Department of Economics
The Ohio State University
Midterm Exam Questions and Answers–Econ 8712

Prof. James Peck
Fall 2015

Directions: *Answer all questions, be neat, and show all work. If it is appropriate, you are allowed to use propositions presented in class without proving them here.*

1. (30 points)
A decision maker (DM) is a von Neumann-Morgenstern expected utility maximizer with Bernoulli utility function over final wealth x given by the strictly concave function $u(x)$ and initial wealth W. Suppose that the DM must choose one of the following options.

Option 1: Place a bet of size b, which wins with probability $\frac{1}{2}$ and loses with probability $\frac{1}{2}$. That is, the DM's final wealth is $W + b$ if he wins, and his final wealth is $W - b$ if he loses.

Option 2: Place two bets, each of size $\frac{b}{2}$. Each bet wins (wealth is increased by $\frac{b}{2}$) with probability $\frac{1}{2}$ and loses (wealth is decreased by $\frac{b}{2}$) with probability $\frac{1}{2}$, and the outcomes across bets are independent of each other.

Indicate which option the DM will prefer, and formally justify your answer.

Answer:
The DM will prefer option 2. The utility of option 1 is given by

$$\frac{1}{2}u(W + b) + \frac{1}{2}u(W - b).$$

With option 2, the DM will win both bets w.p. $\frac{1}{4}$, lose both bets w.p. $\frac{1}{4}$, and break even by winning one bet and losing one bet w.p. $\frac{1}{2}$. Therefore, the utility of option 2 is given by

$$\frac{1}{4}u(W + b) + \frac{1}{4}u(W - b) + \frac{1}{2}u(W).$$

The utility of option 2 exceeds the utility of option 1 by

$$\begin{aligned} & \frac{1}{2}u(W) - \frac{1}{4}u(W + b) - \frac{1}{4}u(W - b) \\ = \quad & 2[u(W) - \frac{1}{2}u(W + b) - \frac{1}{2}u(W - b)]. \end{aligned}$$

The term in brackets is strictly positive whenever $b > 0$, due to the strict concavity of the utility function.

2. (30 points)

The following partial equilibrim economy has F firms, where for $f = 1, ..., F$, firm f has the cost function $c_f(y_f) = (y_f)^2$. There are n consumers, each of whom has an initial endowment of money (numeraire), $\omega_i^m = 1$ and a zero endowment of the good. Also, each consumer owns a share, $\frac{1}{n}$, of each firm's profits. Letting x_i and m_i denote consumer i's consumption of the good and money, respectively, for $i = 1, ..., n$, consumer i has the utility function $\log(x_i)+m_i$.

(a) (15 points) Calculate the competitive equilibrium price.

(b) (10 points) Calculate the competitive equilibrium allocation, including the money consumption of each consumer.

(c) (5 points) How does the competitive equilibrium price vary with the parameter F? Give an economic intuition for this relationship.

Answer:

(a) Utility maximization gives the MRS condition,

$$\frac{1}{x_i} = p$$

so the demand function is $x_i = \frac{1}{p}$ and the market demand function is $\frac{n}{p}$.

Profit maximization gives the f.o.c.

$$2y_f = p$$

so the supply function is $y_f = \frac{p}{2}$ and the market supply function is $\frac{Fp}{2}$. Equating supply and demand yields the market clearing condition

$$\begin{aligned} \frac{n}{p} &= \frac{Fp}{2}, \text{ so} \\ p^* &= \sqrt{\frac{2n}{F}}. \end{aligned}$$

(b) From the demand function and supply function, we have

$$\begin{aligned} x_i^* &= \sqrt{\frac{F}{2n}}, \\ y_f^* &= \sqrt{\frac{n}{2F}}. \end{aligned}$$

To find the numeraire consumption, we can compute the cost of output and use market clearing for the numeraire,

$$
\begin{aligned}
nm_i + F\frac{n}{2F} &= n \\
m_i &= \frac{1}{2}.
\end{aligned}
$$

(Alternatively, you can compute profits and use the budget constraint.)

(c) The price is a decreasing function of F. The reason is that the individual firm's supply curve is upward sloping, due to decreasing returns to scale in production. As a result, when more firms are added, the market supply curve shifts to the right, leading to a lower equilibrium price. Intuitively, each firm is operating at a smaller scale, where marginal costs are lower. This is certainly not the result of more firms leading to more competition and less market power. (If that is not what you mean by "more competition" it is not clear what you do mean.)

3. (40 points)

Consider the following pure-exchange economy with two types of consumers and two goods. There are 100 type-1 consumers and 200 type-2 consumers. If consumer i is of type 1, she has the utility function

$$u_i(x_i^1, x_i^2) = \log(x_i^1) + \log(x_i^2)$$

and the initial endowment vector, (ω_1^1, ω_1^2). If consumer i is of type 2, she has the utility function

$$u_i(x_i^1, x_i^2) = \log(x_i^1) + \log(x_i^2)$$

and the initial endowment vector, (ω_2^1, ω_2^2).

(a) (5 points) Define a competitive equilibrium for this economy.

(b) (20 points) Compute the competitive equilibrium price vector as a function of the endowment parameters.

(c) (15 points) Derive a simple condition that characterizes the values of the endowment parameters for which type-1 consumers are net suppliers of good 1 in the CE.

Answer:

(a) A CE is a price vector, (p^{*1}, p^{*2}), and an allocation, (x_i^{*1}, x_i^{*2}) for $i = 1, ..., 300$, such that

(i) for type 1 consumers, (x_i^{*1}, x_i^{*2}) solves

$$
\begin{aligned}
& \max \log(x_i^1) + \log(x_i^2) \\
& \text{subject to} \\
p^{*1}x_i^1 + p^{*2}x_i^2 &\leq p^{*1}\omega_1^1 + p^{*2}\omega_1^2 \\
x_i &\geq 0
\end{aligned}
$$

(ii) for type 2 consumers, (x_i^{*1}, x_i^{*2}) solves

$$\begin{aligned} & \max \log(x_i^1) + \log(x_i^2) \\ & \text{subject to} \\ p^{*1}x_i^1 + p^{*2}x_i^2 &\leq p^{*1}\omega_2^1 + p^{*2}\omega_2^2 \\ x_i &\geq 0 \end{aligned}$$

(iii) market clearing:

$$\begin{aligned} \sum_{i=1}^{300} x_i^{*1} &\leq 100\omega_1^1 + 200\omega_2^1 \\ \sum_{i=1}^{300} x_i^{*2} &\leq 100\omega_1^2 + 200\omega_2^2. \end{aligned}$$

(b) Normalize the price vector to be of the form $(p, 1)$. Use the subscript "1" to denote type 1 consumers and the subscript "2" to denote type 2 consumers. Because utility is strictly monotonic, budget constraints and market clearing conditions will hold with equality.

For type 1 consumers, utility maximization is characterized by the MRS condition and budget equation

$$\begin{aligned} px_1^1 &= x_1^2 \\ px_1^1 + x_1^2 &= p\omega_1^1 + \omega_1^2 \end{aligned}$$

from which we have the demand functions

$$\begin{aligned} x_1^1 &= \frac{p\omega_1^1 + \omega_1^2}{2p} \\ x_1^2 &= \frac{p\omega_1^1 + \omega_1^2}{2}. \end{aligned}$$

For type 2 consumers, solving the utility maximization problem yields the demand functions

$$\begin{aligned} x_2^1 &= \frac{p\omega_2^1 + \omega_2^2}{2p} \\ x_2^2 &= \frac{p\omega_2^1 + \omega_2^2}{2}. \end{aligned}$$

Market clearing for good 2 can be used to solve for the price

$$\begin{aligned} 100\frac{p\omega_1^1 + \omega_1^2}{2} + 200\frac{p\omega_2^1 + \omega_2^2}{2} &= 100\omega_1^2 + 200\omega_2^2, \\ p &= \frac{\omega_1^2 + 2\omega_2^2}{\omega_1^1 + 2\omega_2^1}. \end{aligned}$$

(c) Consumer 1 is a net supplier of good 1 if $x_1^1 < \omega_1^1$, or

$$\begin{aligned} \frac{p\omega_1^1 + \omega_1^2}{2p} &< \omega_1^1 \text{ or} \\ \frac{\omega_1^2}{\omega_1^1} &< p. \end{aligned}$$

Substituting the equilibrium price into the previous inequality, we have

$$\frac{\omega_1^2}{\omega_1^1} < \frac{\omega_1^2 + 2\omega_2^2}{\omega_1^1 + 2\omega_2^1}.$$

To find a "simple" condition, cross multiply and simplify, yielding

$$\frac{\omega_1^1}{\omega_1^2} > \frac{\omega_2^1}{\omega_2^2}.$$