

Department of Economics
The Ohio State University
Econ 817–Game Theory

Homework #1–Due Wednesday October 14

Directions: Answer all questions, and be neat. If you discuss the questions in study groups, list the members of your study group below your name, and make sure that the writeup is your own work. Do not look at answer sheets from previous years' homeworks or the Osborne-Rubinstein solutions manual. Here is a hint for showing that there is no Nash equilibrium or that a given strategy profile is the unique Nash equilibrium: Show that for any "candidate" strategy profile, some player must have a profitable deviation. This approach is usually easier than constructing the best reply correspondence and showing there is no fixed point.

1. Consider the following cooperative game involving 5 political parties. The number of votes controlled by party i is denoted by x_i , where we have $x_1 = 45$, $x_2 = 45$, $x_3 = 3$, $x_4 = 3$, and $x_5 = 4$. Any coalition receiving a majority of the votes will be able to form a government and divide the surplus. Thus, the characteristic function is given by

$$\begin{aligned} v(S) &= 1 \quad \text{if } \sum_{i \in S} x_i \geq 51 \\ &= 0 \quad \text{otherwise.} \end{aligned}$$

- (a) Find the core of this game.
- (b) Find the Shapley value of this game.
- (c) What would be the Shapley value of the game in which the three smaller parties (3, 4, and 5) merged into a single party? Is it in their interest to merge?

2. O-R, exercise 56.4.

3. O-R, exercise 19.1.