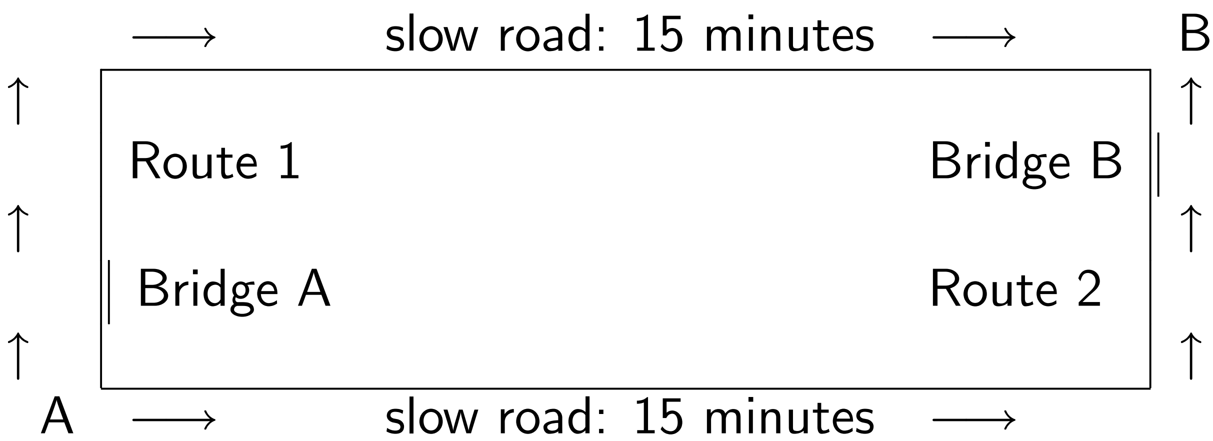
The (Braess) Transportation Paradox



1000 cars must commute from point A to point B.

East-West roads take 15 minutes, no matter how many cars are on the road.

North-South roads are fast, but the bridges can get congested.

A bridge with F cars takes $\frac{F}{100}$ minutes to cross.

Modeling the Commuting Problem as a Strategic Game

$N = \{1, 2, ..., 1000\}$

$A_i = \{1, 2\}$

For $j = 1, 2$, define the number of cars taking Route j

$$R_j = \sum_{i=1}^{1000} 1_{a_i = j}$$

$u_i(a) = -[15 + \frac{R_1}{100}] \quad \text{if} \quad a_i = 1$

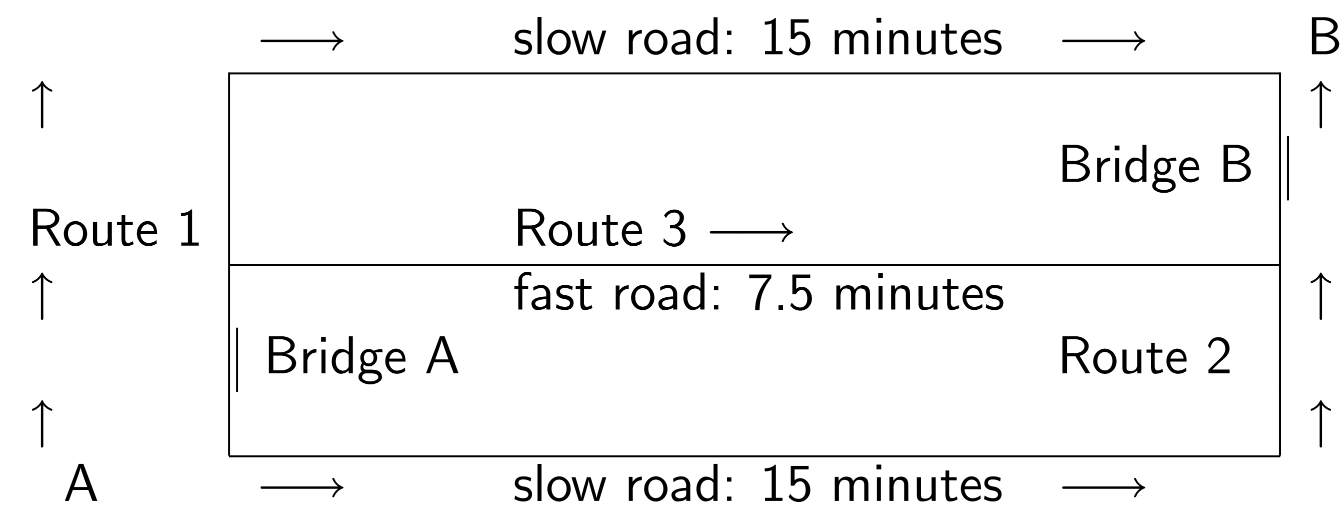
$u_i(a) = -[15 + \frac{R_2}{100}] \quad \text{if} \quad a_i = 2$

It is easy to see that any pure strategy Nash equilibrium has 500 cars taking each route, with a payoff of -20 (commuting time of 20 minutes). For example,

$a_i = 1$ if i is odd, and $a_i = 2$ if i is even.

Now suppose that we add a new East-West road, which takes 7.5 minutes, no matter how many cars use the road.

This adds another possible route, Route 3, which uses the faster East-West road but requires commuters to take both bridges. (Notice that no one would want to take this road the opposite direction.)



The new problem is modeled as

$N = \{1, 2, ..., 1000\}$

$A_i = \{1, 2, 3\}$

$u_i(a) = -[15 + \frac{R_1+R_3}{100}] \quad \text{if} \;\; a_i = 1$

$u_i(a) = -[15 + \frac{R_2+R_3}{100}] \quad \text{if} \;\; a_i = 2$

$u_i(a) = -[7.5 + \frac{R_1+R_3}{100} + \frac{R_2+R_3}{100}] \quad \text{if} \;\; a_i = 3$

(For example, the flow on Bridge A is $R_1 + R_3$)

If we can find numbers of cars on each route such that utilities are equalized, this is a NE distribution of route choices. This is because any deviation to another route cannot decrease that route's commuting time.

Based on symmetry of Route 1 and Route 2, we can solve by setting $R_1 = R_2 = r$ and $R_3 = 1000 - 2r$, then solving for r by setting the utility of Route 1 equal to the utility of Route 3.

Solving, we have: $R_1 = R_2 = 250$ and $R_3 = 500$.

The NE commuting time is 22.5, so everyone is worse off with the new option!

If travel time on Route 3 can be brought down to 5 minutes, everyone takes Route 3 and commuting time increases to 25 minutes!

This is a real phenomenon. When 42nd Street in New York City was closed for repairs, traffic flows improved dramatically. A new highway in Stuttgart, Germany caused commuting times to worsen until the highway was ripped up.