Milgron and Stokey

"Information, Trade, and Common Knowledge"

$\hat{P}_i$ partition before trade

P_i partition at the time of trading (including inf. from prices)

REE Common Knowledge that the trade is feasible and mutually acceptable.

n traders

Ω finite $\Omega = \Theta \times X$

What is a def. of payoff relevent events?

Θ payoff - relevant events

X payoff - irrelevant events, don't affect endowments or tastes

but x and θ may be correlated

$\mathbb{R}^l_+$ consumption set

$e_i : \Theta \to \mathbb{R}^l_+$ endowments

$U_i : \Theta \times \mathbb{R}^l_+ \to \mathbb{R}$ utility

$p_i(\cdot)$ prior beliefs about ω

$\hat{P}_i$

assume $U_i(\theta, \cdot)$ is increasing $\forall\, i, \theta$

Def If $U_i(\theta, \cdot)$ is (strictly) concave for all θ, i is (strictly) weakly risk averse.

Milgrom & Stokey

Thm If we start with an ex ante Pareto optimal allocation and have common priors, then risk averse traders cannot have common knowledge of gains from trade (after receiving private information)

— Does not depend on the rules of trade, price taking, etc.

— When we see retrade, there must not have been complete markets, ex ante, or else bounded rationality.

— Noise trader models in finance.

Def A trade $t = (t_1, \ldots, t_n)$ is a function from $\Omega \to \mathbb{R}^{nl}$, specifying the net trade of commodities in each state.

If $t : \Theta \to \mathbb{R}^{nl}$, we have a θ-contingent trade

t is feasible if

$$e_i(\theta) + t_i(\theta, X) \geqq 0 \qquad \forall i, \theta, X$$

$$\sum_{i=1}^{n} t_i(\theta, X) \leqq 0 \qquad \forall \theta, X$$

Def Beliefs are concordant if

$$p_1(X|\theta) = p_2(X|\theta) = \cdots = p_n(X|\theta) \qquad \forall X, \theta$$

[But traders can disagree about the probability of θ.]

Note: If agents are risk averse and have concordant beliefs, θ-contingent $t^* = (E_1(t_1|\theta); \ldots; E_n(t_n|\theta))$ is feasible + is weakly preferred to any trade t ~~that~~ is feasible.

t can include side bets about X, when all relevant uncertainty is insurable.

Thm 1

Proof

Let R be the meet of $P_1 \cdots P_n$.

Suppose it is common knowledge at ω' that t is feasible and mutually acceptable.

Then $\forall i$ and $\omega \in R(\omega')$ we have

(1) $$E_i\left[U_i(\theta, e_i + t_i) \mid P_i(\omega)\right] \geq E_i\left[U_i(\theta, e_i) \mid P_i(\omega)\right]$$

Suppose (1) is strict for trader j at ω', and consider the (θ, x) trade t^*:

$$t_i^* \equiv t_i \, 1_{R(\omega')} \qquad \forall i$$

$1_{R(\omega')} = \begin{cases} 1 & \omega \in R \\ 0 & \text{other} \end{cases}$

Obviously, t^* is feasible.

($R(\omega)$ itself does not change)

Taking expectations ex ante, for each i we have

$$E_i\left[U_i(\theta, e_i + t_i^*)\right] = E_i\left[E_i\left[U_i(\theta, e_i + t_i 1_{R(\omega')}) \mid P_i\right]\right] =$$

(2) $$E_i\left[E_i\left[U_i(\theta, e_i) 1_{R^c(\omega')} \mid P_i\right]\right] + E_i\left[E_i\left[U_i(\theta, e_i + t_i) 1_{R(\omega')} \mid P_i\right]\right]$$

$\therefore 1_{R^c(\omega')}$ is constant over elements of P_i

R is coarser than P_i, so (2) equals

(3) $$E_i\left[1_{R^c(\omega')} E_i\left[U_i(\theta, e_i) \mid P_i\right]\right] + E_i\left[1_{R(\omega')} E_i\left[U_i(\theta, e_i + t_i) \mid P_i\right]\right]$$

it is c.k. that t is mutually acceptable, ① $\Rightarrow$ so (3)

$$\geq E_i\left[\qquad \qquad \right] + E_i\left[1_{R(\omega')} E_i\left[U_i(\theta, e_i) \mid P_i\right]\right]$$

$$= E_i\left[U_i(\theta, e_i)\right]$$

$$\therefore \quad E_i\left[u_i\left[\theta, e_i + t_i^*\right]\right] \geq E_i\left[u_i(\theta, e_i)\right] \quad \forall i$$

and strict inequality for j.

note t^* is not necessarily a θ-trade, so we cannot contradict Pareto optimality yet.

By concordant beliefs, the θ-trade

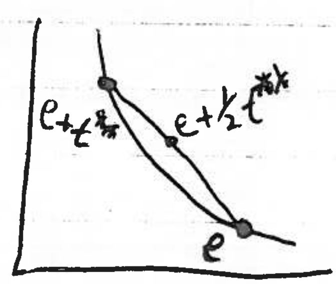
$$t^{**} = E\left[t^* \mid \theta\right]$$

is feasible, independent of X, and ex ante weakly preferred to t^* so it Pareto dominates the null trade.

$\therefore$ t cannot be strictly preferred in (1).

— All consumers are indifferent

If consumers are strictly risk averse, and t is not null, then $\frac{1}{2} t^{**}$ is a Pareto improving θ-trade.



Thm 2 Let e be an ex ante P.O. allocation relative to θ-trades, supported by prices $q(\theta)$, and assume concordant beliefs. If $\hat{P}_1(\omega) \ldots \hat{P}_n(\omega)$ is revealed with markets reopened,

$$\hat{q}(\theta \mid \overset{\omega}{x}) = q(\theta)\, p(Q(\omega) \mid \theta)$$

together with the initial allocation, constitutes a fully revealing REE.

where Q is the join of $\hat{P}_1 \cdots \hat{P}_n$ (coarsest common refinement)

Proof Through prices, each agent observes $p(Q(\omega) \mid \theta)$.

Each agent can calculate $p_i(\theta \mid Q(\omega)) = \dfrac{p(Q(\omega) \mid \theta)\; p_i(\theta)}{p_i(Q(\omega))}$

so that each agent updates priors based on complete information.

"Straightforward" to check we have a Competitive Equilibrium.

note — there may be other REE that are not fully revealing.

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Thm 2 (REE)

$\boxed{\text{say, good 1}}$

$$\frac{\hat{q}(\theta|\overset{\omega}{\cancel{*}})}{\hat{q}(\theta'|\underset{\omega}{*})} = \frac{q(\theta)\, P(Q(\omega)|\theta)}{q(\theta')\, P(Q(\omega)|\theta')}$$

since $q(\theta)$ is a C.E. ex ante

$$= \underbrace{\left(\frac{P_i(\theta)\, u_i'(\theta, e_i(\theta))}{P_i(\theta')\, u_i'(\theta', e_i(\theta'))}\right)}_{\text{C.E.}} \underbrace{\left[\frac{P_i(\theta|Q(\omega))\, P_i(\theta')}{P_i(\theta'|Q(\omega))\, P_i(\theta)}\right]}_{\text{Bayes' Rule}}$$

$$= \frac{P_i(\theta|Q(\omega))\, u_i'(\theta, e_i(\theta))}{P_i(\theta'|Q(\omega))\, u_i'(\theta', e_i(\theta'))}$$

If fully revealing, then for any x, MRS equals price ratio.

$$P_i(\theta|Q(\omega)) = \frac{P(Q(\omega)|\theta)\, P_i(\theta)}{P_i(Q(\omega))}$$

Thm 3

First round — trade to a P.O. allocation e.
receive signals and markets reopen — price vector $\hat{q}(\theta \mid x)$

$$p_i(\theta \mid P_i(\omega), \hat{q}) = p_i(\theta \mid \hat{q}) \qquad \forall\, \theta, i$$

[There is no information available in the private signal that is not in the price.]

[Changes in prices depends only on the information and not on ~~prices~~ endowments, preferences, allocation e]

Value of Information

Any attempt to speculate on new information causes that information to be revealed by prices.

Paradox: How do prices reflect the information if informed traders either don't trade or don't bother to consider their private signal? incentive to gather info.?

Must look at the price process to know how informative prices are. Prices (once formed) may be informative, but after trade has taken place (too late to change actions).

- Ex ante market may not exist —

Incomplete markets and better information could be worth gathering.