“A Battle of Informed Traders and the Market Game Foundations for Rational Expectations Equilibrium”

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During the 19th century, Jacob Little, who was nicknamed the "Great Bear of Wall Street," would sell short shares that he did not own and then spread rumors about the insolvency of the company. After he had forced the price down, he would cover his short position.

In 1901 the managers of American Steel shorted the firm's stock and then closed its steel mills. When the price fell from \$60 to \$40, the managers covered their short positions and reopened the mills.

(examples taken from Allen and Gale (RFS 1992))

This paper makes two contributions:

1. Models dynamic manipulation of prices in a model with *fully rational* traders.

An informed trader who ultimately wants to sell might buy in period 1 and push up the price, thereby favorably influencing the price at which he sells in period 2.

If there are no noise traders to absorb losses, what will happen?

2. Provides a mechanism based on the Shapley-Shubik market game model, whose equilibrium allocation converges to the competitive rational expectations equilibrium.

The market structure is based on the Shapley-Shubik market game.

Think of a "trading post" where on post j traders place *bids* of a numeraire commodity (commodity money) and *offers* of commodity j.

Two goods per period: good x is "the good" being offered and good y is "money."

There are two periods of trading and utility only depends on the final holdings and the state of nature.

The Players

Bulls observe the state, are endowed with a positive amount of the good, and only care about money.

Bears observe the state, are endowed with a negative amount of the good, and only care about money.

Consumers are uninformed and their utility depends on consumption of both goods and the state of nature, θ.

Preview of Results:

When short-sale constraints are imposed, there is an equilibrium in which:

(i) in period 1, bears and consumers sell up to their short sale constraints, and bulls buy,

(ii) the period 1 price reveals θ, and the period 2 price equals the fully revealing REE price,

(iii) $p^1(\theta) > p^2(\theta)$, so bulls lose money on the goods that they first buy and then sell,

(iv) as the economy is replicated, the allocation converges to the REE allocation.

When short-sale constraints are eliminated, there is no type-symmetric fully revealing equilibrium. Bulls and bears each seek to push the price in opposite directions. But there is a non-revealing equilibrium corresponding to the non-revealing REE allocation.

Literature

1. Models of price manipulation based on the Kyle (1985) noise trader model: Vila (1989), Kumar and Seppi (1992), . . .

2. Allen and Gale (1992). A large trader can either be informed or an uninformed manipulator, and the uninformed manipulator makes profits. Price formation is not modeled explicitly. Allen, Litov, and Mei (2006) model attempts to corner the market, with an exogenous settlement price in the event of a corner.

3. Large literature on REE in general equilibrium with asymmetric information, starting with Radner (1979). All consumers know the equilibrium price as a function of the state, so when they see the price they update their information. In a fully revealing REE, all consumers maximize as if they know the state, we find the market clearing prices, and we verify that the resulting price function can be inverted. Paradoxes based on the lack of a price-formation process.

4. Reny and Perry (2006) show convergence to REE in a static auction model with unit demands.

5. Vives (2011a,b) models a static supply-function submission game, shows convergence to REE. The strategic choice is different: limit orders vs. market orders. Also, the present model considers the "pure common value case" where his revealing equilibrium breaks down.

6. Large literature on strategic market games: Shapley and Shubik (1977) and many others. Dubey, Geanakoplos, and Shubik (1987) use a two-period model, but where utility depends on consumption in each period. There are no large traders, so no strategic manipulation of prices. Prices reveal the state but the REE outcome does not obtain.

7. Forges and Minelli (1997) use a repeated game model with an infinite number of consumption periods, and show that the REE is obtained.

8. Hu and Wallace (2012) show convergence to REE in a two-period Shapley Shubik model with one consumption period (like the present paper), but some agents are assigned to period 1 and are not allowed to trade afterwards.

Model Details

The state is drawn from a continuous and strictly increasing c.d.f., $G(\theta)$, with support $[\underline{\theta}, \bar{\theta}]$.

The set of consumers is denoted by the unit interval, $C = [0, 1]$, where consumer $h \in C$ has the endowment vector, (ω_h^x, ω_h^y), and is a von Neumann-Morgenstern expected utility maximizer with the concave and quasi-linear Bernoulli utility function, $u_h(x_h, \theta) + y_h$ satisfying concavity, Inada, and single crossing.

There are n bulls and n bears, who care only about consumption of good y (as long as x consumption is non-negative). Denote the set of bulls as A_+ and the set of bears as A_-. Bulls begin the game with a positive endowment of good x, $\omega \geq 0$, and bears begin the game with a negative endowment of good x, $-\omega$.

The competitive economy has a fully revealing REE, where the price of good x in terms of good y is given by the strictly increasing and continuously differentiable function, $f(\theta)$. Denote the REE consumption for consumer h in state θ as $(\widehat{x}_h(\theta), \widehat{y}_h(\theta))$.

Consumer h's action set in period 1 is given by

$$\{(b_h^1, q_h^1) \in \Re_+^2 : q_h^1 \leq \omega_h^x, b_h^1 \leq \omega_h^y, q_h^1 b_h^1 = 0\}.$$

Bull i in state θ has an action set in period 1 given by

$$\{(b_i^1(\theta), q_i^1(\theta)) \in \Re_+^2 : q_i^1 b_i^1 = 0, q_i^1(\theta) \leq \overline{\omega} + \omega\},$$

Bear j in state θ has an action set in period 1 given by

$$\{(b_j^1(\theta), q_j^1(\theta)) \in \Re_+^2 : q_j^1 b_j^1 = 0, q_j^1(\theta) \leq \overline{\omega} - \omega\}.$$

Denoting a strategy profile for the entire game as σ, the price of good x in period 1 when the state is θ is given by

$$p^1(\theta, \sigma) = \frac{\sum_{i \in A_+} b_i^1(\theta) + \sum_{j \in A_-} b_j^1(\theta) + \int_{h \in C} b_h^1 dh}{\sum_{i \in A_+} q_i^1(\theta) + \sum_{j \in A_-} q_j^1(\theta) + \int_{h \in C} q_h^1 dh}.$$

In period 2, a consumer's bid and offer can depend on the period 1 price and the chosen period 1 action, and a bull's or a bear's bid and offer can depend on the period 1 action profile, s^1, and the observed state. We denote period 2 actions as $(b_h^2(p^1, b_h^1, q_h^1), q_h^2(p^1, b_h^1, q_h^1))$ for consumer h, $(b_i^2(\theta, s^1), q_i^2(\theta, s^1))$ for bull i, and $(b_j^2(\theta, s^1), q_j^2(\theta, s^1))$ for bear j.

Since we must evaluate sequential rationality, we will need notation for the price in period 2 in state θ under strategy profile σ, following an arbitrary action profile in period 1, s^1, which determines a period 1 price, p^1. The period 2 price is the sum of the bids divided by the sum of the offers, given by

$$p^2(\theta, \sigma; s^1) =$$

$$\frac{\sum_{i\in A_+} b_i^2(\theta, s^1) + \sum_{j\in A_-} b_j^2(\theta, s^1) + \int_{h\in C} b_h^2(p^1, b_h^1, q_h^1)dh}{\sum_{i\in A_+} q_i^2(\theta, s^1) + \sum_{j\in A_-} q_j^2(\theta, s^1) + \int_{h\in C} q_h^2(p^1, b_h^1, q_h^1)dh}$$

For consumer h, final consumption in state θ under strategy profile σ is given by

$$\begin{aligned} x_h(\theta,\sigma) &= \omega_h^x + \frac{b_h^1}{p^1(\theta,\sigma)} - q_h^1 + \\ & \frac{b_h^2(p^1,b_h^1,q_h^1)}{p^2(\theta,\sigma)} - q_h^2(p^1,b_h^1,q_h^1) \end{aligned}$$

$$\begin{aligned} y_h(\theta,\sigma) &= \omega_h^y - b_h^1 + q_h^1 p^1(\theta,\sigma) - \\ & b_h^2(p^1,b_h^1,q_h^1) + q_h^2(p^1,b_h^1,q_h^1) p^2(\theta,\sigma). \end{aligned}$$

For bull i and bear j, the final allocation, net of the endowment of good y, is given by

$$x_i(\theta, \sigma) = \omega + \frac{b_i^1(\theta)}{p^1(\theta, \sigma)} - q_i^1(\theta) + \frac{b_i^2(\theta, s^1)}{p^2(\theta, \sigma)} - q_i^2(\theta, s^1)$$

$$y_i(\theta, \sigma) = -b_i^1(\theta) + q_i^1(\theta)p^1(\theta, \sigma) - b_i^2(\theta, s^1) + q_i^2(\theta, s^1)p^2(\theta, \sigma)$$

$$x_j(\theta, \sigma) = -\omega + \frac{b_j^1(\theta)}{p^1(\theta, \sigma)} - q_j^1(\theta) + \frac{b_j^2(\theta, s^1)}{p^2(\theta, \sigma)} - q_j^2(\theta, s^1)$$

$$y_j(\theta, \sigma) = -b_j^1(\theta) + q_j^1(\theta)p^1(\theta, \sigma) - b_j^2(\theta, s^1) + q_j^2(\theta, s^1)p^2(\theta, \sigma).$$

The maximum possible offer by consumers and bears in period 1, denoted by $\overline{x}^1$, is given by

$$\overline{x}^1 = n(\overline{\omega} - \omega) + \int_{h \in C} \omega_h^x dh.$$

Solution concept is weak perfect Bayesian equilibrium (WPBE).

Consumer beliefs will assign probability one to a single state following any p^1, denoted by $\theta^e(p^1)$.

Proposition 1: *The following strategy profile and beliefs constitute a symmetric WPBE:*

$$\begin{aligned}
\text{bull } i \quad &: \quad b_i^1(\theta) = \frac{f(\theta)(\overline{x}^1 + \omega)}{n}, \quad q_i^1(\theta) = 0, \\
b_i^2(\theta, s^1) &= 0, \quad q_i^2(\theta, s^1) = \omega + \frac{b_i^1}{p^1} - q_i^1,
\end{aligned}$$

$$\begin{aligned}
\text{bear } j \quad &: \quad b_j^1(\theta) = 0, \qquad q_j^1(\theta) = \overline{\omega} - \omega, \\
b_j^2(\theta, s^1) &= (q_j^1 + \omega - \frac{b_j^1}{p^1}) f(\theta^e(p^1)), \quad q_j^2(\theta, s^1) = 0,
\end{aligned}$$

$$\begin{aligned}
\text{consumer } h \quad &: \quad b_h^1 = 0, \qquad q_h^1 = \omega_h^x, \\
b_h^2(p^1, b_h^1, q_h^1) &= \left[\widehat{x}_h(\theta^e(p^1)) + q_h^1 - \omega_h^x - \frac{b_h^1}{p^1}\right] f(\theta^e(p^1)), \\
q_h^2(p^1, b_h^1, q_h^1) &= 0.
\end{aligned}$$

$$\begin{aligned}
\theta^e(p^1) &= f^{-1}(\frac{p^1 \overline{x}^1}{\overline{x}^1 + \omega}) \quad \text{if} \quad p^1 \leq \frac{\overline{x}^1 + \omega}{\overline{x}^1} f(\bar{\theta}) \\
\theta^e(p^1) &= \bar{\theta} \quad \text{if} \quad p^1 > \frac{\overline{x}^1 + \omega}{\overline{x}^1} f(\bar{\theta})
\end{aligned}$$

Along the equilibrium path, prices are given by

$$p^1(\theta) = \left(\frac{\overline{x}^1 + \omega}{\overline{x}^1}\right) f(\theta) \text{ and}$$
$$p^2(\theta) = f(\theta).$$

Intuition:

–The $p^1(\theta)$ formula is found by substituting period 1 actions into the formula defining prices. Solving the $p^1(\theta)$ formula for θ yields $\theta^e(p^1)$.

–Consumers in period 2 believe that the state is θ^e and the price will be the REE price $f(\theta^e)$. They bid and offer so that consumption of good x is the REE quantity (unaffected by arbitrage profits: quasi-linear assumption).

–Bulls in period 2 offer their entire holdings of good x.

–Some algebra shows that bears exactly close out their positions in period 2 by following their strategy.

–Offering up to their short-sale limits in period 1 is sequentially rational for consumers and bears.

Intuition (continued):

–It turns out that bull i is indifferent as to his bid in period 1. Increasing his bid increases p^2 and his net sales revenue (from selling ω units in period 2), but increasing his bid also increases his arbitrage losses, and these effects exactly balance.

$$\begin{aligned} & \pi_i^{bull}(\theta, b_i^1, q_i^1) \\ = \; & -b_i^1 + q_i^1 p^{1,bull}(\theta, b_i^1, q_i^1) + \\ & \left[\omega + \frac{b_i^1}{p^{1,bull}(\theta, b_i^1, q_i^1)} - q_i^1\right] f(\theta^e(p^{1,bull}(\theta, b_i^1, q_i^1))). \end{aligned}$$

where

$$p^{1,bull}(\theta, b_i^1, q_i^1) = \frac{\frac{(n-1)}{n} f(\theta)(\overline{x}^1 + \omega) + b_i^1}{\overline{x}^1 + q_i^1}.$$

Bulls actually bid up the price in period 1 above the price in period 2, and lose money as a result as bears and consumers take the other side of the transactions. Still, if the bulls do not bid up the price, consumers would mistakenly think that the state is lower than it actually is.

This is reminiscent of Milgrom and Roberts (1982) on limit pricing. (Monopolist's period 1 price is below the static monopoly price, but it still reveals the cost type.)

Proposition 2: *Consider an r-fold replication of the economy. Then the equilibrium converges to the REE as $r \to \infty$, in the following sense. For all θ, $p^1(\theta)$ converges to the REE price, $p^2(\theta)$ is exactly the REE price, and the allocation uniformly converges to the REE allocation.*

It is interesting to note that the convergence result in Proposition 2 applies to the most paradoxical environment discussed in the REE literature, in which the net trades of all informed agents do not depend on the state of nature. Here, bulls sell ω units and bears buy ω units in all states.

The Model Without Short-Sale Restrictions

In the equilibrium of Proposition 1 as $\overline{\omega} \to \infty$, we approach the revealing REE but $b_i^1(\theta) \to \infty$ and $q_j^1(\theta) \to \infty$.

Without short-sale restrictions, it turns out that the resulting economy is unstable, with no revealing equilibrium. Loosely speaking, bulls and bears want to manipulate consumer beliefs in opposite directions, with no solution with finite bids and offers.

Proposition 3. *In the model without short-sale restrictions and $\omega > 0$, there does not exist an open-market, fully revealing, type-symmetric WPBE with bid and offer functions that are continuously differentiable in θ.*

There is a non-revealing REE with price p^{nr}. There is a non-revealing WPBE where only consumers trade in period 1, and prices on the equilibrium path are $p^1(\theta) = p^2(\theta) = p^{nr}$.

Concluding Remarks

It is not so easy to profitably manipulate prices when all traders are rational. Advocates of noise-trader models argue that we can think of noise traders as fully rational but constrained. Yes, but the models only look at particular constraints, like being forced to buy or sell in a particular period. Here, if we constrained consumers to buy or sell in a particular period the bulls and bears could profit as a result.

Convergence to REE is shown using a mechanism in which traders have free access to markets. The dynamic structure resolves one of the paradoxes associated with REE.

Without short-sale restrictions, there is a non-revealing but no revealing equilibrium.

Future work: (i) conditionally independent signals, (ii) endogenous and costly information acquisition, (iii) endogenous "initial" positions.