Chamley and Gale, "Information Revelation and Strategic Delay in a Model of Investment"

Econometrica 1994

Imagine that investment returns collapse and we enter a recession. After a while, some firms observe risky but profitable investment opportunities. Will succeed if investment returns are high again, fail otherwise.

Observing the opportunity is a signal that investment returns are high, but it might be even more profitable to wait and see if others invest. But if everyone waits, no one will learn anything and the recession continues.

Chamley-Gale model this situation. Pure informational externality, so investment payoff does not depend on others' actions.

Equilibrium features: strategic delay, risk of investment collapse (herd behavior), bias towards underinvestment.

The Model (Static Analysis)

N agents

n agents receive an investment opportunity–risky but positive expected profits.

investment opportunity is a real option–agent maintains the option to wait and invest later.

investment opportunity is private information–only revealed to the market when investment occurs.

type-1 agents have an opportunity, and type-0 agents do not.

Process Generating Investment Opportunities:

nature chooses n with probability $g_0(n)$

$pr(\text{type-1}|n) = \frac{n}{N}$

From Bayes' rule, the posterior probability of n (given type-1) is

$$g(n) = \frac{g_0(n)n}{\sum_{n'=0}^{N} g_0(n')n'}.$$

Payoffs for type-1 agents:

$v(n)$ denotes the expected payoff conditional on n agents with investment opportunities.

Assume that $v(n)$ is strictly increasing and is negative for some n in the support of $g_0(n)$.

Assume that investment is profitable, ex ante

$$V \equiv \sum_{n=0}^{N} g(n)v(n) > 0$$

Example: 2 equally likely states, conditional independence

$v_0 = -c$ (low state)

$v_1 = 1 - c$ (high state)

$pr(\text{type-1}|\text{state 1}) = pr(\text{type-0}|\text{state 0}) = \alpha$

Then we have:

$$\begin{aligned}
pr(n|\text{state 1}) &= \frac{N!}{n!(N-n)!}\alpha^n(1-\alpha)^{N-n} \\
pr(n|\text{state 0}) &= \frac{N!}{n!(N-n)!}(1-\alpha)^n\alpha^{N-n} \\
g_0(n) &= \frac{1}{2}pr(n|\text{state 1}) + \frac{1}{2}pr(n|\text{state 0})
\end{aligned}$$

Example (continued)

$v(n) = pr(\text{state } 1|n) - c$

From Bayes' rule, we can compute $v(n) =$

$$
\begin{aligned}
& \frac{\frac{N!}{n!(N-n)!}\alpha^n(1-\alpha)^{N-n}}{\frac{N!}{n!(N-n)!}\alpha^n(1-\alpha)^{N-n} + \frac{N!}{n!(N-n)!}(1-\alpha)^n\alpha^{N-n}} - c \\
= \; & \frac{1}{1 + \left(\frac{\alpha}{1-\alpha}\right)^{N-2n}} - c
\end{aligned}
$$

Ex ante payoffs are

$$V = \alpha - c$$

Suppose that the type-1 agents invest w.p. λ and type-0 agents cannot invest.

Let $g(n|k)$ denote the probability of n opportunities, given mixing probability λ and that k agents invest.

If an outside observer sees k out of N invest, her profits are

$$V(k) \equiv \sum_n g(n|k)v(n).$$

Lemma: Chamley and Gale prove that when $g(n)$ is non-degenerate and $\lambda > 0$, then $V(k)$ is strictly increasing in k.

The outside observer's ex ante expected profits, when she will observe k before deciding whether to invest, are

$$W(\lambda) = \sum_k p(k) \max[V(k), 0],$$

where $p(k)$ is the probability that k agents invest, given λ,

$$p(k) = \sum_n pr(k|n, \lambda)g(n).$$

The value of the option to delay investment until after observing k is $W(\lambda) - V$.

If $V(k) \geq 0$ for all k, then $W(\lambda) = V$.

If $V(k) < 0$ for some k, then $W(\lambda) > V$.

Proposition: $W(\lambda)$ is strictly increasing in λ when $W(\lambda) > V$.

Intuition is that for higher λ, k provides more precise information about n, so the option of observing k is more valuable.

proof sketch: Suppose $\lambda' < \lambda$. Think of n agents flipping a coin with heads probability λ, then the k who come up heads flip another coin, with heads probability λ'/λ. It is more informative to observe the outcome of the first coin flip than the noisier outcome of who flips heads twice. Therefore, $W(\lambda) > W(\lambda')$.

The Bayesian Extensive Game (with observable actions)

N players

Timing:

1. Nature provides agents with investment opportunities, which determines n.

2. In period $t \geq 1$, type-1 agents observe the history of previous investment and simultaneously decide whether to invest in period t or wait. (Type-0 agents will be passive.)

Agents who do not invest receive a payoff of zero. Agents who invest in period t receive a payoff of

$$\delta^{t-1} v(n),$$

where $\delta < 1$ is the discount factor.

Denote the number of agents who invest in period t as k_t, and a history for period t as

$$h_t = (k_1, ..., k_{t-1}).$$

Denote a history of arbitrary length as h.

A symmetric PBE is a mixing probability after each history, $\lambda(h)$, and beliefs about n, $\mu(n|h)$.

$V(h)$ denotes the expected payoff of investment.

$W(\lambda, h)$ denotes the undiscounted payoff of waiting, then investing next period if and only if investment is profitable, when others invest this period with probability λ.

$W^*(h)$ denotes the equilibrium (undiscounted) payoff from waiting.

Characterizing the symmetric PBE

Proposition: The one-step-property holds,

$$W^*(h) = W(\lambda(h), h) = \sum_k p(k|h)\,\mathrm{max}[V(h,k), 0].$$

Proof sketch: If $V(h,k) > 0$ holds, then we must have $\lambda(h,k) > 0$. Otherwise, for the next period in which the investment probability is positive, i.e. $\lambda(h,k,0,...,0) > 0$, agents willing to mix would be better off investing after (h,k). Also, if the investment probability is zero forever afterwards, some agent would be better off investing.

If $V(h,k) \leq 0$ holds, no one will ever invest.

Since it is optimal to invest after profitable (h,k) and never invest after unprofitable (h,k), the one-step-property follows.

Note: The one-step-property allows us to characterize the equilibrium mixing probability based on a comparison to the expected profits of making a final decision next period. It does not say that all agents who wait will make a final decision next period.

Theorem: There is a unique symmetric PBE outcome, based on the following 3 types of histories.

(a) If $V(h) \leq 0$ holds, then investment is unprofitable, and $\lambda(h) = 0$.

(b) If $V(h) \geq \delta \sum_n \mu(n|h) \max[v(n), 0]$, then investment is more profitable than waiting one period to learn all private information, and $\lambda(h) = 1$.

(c) Otherwise, $0 < \lambda(h) < 1$ is the unique value for which type-1 agents are indifferent, solving

$$V(h) = \delta W(\lambda, h)$$

Proof sketch: To show that there is a unique mixing probability for case (c), remember that $W(\lambda, h)$ is strictly increasing in λ.

For $\lambda = 0$, we have $W(\lambda, h) = V(h)$, so $V(h) > \delta W(\lambda, h)$.

For $\lambda = 1$, we have $W(\lambda, h) = \sum_n \mu(n|h) \max[v(n), 0]$, so $V(h) < \delta W(\lambda, h)$.

Thus, there is a unique λ for which $V(h) = \delta W(\lambda, h)$ holds.

Beliefs off the equilibrium path play no role, so any consistent beliefs can be chosen. The only events off the equilibrium path are:

1. Some agents invest when $\lambda(h) = 0$. This is never sequentially rational, for any continuation strategies.

2. We have $\lambda(h) = 1$, but some agents wait and invest later. Again, this is never sequentially rational, for any continuation strategies.

Properties of the Equilibrium

Because either type-1 agents always invest or mix in period 1, equilibrium profits are V, just as if no one could observe market activity. Strategic delay dissipates all of the gains from learning from market activity.

As $N \to \infty$, after histories involving mixing we have $\lambda(h) \to 0$, but the probabilities of $k_t = 0, 1, 2$, etc. converge to well defined limits. The rate of investment is independent of N for large economies.

As $N \to \infty$, once a non-negligible fraction of agents has invested we can infer n/N, by the law of large numbers. Remaining agents act with full information. Therefore, the equilibrium has a phase of "negligible" investment followed by either a surge or a collapse.

If $N \to \infty$ and $\delta \simeq 1$, the probability of unprofitable investment approaches zero, because agents do not invest until they are almost sure that the investment return is positive. However, there is a positive probability of investment collapse when the return is positive. Thus, overinvestment is impossible but underinvestment is possible.