Ennis and Keister, "Run equilibria in the Green-Lin model of financial intermediation"

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In Diamond-Dybvig, we see run equilibria in the optimal *simple* contract.

When the fraction of impatient traders is constant, suspension of convertibility can achieve the first-best without a run equilibrium.

Peck and Shell (*JPE* 2003) construct examples in which the fraction of impatient is random, and the optimal contract (which allows for withdrawal amounts to depend on the history) also has a run equilibrium.

In Peck and Shell, patient and impatient traders have different marginal utilities of consumption.

Green and Lin (*Journal of Economic Theory* 2003) assume that:

1) patient and impatient traders have the same marginal utilities of consumption.

2) the distribution of types across traders is i.i.d.

3) traders observe their place in line (clock assumption).

Green and Lin show that the first-best allocation, subject to resource and sequential service but not incentive constraints, can be uniquely implemented. IC constraint does not bind.

Ennis and Keister clarify which of Green and Lin's assumptions are needed to rule out equilibrium bank runs.

They provide a correlated types example, in which the optimal contract also has a partial run equilibrium, satisfying all of the assumptions except independence.

They provide an example in which the optimal contract also has run equilibrium, satisfying all of the assumptions except the assumption that traders observe their place in line.

The Model

Two consumption periods, 0 and 1.

Finite number of traders, I.

Consumption vector (across periods 0 and 1) of trader i is denoted as $a_i = (a_i^0, a_i^1)$.

Aggregate consumption is given by $a = (a_1, ..., a_I)$.

We assume a CES utility function with parameter $\gamma > 1$:

$$v(a_i^0, a_i^1; \omega_i) = \frac{1}{1-\gamma}(a_i^0 + \omega_i a_i^1)^{1-\gamma} \tag{1}$$

In (1), ω_i refers to the trader's type, where a trader is *impatient* if $\omega_i = 0$ and *patient* if $\omega_i = 1$. The set of types is $\Omega \equiv \{0, 1\}$.

A trader's type is private information.

A state of nature is a specification of all agents' types: $\omega = (\omega_1, ..., \omega_I) \in \Omega^I$.

Denote the number of patient traders as $\theta(\omega) = \sum_{i=1}^I \omega_i$.

Let $p(\theta)$ denote the probability that $\theta(\omega) = \theta$.

Think of the following process for determining types:

1. Nature chooses θ based on the probabilities $p(\theta)$.

2. Nature chooses ω in such a well that the number of patient traders is θ and all permutations are equally likely. That is, we have

$$pr(\omega_i = 1|\theta) = \frac{\theta}{I}.$$

A special case is independent types, where traders are *impatient* w.p. π:

$$p(\theta) = \frac{I!}{\theta!(I-\theta)!}(1-\pi)^{\theta}\pi^{(1-\theta)}.$$

Technology

Each trader deposits 1 unit of the consumption good, so aggregate deposits are I.

Each unit liquidated in period 0 yields 1 unit of consumption.

Each unit not liquidated yields $R > 1$ units of consumption in period 1.

Feasible Allocations

The set of *ex post* allocations is

$$A = \{a : \sum_{i=1}^{I} (a_i^0 + \frac{a_i^1}{R}) \leq I\}.$$

A *state-contingent* allocation maps states to ex post allocations. The set of state-contingent allocations is:

$$F = \{\mathbf{a} : \Omega^I \to A\}.$$

Sequential Service Constraint

Traders contact the bank in period 0 in a fixed order, $1, 2, ..., I$. Ennis and Keister usually assume that traders know their place in line.

To capture the idea that impatient traders require immediate consumption, the period 0 consumption of trader i can only depend on $\omega^i = (\omega_1, ..., \omega_i)$.

Thus, the sequential service constraint is:

$$\mathbf{a}_i^0(\omega) = \mathbf{a}_i^0(\widehat{\omega}) \quad \text{for all } \omega, \widehat{\omega} \text{ s.t. } \omega^i = \widehat{\omega}^i. \qquad (2)$$

Define

$$F' = \{\mathbf{a} \in F : \quad (2) \text{ holds}\}.$$

The Efficient Allocation

Expected utility, given the state-contingent allocation $\mathbf{a}$, and given the true state ω^*, is denoted by

$$U_i(\mathbf{a}, \omega^*) = E[v(\mathbf{a}_i(\omega), \omega_i) | \omega_i = \omega_i^*].$$

The efficient allocation, if the planner could observe types as they show up in line, solves

$$\max_{\mathbf{a} \in F'} \sum_{i=1}^{I} E[U_i(\mathbf{a}, \omega)]. \tag{3}$$

The efficient allocation will have:

–impatient consume only in period 0 and patient consume only in period 1,

–remaining resources in period 1 evenly divided among the patient

$$\mathbf{a}_i^1(\omega) = \frac{R[I - \sum_{i=1}^{I} \mathbf{a}_i^0(\omega)]}{\theta(\omega)}$$

What remains to be characterized is $\mathbf{a}_i^0(\omega)$ for histories with $\omega_i = 0$.

Proposition 1 characterizes the efficient allocation and shows how to compute it recursively.

To see whether the efficient allocation can be implemented, it is without loss of generality to look at direct mechanisms (revelation principle). Traders report their type, so the message space is

$$m_i \in \{0, 1\} \equiv M.$$

Trader i's *communication strategy* must be measurable with respect to his/her informaton, and is a function from states to messages

$$\mu_i : \Omega^I \to M.$$

We denote the profile of messages in state ω as $\mu(\omega) = (\mu_1(\omega), ..., \mu_I(\omega))$.

Note: When we say that a communication strategy must be measurable w.r.t. trader i's information, this must include ω_i and not the types of the other traders. However, some mechanisms give traders additional information or cannot prevent traders from having additional information. For example, the mechanism can

1. Not give any additional information, so that type is all a trader knows. (Peck-Shell)

2. Let traders know their type and place in line (Green-Lin).

3. Let traders know their type and the entire history of reports.

An *allocation rule*, α, specifies an allocation for each message profile:

$$\alpha : M^I \to A.$$

Feasible allocation rules must satisfy the sequential service constraint (2), so period 0 consumption for trader i cannot depend on messages after i.

The state contingent allocation that traders receive is determined from the communication strategies and the allocation rule:

$$\mathbf{a}(\omega) = \alpha(\mu(\omega)).$$

Def. An allocation is *implementable* if it is the outcome of a set of Bayesian NE communication strategies for some allocation rule α. That is, $\mathbf{a}$ is implementable if there exists α and an equilibrium strategy profile μ^*, such that

$$\mathbf{a}(\omega) = \alpha(\mu^*(\omega)) \quad \text{for all } \omega.$$

Def. An allocation is *truthfully implementable* (or Bayesian incentive compatible) if it is implementable and $\mu_i^*(\omega) = \omega_i \quad$ for all ω.

A key allocation rule treats all messages as truthful and assigns consumption according to the planner's problem, (3). Call this allocation rule α^*.

If the efficient allocation is truthfully implementable using α^*, are there other equilibria to α^* in which some *patient* traders report $\mu_i(\omega) = 0$ (that they are impatient)?

Example (where traders do not observe their place in line):

$I = 15, R = 1.1, \gamma = 6$, independent and equally likely types.

Given the allocation rule α^*, if all other patient traders claim to be impatient, a patient trader is better off running (claiming to be impatient) if among the first 12, and is better off telling the truth if among the last 3. Overall, better off running.

Reducing the probability of being impatient below $\frac{1}{2}$ or increasing I makes a run equilibrium more likely.

Proposition (Green and Lin): If types are independent *and traders know their place in line*, α^* has a unique Bayesian NE, in which the efficient allocation obtains.

The proof is by iterated elimination of strictly dominated strategies. If you know that everyone after you will report truthfully, you want to report truthfully.

What if traders know their place in line but types are not independent?

Example: $I = 4, R = 2, \gamma = 6$

$p(0) = p(1) = p(3) = p(4) = \frac{\varepsilon}{4}$

$p(2) = 1 - \varepsilon$

For this example, if ε is very small, the optimal contract α^* will be similar to the optimal contract in the standard Diamond-Dybvig model where the fraction of impatient is known to be one half. This reflects the near certainty that $\theta = 2$.

Give the first two traders claiming to be impatient approximately $c^0 = 1.25$, planning to give the two patient traders approximately $c^1 = 1.5$.

Not surprisingly, α^* is truthfully implementable when ε is small.

The Example also has a *partial run equilibrium*:

The first two traders always report impatient, $\mu_i(\omega) = 0$ for all ω.

Traders 3 and 4 always tell the truth, $\mu_i(\omega) = \omega_i$ for all ω.

Reason for the partial run equilibrium:

Ennis and Keister prove a general result that the last two traders always report truthfully under α^*.

Consider a patient trader 2, after trader 1 reports impatient, $\mu_1 = 0$. She know that it is very likely that exactly two of the other traders are impatient.

There is approximately a $\frac{2}{3}$ chance that trader 1 is impatient, in which case trader 2 receives approximately $c^0 = 1.25$ by claiming to be impatient and approximately $c^1 = 1.5$ by claiming to be patient.

But there is approximately a $\frac{1}{3}$ chance that trader 1 is patient and misreports, in which case trader 2 receives approximately $c^0 = 1.25$ by claiming to be impatient and approximately $c^1 = 1.1$ by claiming to be patient.

Because of risk aversion, it is better for trader 2 to report impatient. Similar logic for trader 1.

Intuition: After trader 1 reports impatient, trader 2 is much more pessimistic about the types of traders 3 and 4 than the bank. The bank, by choosing a contract that assumes the truth telling equilibrium, is being far too generous to consumer 3 when he reports impatient.

With independent types, the bank and trader 3 will have the same beliefs about the types of traders 3 and 4.

Concluding Remarks

1. If we think that bank runs are a possibility, then even if α^* is truthfully implementable, the planner might want to choose a different contract. Either eliminate the run equilibrium, or adjust the consumptions to better insure those who would suffer.

Concluding Remarks

2. Suppose types are independent but utility depends on the type. Andolfatto, Nosal, and Wallace (*JET* 2007) show that a mechanism where traders observe the history of reports uniquely implements the planner's problem, with no run equilibrium.

However, there are more incentive compatibility constraints to satisfy when traders observe the history of reports, or their place in line, than when traders do not observe anything. Thus, giving more information to traders can eliminate bank runs, but the overall welfare can be lower.

Nosal and Wallace (2009) show that this is in fact the case for an example from Peck and Shell (2003). As long as the probability of a bank run is low enough, it can be better to take that risk.