of the utility function, recall the restriction that the buyer with the higher signal must have weakly greater value for the object, i.e., $v(x, z) \geq v(z, x)$ for $x > z$. Suppose the function $v(x, z)$ is fixed for $x > z$, but allowed to vary for $x < z$ in ways that preserve this monotonicity. The boundary of such variations is when $v(x, z) = v(z, x)$ for all (x, z), which is exactly the pure common values case. However, this is also the case in which the posted-price mechanism performs best relative to the optimal standard mechanism, the performance of which does not change with these variations in the utility function. Thus, the optimal standard mechanism is less likely to outperform a posted-price mechanism the more common are the buyers' valuations.

A more general sufficient condition for the superiority of the posted-price mechanism to the standard mechanism can be derived as follows. The difference between expected revenue for the candidate posted price mechanism and for the optimal standard mechanism is $(B) - (A)$, or

$$\frac{\int_{t^*} v(t^*;t)f(t)dx}{(1-F(t^*))} - \frac{2\int_{t^*}\int_{t^*} tf(t)dtf(z)dz}{(1-F(t^*))^2}.$$

Integration by parts yields

$$t^* + \int_{t^*}^{\infty} \frac{\partial}{\partial t}v(t^*;t)\frac{1-F(t)}{1-F(t^*)}dt - (t^* + \int_{t^*}^{\infty} \frac{(1-F(t))^2}{(1-F(t^*))^2}dt)$$

$$= \int_{t^*}^{\infty} \left(\frac{\partial}{\partial t}v(t^*;t) - \frac{1-F(t)}{1-F(t^*)}\right)\frac{1-F(t)}{1-F(t^*)}dt. \qquad \text{(C)}$$

If (C) is positive, then the posted-price mechanism yields a higher expected payoff to the seller. A sufficient condition for this is that the integrand of (C) be nonnegative for all relevant values of t, or

$$\frac{\partial}{\partial t}v(t^*;t) \geq \frac{1-F(t)}{1-F(t^*)}$$

for all $t \geq t^*$, with the inequality strict for some positive measure of such t. In other words, posted prices are necessarily preferred if the effect on own valuation of the other buyer's signal when it is larger than one's own signal is sufficiently large. Note that in the special case of $v(t_n;t_{-n}) = \max\{t_n;t_{-n}\}$, $\frac{\partial}{\partial t}v(t^*;t) = 1$ for all $t > t^*$, so the sufficient condition is satisfied. However, there is clearly a range of functions $v(\cdot;\cdot)$ that also satisfy the condition, for any $F(\cdot)$. We also note that the condition is not necessary; for any $F(\cdot)$, nonnegativity of (C) amounts only to a requirement on the average value of $\frac{\partial}{\partial t}v(t^*;t)$ for $t > t^*$, weighted by $1 - F(t)$. So for example, while the sufficient condition imposes the requirement that $\frac{\partial}{\partial t}v(t^*;t) = 1$ for $t = t^*$, this need not be true in general for the posted-price mechanism to be preferred.

To generate additional comparative statics, we consider a class of specifications for $v(\cdot;\cdot)$: for $x < z$, $v(x;z) = \alpha z + (1-\alpha)x$, $\alpha \in [0;1]$. Above, we showed that if $\alpha = 1$, then the posted-price mechanism yields a higher expected revenue for all distributions on the signals. If $\alpha = 0$, then it is as in the private values case, and the optimal standard mechanism performs better than the posted-price mechanism for all distributions. Since the performance of the posted-price mechanism is continuous and strictly increasing in α, there is a unique α^* for any distribution such that the posted-price mechanism performs strictly better if and only if $\alpha > \alpha^*$. To solve for α^*, we equate the expected revenue of the two mechanisms:

$$\int_{t^*}^{1} t\frac{2(1-F(t))f(t)}{(1-F(t^*))^2}dt = \alpha^* \int_{t^*}^{1} t\frac{f(t)}{(1-F(t^*))}dt + (1-\alpha^*)t^*;$$

or

$$\alpha^* = \frac{\int_{t^*}^{1} (t-t^*)\frac{2(1-F(t))f(t)}{(1-F(t^*))^2}dt}{\int_{t^*}^{1} (t-t^*)\frac{f(t)}{(1-F(t^*))}dt}$$

Thus, α^* is the ratio of the expected first of two order statistics of the variable $t - t^*$, conditional on $t \geq t^*$, to the expectation of $t - t^*$ conditional on $t \geq t^*$.

As a comparative static, consider variations in the function $F(\cdot)$ that preserve two features: (1) the cutoff t^* in the optimal auction remains constant,[6] and (2) the expectation of t conditional on being greater than or equal to t^*, $\int_{t^*}^{1} t(f(t)/(1-F(t^*)))dt$, remains constant. Such variations do not change the expected revenue from the posted-price mechanism. However, they do change the expected revenue from the optimal standard mechanism insofar as they change the expectation of the first order statistic conditional on it being at least t^*. In particular, when comparing any two distributions $F_1(\cdot)$ and $F_2(\cdot)$ that satisfy (1) and (2) above, if $F_1(\cdot)$ has a lower associated conditional expectation of its first order statistic, then the posted-price mechanism is preferred to the auction for a larger range of α under $F_1(\cdot)$ than under $F_2(\cdot)$. For example, suppose the truncated distributions $(F_1(t) - F(t^*))/(1-F(t^*))$ and $(F_2(t) - F(t^*))/(1-F(t^*))$ have identical means and are such that the latter second-order stochastically dominates the former (i.e., the former is a mean-preserving spread of the latter). Then the expectation of the first order statistic from a sample of two is smaller for the former than for the latter,[7] and α^* is smaller for $F_1(\cdot)$ than for $F_2(\cdot)$. The posted-price mechanism performs relatively better for high-variance distributions because for a given mean, a high variance depresses the expectation of low order statistics, which tend to fall in the lower tail of the distribution, and hence of the selling price in an auction.

An additional comparative static on the distribution of signals can be derived by working directly with the virtual valuations, recalling that expected revenue is the expectation of

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[6] A simple way to satisfy this is to hold $F(t)$ constant for all $t \quad t^*$.

[7] Proof omitted but available on request.

the virtual valuation of the winning buyer. Take the common values case, in which ex-post valuation is the same for all buyers. Since our posted-price mechanism is constructed to sell the object for exactly the same realizations of signals as under the optimal standard mechanism, the first term in the virtual valuation can be ignored for the purpose of comparing revenues. Thus, the mechanism that raises more revenue is the mechanism that yields buyers lower expected information rent. Consider a given pair of signals $(t; t')$ satisfying $t > t' \geq t^*$. Under a standard mechanism, the information rent for this realization is $\frac{1-F(t)}{f(t)} \frac{\partial}{\partial t} v(t; t')$. Under the posted price mechanism, the expected information rent is $\frac{1}{2} \frac{1-F(t)}{f(t)} \frac{\partial}{\partial t} v(t; t') + \frac{1}{2} \frac{1-F(t')}{f(t')} \frac{\partial}{\partial t'} v(t'; t)$: Any change in distribution that expands the difference between the first and second makes the posted-price mechanism relatively more attractive. Suppose that $t > t'$ implies $\frac{\partial}{\partial t} v(t; t') \geq \frac{\partial}{\partial t'} v(t'; t)$ (the interesting case, as auctions tend to be optimal under the reverse assumption). Then for any $G(\cdot)$ satisfying $\frac{1-G(t)}{g(t)} \quad \frac{1-F(t)}{f(t)}$ for all t and

$$\frac{\frac{1-F(t)}{f(t)} - \frac{1-G(t)}{g(t)}}{\frac{1-F(t')}{f(t')} - \frac{1-G(t')}{g(t')}} \quad \frac{\frac{\partial}{\partial t'} v(t'; t)}{\frac{\partial}{\partial t} v(t; t')}$$

for all $t \geq t'$, the difference is larger for $G(\cdot)$ than for $F(\cdot)$, and if the posted-price mechanism generates more expected revenue for $F(\cdot)$ it must also do so for $G(\cdot)$. Assuming that the inverse hazard rate is nonincreasing for both $F(\cdot)$ and $G(\cdot)$, the condition implies that the inverse hazard rate is smaller, meaning a greater likelihood of low signals, and varies less across signals for $G(\cdot)$. The importance of these factors is clear. The greater the likelihood of low signals, the more important it is to allow buyers with low signals to win given that it is desirable to do so. And the less the hazard rate varies across signals, the more important it is to award the object to the buyer with the smaller marginal effect of information on his valuation, which we took to be the buyer with the lower signal by assumption. Taken together, our comparative statics suggest that in a common-value environment, auctions perform poorly relative to posted prices when there is high variation among signals, or when low signals are relatively likely and hazard rates vary little across signals.

A natural question is whether and how these results generalize to more than two buyers. Recall that we have not claimed that the posted-price mechanism is the optimal selling mechanism for our environment, merely that it may be superior to the best auction mechanism. Auctions can similarly be shown to be suboptimal for the general n buyer case via judicious choice of the mechanism against which to compare the optimal standard mechanism. An obvious candidate for such a comparison would be an analogous posted-price mechanism, with allocation via uniform randomization over the set of buyers who express willingness to buy at the posted price. However, one can construct examples in which any such mechanism must fare worse than the optimal standard mechanism for some distributions $F(\cdot)$, even when the valuation function is the analog $u(t_n;t_{-n}) = \max_m \{t_m\}$.[8]

To show that auctions are suboptimal for all $F(\cdot)$ when $u(t_n;t_{-n}) = \max_m \{t_m\}$, consider the following mechanism, which can be described as an auction-posted price hybrid. Each buyer who chooses to participate submits a price greater than some prespecified lowest acceptable price $\underline{p}$. The seller chooses the third-highest submitted price, or a reserve price (in general different and less than $\underline{p}$) p^* if fewer than two prices were submitted, and randomizes between the buyers who submitted the two highest prices to determine which buys the object at the chosen price.

For appropriate choice of p^* given $\underline{p}$, this game has an equilibrium as follows. Each buyer calculates the expectation of $u(t_n;t_{-n})$ conditional on his signal being tied for the second-highest. If this value is greater than or equal to $\underline{p}$ he submits it in the mechanism; if not, he does not participate. Define $\hat{t}$ as the infimum of types who submit bids under this strategy, and choose p^* to be the expectation of $u(\hat{t};t_{-n})$ given that $\hat{t}$ is at least the second largest signal realization. To see that this supports an equilibrium, first note that since $u(\cdot;\ldots;\cdot)$ is increasing in all arguments, under this strategy profile every type of buyer earns a nonnegative expected payoff. Just as in a second-price auction, the price a given buyer announces has no marginal bearing on the price he pays if he wins the object. Thus, no buyer has an incentive to submit a price lower than his equilibrium submission. If a buyer who does not have one of the two highest signals submits one of the two highest bids, his expected payoff is negative conditional on these events given the equilibrium strategies of others, and thus he does not wish to submit a price greater than his equilibrium submission, establishing equilibrium. Now choose $\underline{p}$ so that $\hat{t} = t^*$. For the specification $u(t_n;t_{-n}) = \max_m \{t_m\}$, the selling price in the hybrid mechanism when at least three buyers submit prices, conditional on the realization of the third-highest signal, is the expectation of a randomly drawn signal

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[8] Our gratitude to Vladimir Mares for providing such an example, omitted here.

from $F(\cdot)$ conditional on it being greater than the realization of the third-highest signal. In the optimal standard mechanism, it is the expectation of the lower of two random draws from that same conditional distribution. Since the former is greater for all realizations of the third-highest signal, it is unconditionally greater, for all distributions $F(\cdot)$, and the hybrid mechanism yields more expected revenue than the optimal auction.

4. SOME IMPLICATIONS FOR MULTI-UNIT AUCTIONS

The revenue consequences of valuation interdependencies also have force when a seller has multiple objects to sell. While there has been less focus on optimal selling mechanisms for multiple objects in the literature, largely because of complexities associated with multi-dimensional information for buyers, in certain simplified settings the traditional analysis of optimal mechanism design extends. Here we will focus on one such setting, in which objects are identical and each buyer only desires one object. In this setting, if nondecreasingness of virtual valuations in signals holds, then the optimal mechanism entails that the seller sell objects to the buyers with the highest signals, either until all objects are assigned, or until signals fall below some cutoff. This can be implemented by, for instance, a "highest rejected bid" uniform-price auction with a reserve price.

To emphasize our point, we again focus on a sample specification in which monotonicity of virtual valuation in signal may be violated. Let there be four buyers and two objects for sale. As before, buyers receive independent and identically distributed signals, each drawn from $[0,1]$ according to a continuous distribution $F(\cdot)$. Generally, buyer n's valuation for one object is a function $u(t_n, t_{-n})$, with t_{-n} now a triple of signals received by the other buyers. For the purposes of our demonstration, we focus on a particular specification, in which each object has the same final value to all buyers (pure common values), and that value is a function only of the second- and third-highest realized signals. Calling these signals y_3 and y_2, respectively (in keeping with our convention; see footnote 3), we have that $u(t_n, t_{-n}) = v(y_3, y_2)$ for all n. For convenience, we assume that the univariate function $v(y, y)$ is strictly increasing in y, in which case $v(y, y) = y$ may be assumed without additional loss of generality.

First, we consider the case in which the common value is equal to the realization of the third-highest signal, y_2. In such an environment, the seller can extract all surplus from trade by holding a single highest-rejected-bid uniform-price auction with no reserve price. In such an auction, it is an equilibrium for each buyer to bid his signal. The price-setting bid is the third-highest, which equals exactly the common value y_2 in this equilibrium. Thus, a traditional multi-unit auction is necessarily optimal for this specification.

Next, suppose the common value is equal to the realization of the second-highest signal, y_3. If the seller holds a uniform price auction with no reserve price as above, it remains an equilibrium for buyers to bid their signals; however, since the third-highest bid sets the price, the seller now does not extract all surplus with this mechanism. As an alternative to such a mechanism, suppose the seller randomly pairs the buyers, and holds a distinct second-price auction with no reserve price for each pair. For example, if buyers 1 and 2 are paired and buyers 3 and 4 are paired, then 1 and 2 bid for one of the objects, and 3 and 4 bid for the other. An implication of this selling scheme is that unlike in the single uniform-price auction for two objects, the buyers who ultimately win objects may not be those with the highest signals, because one-third of the time the buyers with the two highest signals will be paired together. In a private values setting with the standard hazard rate assumption this feature would be strictly detrimental to the seller (assuming monotonicity of virtual valuations in signals); we will show that under our particular interdependency specification, it is unambiguously beneficial.

In the alternative mechanism in which buyers are randomly paired and markets for the objects are segmented, the equilibrium bid function follows the traditional logic for second-price single object auctions: bid the expected value of the object conditional on having the same signal as the highest among all competitors for that object. Here, for a buyer with signal t, this is

$$t + \int_t^1 (z - t)2(1 - F(z))f(z)dz.$$

The latter term occurs because if both buyers in the other pair have signals greater than t, then the true value of the object is the lower of their signals; however, because they are not competing for the same object, the buyer in question does not condition on having a signal at least as large as theirs. Thus, buyers bid more in the selling mechanism with segmented markets than in the single uniform-price auction. However, these prices are being set by the buyer with the lowest signal and either the buyer with the second-lowest signal (two-thirds of the time) or the second-highest signal (one-third of the time), rather than always being set by the buyer with the second-lowest signal in the uniform-price auction.

The relevant question is whether increased bidding by all buyers compensates the seller for having the price determined by signals that are lower on average. The seller's expected revenue in the segmented auctions is

$$
\begin{aligned}
& \int_0^1 2[t + \int_t^1 (z - t)2(1 - F(z))f(z)dz]2(1 - F(t))f(t)dt \\
= & \int_0^1 2t(1 - (1 - F(t))^2)2(1 - F(t))f(t)dt + 2\int_0^1 \int_t^1 z2(1 - F(z))f(z)dz2(1 - F(t))f(t)dt.
\end{aligned}
$$

Note that the second term in the sum of the second expression is the expectation of the larger of two independent draws from the distribution $1 - (1 - F(t))^2$. Thus, the second term is equal to

$$\int_0^1 t4(1 - (1 - F(t))^2)(1 - F(t))f(t)dt,$$

which is the same as the first term in the sum. Thus, expected revenue in the segmented auctions is

$$
\begin{aligned}
& \int_0^1 2t4(1 - (1 - F(t))^2)(1 - F(t))f(t)dt \\
= & \int_0^1 2t4(2 - F(t))F(t)(1 - F(t))f(t)dt \\
= & \int_0^1 2t4(2 - 2F(t) + F(t))F(t)(1 - F(t))f(t)dt \\
= & \int_0^1 2t8(1 - F(t))^2F(t)f(t)dt + \int_0^1 2t4(1 - F(t))F(t)^2f(t)dt \\
= & \frac{2}{3}\int_0^1 2t12(1 - F(t))^2F(t)f(t)dt + \frac{1}{3}\int_0^1 2t12(1 - F(t))F(t)^2f(t)dt.
\end{aligned}
$$

The last sum demonstrates that the expected per-object revenue from this selling scheme is a mixture, with weights of two-thirds and one-third, of the third-largest signal y_2 and the second-largest signal y_3. This is clearly greater than an expected per-object revenue of y_2, as the seller obtains in the single uniform price auction, for all $F(\cdot)$. Thus, the alternative mechanism yields the seller a greater expected revenue than the uniform price auction for all $F(\cdot)$.

A legitimate reservation about the above is that the uniform-price auction we examine does not meet the standard of being optimal within a particular class of mechanism, as our benchmark did in the analysis of Section 2. Our definition of a standard mechanism is readily extended to the multiple object case by the characterization that for any buyer who wins an object, all buyers with greater signals must also win objects. The optimal standard mechanism in this setting may look a bit more complicated than a uniform price auction with a single reserve price. For instance, if there are common values and the common value is the realization of the second-highest signal, then the optimal standard mechanism is as follows: there is a reserve price, and if exactly two buyers bid more than the reserve price, then each receives one object for the reserve price. If more than two buyers bid more than the reserve price, then the third-highest bid sets the price. However, if fewer than two buyers bid more than the reserve price, then the seller sells one object to the highest bidder for the second-highest bid, whether or not the highest bid is above the reserve price. Even if the seller uses this optimal standard mechanism, there is an alternative that necessarily yields greater expected revenue. This alternative uses the random pairing technique as above and segmented auctions with identical reserve prices, with two exceptions. If fewer than two buyers bid above the reserve price, then the seller sells one object to the high bidder at the second-highest bid, as in the standard mechanism. And if exactly two buyers bid above the reserve price, they each receive an object at a fixed price (different from the reserve price), even if they are randomly paired together. This mechanism raises greater expected revenue than the optimal standard mechanism for all distributions $F(\cdot)$.[9] For a given $F(\cdot)$, a simpler alternative mechanism may better the optimal standard mechanism; for instance, if $F(\cdot)$ is uniform, then random pairings and segmented auctions with no reserve prices outperform the optimal standard mechanism.

To understand the results of this section, consider the virtual valuations for the two cases we analyze. When the common value is the third-highest signal y_2, then the realizations

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[9] Proof of the results of this paragraph is available from the authors on request.

of the two highest signals have no marginal effect on the common value, and the virtual valuations of the buyers with the two highest signals are simply the common value. Thus, the seller unambiguously wishes to sell to them, and extracts all buyer surplus by doing so. When the common value is the second-highest signal y_3, the virtual valuation of the buyer with the second-highest signal is strictly smaller than the virtual valuations of the other buyers. Thus, the seller benefits from selling an object to the buyer with the third-highest signal rather than the buyer with the second-highest signal, as occurs with probability $1/3$ in the pairing mechanism. In fact, the seller would do even better using a mechanism that gave an object to the buyer with the third-highest signal with probability $1/2$ and an object to the buyer with the second-highest signal with probability $1/2$; such a mechanism is easy to construct, though perhaps not as transparent as segmented auctions with random pairings.

If the common value depends on the realizations of both y_2 and y_3, then whether the uniform-price auction betters the alternative mechanism will depend on their relative weights in $v(\cdot;\cdot)$. For instance, if the common value equals $\alpha y_2 + (1 - \alpha)y_3$, $\alpha \in [0;1]$, then there exists a critical α^* such that for $\alpha > \alpha^*$, the uniform-price auction yields higher revenue, and for $\alpha < \alpha^*$, the segmented auctions perform better.

5. CONCLUSION

Issues of optimality have been somewhat incidental to the tremendous research interest in and use of auctions. Although the optimality of auctions in the canonical independent private values case with symmetric buyers is unquestionably a strong point in auctions' favor, interest in auctions has remained high even for circumstances in which they are known to be suboptimal, as in, e.g., the case of statistically dependent buyer information, or of asymmetric buyers. A possible reason for this is that the selling mechanisms that are demonstrably superior to auctions under these conditions entail rules lacking the simple transparency that auctions offer.

This paper has shown that when buyer valuations are interdependent, the case emphasized in the current auction literature, auctions are liable not only to be suboptimal, but to perform worse than other straightforward and transparent mechanisms, the posted price being a specific example. Such a conclusion may have important implications for the choice of selling mechanism even in environments in which the discovery and implementation of a theoretically optimal mechanism is infeasible.

Although our analysis employs examples rather than very general specifications, as possibility results they exhibit a certain robustness. In the core private values case, optimality of auctions depends on the distribution of signals $F(\cdot)$ satisfying an innocuous hazard rate condition; we have shown that for certain specifications of valuation interdependencies, an auction may be an inferior way to sell an object irrespective of $F(\cdot)$. Thus, the nature of the interdependencies ought to be a nontrivial consideration in the determination of whether or not to auction.

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