Department of Economics
The Ohio State University
Midterm Exam Questions and Answers–Econ 8712

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1. (35 points)

A decision maker (DM) is a von Neumann-Morgenstern expected utility maximizer with Bernoulli utility function over final wealth x given by the function $u(x) = \log(x)$, and initial wealth W. Suppose that this wealth includes a car and a truck. The car will have an accident with probability π, and not have an accident with probability $1-\pi$; the truck will have an accident with probability π, and not have an accident with probability $1-\pi$. Also, the two accidents are independent of each other. Each accident results in a loss of wealth of D, and assume that $W > 2D$ holds.

Assume that the DM is able to purchase as much fair insurance as she wishes for the car, so that a policy of size α requires a premium of $\alpha\pi$ to be paid by the DM in all states, and the policy pays the DM α in the event that she has a car accident. Also assume that no insurance is available for the DM's truck.

(i) (20 points) Set up the optimization problem faced by the DM and find the first order conditions for an interior solution.

(ii) (15 points) Demonstrate whether the optimal (utility maximizing) insurance policy satisfies $\alpha < D$, $\alpha = D$, or $\alpha > D$.

Answer:

(i) There are 4 states of nature. With both accidents, consumption is $W - \alpha\pi - 2D + \alpha$. With only a car accident, consumption is $W - \alpha\pi - D + \alpha$. With only a truck accident, consumption is $W - \alpha\pi - D$. With no accident, consumption is $W - \alpha\pi$. Therefore, the optimization problem is

$$\max_{\alpha} \left[\begin{array}{c} \pi^2 \log(W - \alpha\pi - 2D + \alpha) + \pi(1-\pi)\log(W - \alpha\pi - D + \alpha) \\ +\pi(1-\pi)\log(W - \alpha\pi - D) + (1-\pi)^2 \log(W - \alpha\pi) \end{array} \right].$$

The first order condition is

$$\frac{\pi^2(1-\pi)}{W - \alpha\pi - 2D + \alpha} + \frac{\pi(1-\pi)^2}{W - \alpha\pi - D + \alpha} + \frac{-\pi^2(1-\pi)}{W - \alpha\pi - D} + \frac{-\pi(1-\pi)^2}{W - \alpha\pi} = 0,$$

which can be simplified to

$$\frac{\pi}{W - \alpha\pi - 2D + \alpha} + \frac{(1-\pi)}{W - \alpha\pi - D + \alpha} + \frac{-\pi}{W - \alpha\pi - D} + \frac{-(1-\pi)}{W - \alpha\pi} = 0. \quad (1)$$

(ii) Evaluating the left side of (1) at $\alpha = D$ yields

$$\frac{\pi}{W - D\pi - D} + \frac{(1-\pi)}{W - D\pi} + \frac{-\pi}{W - D\pi - D} + \frac{-(1-\pi)}{W - D\pi}.$$

The first and third terms cancel, and the second and fourth terms cancel, so the expression is zero. Thus, the f.o.c. hold at $\alpha = D$. Because the objective function is concave, this is the unique optimal solution.

2. (35 points)

The following partial equilibrium economy has 2 consumers and 24 firms. Consumer 1 has the utility function,

$$u_1(x_1, m_1) = \log(x_1) + m_1,$$

the numeraire endowment $\omega_1^m = 3$, and owns all of the firms, $T_{1,f} = 1$ for $f = 1, ..., 24$. Consumer 2 has the utility function,

$$u_2(x_2, m_2) = 2\log(x_2) + m_2,$$

the numeraire endowment $\omega_2^m = 3$, and owns no shares of any firms, $T_{2,f} = 0$ for $f = 1, ..., 24$.

For $f = 1, ..., 24$, each firm f has the cost function given by

$$c_f(y_f) = (y_f)^2.$$

(i) (20 points) Solve for the competitive equilibrium price and allocation.

(ii) (15 points) Derive an expression for the deadweight loss (the reduction in total surplus) that would arise if the total output is one unit less than the competitive equilibrium quantity, assuming that the output is efficiently produced and distributed.

Answer:

(i) Consumer 1's utility maximizing consumption solves

$$\frac{1}{x_1} = p,$$

giving rise to the demand function $x_1 = \frac{1}{p}$. Similarly, consumer 2 has the demand function, $x_2 = \frac{2}{p}$, and the market demand function is $D(p) = \frac{3}{p}$.

For each firm f, profit maximization entails (equating marginal cost to the price) $2y_f = p$, yielding the supply function, $y_f = \frac{p}{2}$, and profits of $p\frac{p}{2} - \frac{p^2}{4} = \frac{p^2}{4}$. Therefore, the market supply function is $S(p) = 12p$.

Equating demand and supply, we have $\frac{3}{p} = 12p$, so the equilibrium price is $p = \frac{1}{2}$. Substituting the price into demand and supply functions, we have $x_1 = 2$, $x_2 = 4$, $y_f = \frac{1}{4}$. Since aggregate profits are $\frac{1}{16} \cdot 24$, consumer 1's numeraire consumption is

$$m_1 = 3 - \frac{1}{2} \cdot 2 + \frac{24}{16} = \frac{7}{2}$$

and consumer 2's numeraire consumption is

$$m_2 = 3 - \frac{1}{2} \cdot 4 = 1.$$

(ii) At the price, $p = \frac{1}{2}$, the total quantity produced and consumed is 6. To find the deadweight loss, we evaluate the area between the demand curve and the supply curve, from a quantity of 5 to a quantity of 6. To do this, we must compute the inverse demand function, $p(x) = \frac{3}{x}$ and the inverse supply function, $p(x) = \frac{x}{12}$We have

$$\begin{aligned}
DWL &= \int_5^6 (\frac{3}{x} - \frac{x}{12})dx \\
&= [3\log(x) - \frac{x^2}{24}]|_5^6 \\
&= [3\log(6) - \frac{36}{24}] - [3\log(5) - \frac{25}{24}] \\
&= 3\log(\frac{6}{5}) - \frac{11}{24}.
\end{aligned}$$

3. (30 points)

The following pure exchange economy has two goods and n consumers, where each utility function is continuous, strictly monotonic, and strictly quasi-concave. The aggregate endowment of each good is 1. The aggregate excess demand function of good 1 is given by

$$Z^1(p^1, p^2) = \frac{a}{1+a} + \frac{bp^2}{(1+b)p^1} - 1,$$

where a and b are positive parameters.

(a) (20 points) Find the aggregate excess demand function of good 2.

(b) (10 points) Calculate the competitive equilibrium price of good 1 relative to good 2, $\frac{p^1}{p^2}$.

Answer:

(a) Using Walras' law, we have

$$p^1[\frac{a}{1+a} + \frac{bp^2}{(1+b)p^1} - 1] + p^2 Z^2(p^1, p^2) = 0.$$

Solving the above equation for $Z^2(p^1, p^2)$ yields (after simplifying)

$$Z^2(p^1, p^2) = \frac{p^1}{p^2(1+a)} - \frac{b}{1+b}.$$

(ii) To find the CE price, we can set $Z^1(p^1, p^2)$ equal to zero. Due to strict monotonicity, we know that in any CE, excess demand for each good must equal zero.

$$Z^1(p^1, p^2) = \frac{a}{1+a} + \frac{bp^2}{(1+b)p^1} - 1 = 0$$

can be solved for $\frac{p^1}{p^2}$, yielding

$$\frac{p^1}{p^2} = \frac{(1+a)b}{1+b}.$$