The Ohio State University
Department of Economics
Econ 808 Midterm

Profs Levin, Morelli, and Peck
Spring 2001

Directions: *Answer all questions and show all work. There are two questions on part 1, and one question on part 2. Each question is worth 33 points.*

Part 1

1. Consider an exchange economy with 2 consumers and one consumption good per state. Both consumers have the (Bernoulli) utility function over certain consumption,

$$u_i(x_i) = \log(x_i),$$

and are von Neumann-Morgenstern expected utility maximizers. Each consumer has an initial wealth of 2, which is his/her endowment in states where he/she does not have an accident. For those states in which a consumer is involved in an accident, he/she loses 1 unit of wealth, so his/her endowment is 1.

With probability $\frac{1}{2}$, there are no accidents. With probability $\frac{1}{2}$, consumer 1 is involved in an accident. (Thus, consumer 1 is involved in all accidents.) Conditional on consumer 1 having an accident, he crashes into a tree with probability $\frac{1}{2}$, and collides with consumer 2 with probability $\frac{1}{2}$. When both consumers are in an accident together, assume that neither can sue the other for damages.

(a) (14 points) Define a competitive equilibrium for this economy, where the two consumers participate on a complete state-contingent commodities market.

(b) (19 points) Calculate the competitive equilibrium price vector and allocation.

2. In the following variant of Spence's signaling model, everything is as in the original game (with the workers receiving education, followed by firms offering wage schedules), except that education increases productivity. Letting $x_i(y)$ denote the productivity of type i with education y, we have

$$\begin{aligned} c_1(y) &= y \\ x_1(y) &= 1 + \sqrt{y} \\ c_2(y) &= \frac{y}{2} \\ x_2(y) &= 2 + \sqrt{y}. \end{aligned}$$

The fraction of type 1 workers in the population is q_1.

(a) (11 points) Assuming full information, where firms can directly identify a worker's type, what will be the equilibrium education chosen and wage received by each type?

(b) (11 points) Assuming as Spence does that a worker's type is her private information, is the full-information first-best allocation from part (a) possible as an equilibrium outcome? Explain.

(c) (11 points) Again assuming that a worker's type is her private information, calculate the education chosen and wage received in the best separating equilibrium.