The Economics of Rumours

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A population of potential investors face a decision about whether or not to invest, in sequence.

The asset return is common to all investors, and takes on one of two values:

$$\begin{array}{lll} a & \text{w.p.} & p \\ b & \text{w.p.} & 1-p. \end{array}$$

The cost of investing is individual-specific, independent, and takes on one of two values:

$$\begin{array}{lll} 0 & \text{w.p.} & q \\ c & \text{w.p.} & 1-q. \end{array}$$

We assume: $a > c > b > 0$.

Investor 1 observes the realized investment return (a or b), and other investors receive no signals at all.

Banerjee distinguishes between herding and a *rumour process*. How well will investor 1's information be transmitted when other investors only observe the previous investor's behavior?

In Model 1, investor t only observes that there is an investment opportunity if investor $t-1$ invested.

In Model 2, investor t knows when the rumour started (knows her number) and observes whether or not investor $t-1$ invested.

In Model 3, the process runs in continuous time and investors know when the rumour started. Transmission is endogenous: the probability of receiving the rumour is proportional to the number of people who have invested in the past.

Model 1

"Unknown" investment opportunity: investor t only observes that there is an investment opportunity if investor $t-1$ invested.

If investor 1 has cost c and observes that the state is b, he does not invest and the game is over.

Otherwise, he invests and investor 2 has a choice. She can use Bayes' rule to compute

$$pr(a|\text{investor 1 invests}) = \frac{p}{p+(1-p)q}.$$

Assumption (*) is the condition that investor 2 can profitably invest with cost c if she observes investor 1 invest:

$$\frac{p}{p+(1-p)q} \geq \frac{c-b}{a-b}.$$

Without this assumption, the game is trivial, with investment only by zero cost investors.

Proposition 2.1: *Under assumption (*), the informativeness of the rumour does not change over time, in the sense that the ex post probability that the return is "a" remains the same. Everyone invests if and only if the first person invests.*

Investor t, whether she knows her number or not, knows that having the opportunity is equivalent to observing that the first person invested. All subsequent investors invest whatever their cost, and the conditional probability of state a remains forever at

$$\frac{p}{p + (1 - p)q}.$$

Model 2

Investor t knows when the rumour started (knows her number) and observes whether or not investor $t-1$ invested.

We make assumption (*) throughout, so investor 1 invests in state a and also when his cost is zero. The equilibrium will depend on whether or not the following additional assumption holds:

Assumption (**) is the condition that an investor with an opportunity, but knowing only the ex ante distribution of returns, would not invest:

$$\frac{c-b}{a-b} > p.$$

Investor $t+1$ will invest if cost is zero or if state a is sufficiently likely.

From Bayes' rule, we have $pr(a|t \text{ invested}) =$

$$\frac{p \cdot pr(t \text{ invested}|a)}{p \cdot pr(t \text{ invested}|a) + (1-p) \cdot pr(t \text{ invested}|b)}$$

$$= \frac{p}{p + (1-p)z(t)}, \tag{1}$$

where

$$z(t) \equiv \frac{pr(t \text{ invested}|b)}{pr(t \text{ invested}|a)}.$$

It can be shown that all equilibria are of the following form:

For $t \leq t^*$, invest iff cost is zero or if investor $t-1$ invested.

For $t > t^*$, invest iff cost is zero.

Given the form of the equilibrium, just one zero-cost investor will start a chain of investment through period t^*. We can compute:

For $t \leq t^*$,

$$\begin{aligned} pr(t \text{ invested}|a) &= 1 \\ pr(t \text{ invested}|b) &= 1-(1-q)^t \\ z(t) &= 1-(1-q)^t. \end{aligned}$$

For $t \leq t^*$, $z(t)$ is increasing in t, so $pr(a|t$ invested) is decreasing in t.

We have $\lim_{t\to\infty} z(t) = 1$, so (1) implies $\lim_{t\to\infty} pr(a|t$ invested) $= p$.

For finite t, we have $pr(a|t$ invested) $> p$.

Therefore, the informational content of the rumour dies out completely, and beliefs converge to the unconditional probability. Intuitively, for large t it is guaranteed that some investor has zero cost and always invests, which cuts off any connection to investor 1.

What is t^*?

If assumption (**) is *not satisfied*, then $pr(a|t$ invested) $>$ p implies investment is always profitable when the previous investor invested, so we have $t^* = \infty$. As soon as someone invests, all subsequent investors will invest.

If assumption (**) is *satisfied*, then $\lim_{t \to \infty} pr(a|t \text{ invested}) = p$ implies t^* is finite, after which only zero cost investors will invest.

Cutoff occurs when

$$\frac{p}{p + (1-p)z(t)} = \frac{c-b}{a-b}. \tag{2}$$

Substituting $z(t) = 1 - (1-q)^t$ into (2) and solving, we have

$$t^* = \frac{\log\left(1 - \frac{(a-c)p}{(c-b)(1-p)}\right)}{\log(1-q)}.$$

Model 3

The process runs in continuous time and investors know when the rumour started. Transmission is endogenous: the probability of receiving the rumour is proportional to the number of people who have invested in the past.

The set of investors is the unit interval, $[0, 1]$.

At time 0, a fraction of the population of measure x observes the investment return and receive an investment opportunity.

If the state is a, then the measure of investment at time zero is x. If the state is b, then the measure of investment at time zero is qx. (only those with zero cost)

Notation:

$N(a,s)$ and $N(b,s)$ denote the measure of investors that have invested by time s, given the state.

$P(a,s)$ and $P(b,s)$ denote the measure of investors that have not yet received an investment opportunity by time s, given the state.

$pr(r|a,s)$ and $pr(r|b,s)$ denote the probability that an investor hears the rumour for the first time during the time interval between s and $s+ds$.

$$z(s) \equiv \frac{pr(r|b,s)}{pr(r|a,s)}.$$

Our initial conditions are

$$\begin{aligned} P(a,0) &= P(b,0) = 1-x, \\ N(a,0) &= x \quad \text{and} \quad N(b,0) = xq. \end{aligned}$$

From Bayes' rule, the probability of state a, given that an investor first observes the rumour between times s and $s+ds$, is given by

$$\begin{aligned} & \frac{p \cdot pr(r|a,s)}{p \cdot pr(r|a,s) + (1-p)pr(r|b,s)} \\ = & \frac{p}{p+(1-p)z(s)}. \end{aligned}$$

Therefore, a high cost investor invests iff

$$\begin{aligned} \frac{p}{p+(1-p)z(s)} &\geq \frac{c-b}{a-b} \quad \text{or} \\ z(s) &\leq \frac{(a-c)p}{(c-b)(1-p)} \equiv z^*. \end{aligned}$$

Equilibrium behavior falls into one of two regimes:

Regime 1: $z(s) \leq z^*$. Both high and zero cost types invest.

Regime 2: $z(s) > z^*$. Only zero cost types invest.

Since we assume that the probability of receiving the rumour is proportional to $N(i,s)$ for $i = a, b$, we have

$$pr(r|i,s) = yN(i,s)P(i,s)ds. \tag{3}$$

In (3), y is the parameter representing speed of transmission. Think of $yN(i,s)$ rumours being spread to random investors per unit of time, some of whom have already heard the rumour. The proportion of those rumors being heard for the first time is proportional to $P(i,s)$.

From (3) and the definition of $z(s)$, we have

$$z(s) = \frac{N(b,s)P(b,s)}{N(a,s)P(a,s)}. \tag{4}$$

From (4) and our initial conditions, we have $z(0) = q$.

By assumption (*), we start in Regime 1.

Regime 1 Dynamics:

The fraction of the population who invest (and fall out of the group who have not heard) is the fraction who first observe the rumour. Therefore,

$$\begin{aligned}
\frac{dP(i,s)}{ds} &= -yN(i,s)P(i,s) \\
\frac{dN(i,s)}{ds} &= yN(i,s)P(i,s) \\
P(a,s) &= 1 - N(a,s) \\
P(b,s) &= 1 - x(1-q) - N(b,s).
\end{aligned}$$

Regime 2 Dynamics:

The fraction of the population fall out of the group who have not heard is the fraction who first observe the rumour. The fraction who invest is $q\cdot$(fraction who first hear the rumour). Therefore,

$$\begin{aligned}\frac{dP(i,s)}{ds} &= -yN(i,s)P(i,s) \\ \frac{dN(i,s)}{ds} &= yqN(i,s)P(i,s).\end{aligned}$$

Proposition 3.2: *$z(s)$ increases monotonically over time and is unbounded. Therefore, for any value of z^* there will be an instant, t^*, at which there will be a transition from Regime 1 to Regime 2. After t^*, the system will remain in Regime 2.*

Intuition: If we stay in Regime 1, the ratio $\frac{N(b,s)}{N(a,s)}$ remains bounded, but the ratio $\frac{P(b,s)}{P(a,s)}$ goes off to infinity. Therefore, $z(s) = \frac{N(b,s)P(b,s)}{N(a,s)P(a,s)}$ also approaches infinity.

If you hear the rumour after a very long time, you should wonder why it took so long to reach you. Eventually, you know that the return is low.

In Model 1, the informativeness of the rumour remained constant over time. In Model 2, the informativeness of the rumour declined over time. In Model 3, the informativeness of the rumour actually increases over time.

The process in Model 3 seems more like a financial contagion than a rumour. Similar equations are used in the literature on infectious diseases, except here we do not allow people to stop transmitting (as they would by recovering from the disease or dying).