
**Answer:** By the symmetry of the game, the set of rationalizable pure actions is the same for both players. Call it $Z$. Consider $m \equiv \inf(Z)$ and $M \equiv \sup(Z)$. Any best response of player $i$ to a belief about player $j$ (whose support is a subset of $Z$) maximizes $a_i(1 - a_i - E(a_j))$, or equivalently, it maximizes $a_i(1 - a_i - E(a_j))$. Thus, player $i$’s best response to a belief about player $j$ depends only on $E(a_j)$, which can be written as $B_i(E(a_j)) = (1 - E(a_j))/2$. Because $m \leq E(a_j) \leq M$ must hold, $a_i \in B_i(E(a_j))$ implies $a_i \in [(1 - M)/2, (1 - m)/2]$. By the best response property of the rationalizable set, we have $m \in [(1 - M)/2, (1 - m)/2]$ and $M \in [(1 - M)/2, (1 - m)/2]$. Therefore, we have

$$m \geq \frac{1 - M}{2} \quad \text{and} \quad (1)$$

$$M \leq \frac{1 - m}{2}. \quad (2)$$

It follows from (1) and (2) that $m \geq M$ holds, which can only occur if $m = M$. From (1) and (2), we have $m = M = 1/3$. Therefore, the only rationalizable strategy is the unique Nash equilibrium strategy, $a_i = 1/3$.

2. O-R, exercise 76.1.

**Answer:** The simplest example, in which it is common knowledge that two players have different posteriors about some event $A$, is the following. There are two states, with prior probability 1/2 for each state. $\Omega = \{1, 2\}$ and $p(1) = p(2) = 1/2$. Player 1 cannot distinguish between the two states, $\varphi_1 = \{\{1, 2\}\}$, and player 2 can distinguish between the two states, $\varphi_2 = \{\{1\}, \{2\}\}$. Therefore, the meet of the two information structures is $\varphi_1 \wedge \varphi_2 = \{\{1, 2\}\}$. Let $A = \{1\}$. At $\omega = 1$, player 1’s posterior is 1, and player 2’s posterior is 1/2. At $\omega = 2$, player 1’s posterior is 0, and player 2’s posterior is 1/2. Because posteriors are different at all states, it is common knowledge that posteriors are different.

Let $E = \{\omega' : q_1(\omega') > q_2(\omega')\}$. Suppose $E$ is common knowledge at $\omega$. Let $M$ be the element of $\varphi_1 \wedge \varphi_2$ containing $\omega$. Then $M = \bigcup_j P^i_j$, where we
have the union of disjoint elements of \( \wp_1 \), and \( M = \bigcup_j P_j^1 \), where we have the union of disjoint elements of \( \wp_2 \).

Because \( E \) is common knowledge at \( \omega \), we must have \( q_1(\omega') > q_2(\omega') \) for all \( \omega' \in M \).

Therefore, for all \( P_j^1 \subseteq M \), and all \( P_j^2 \subseteq M \), we have

\[
\frac{pr(A \cap P_j^1)}{pr(P_j^1)} > \frac{pr(A \cap P_j^2)}{pr(P_j^2)}
\]

Cross multiplying, \( pr(P_j^2)pr(A \cap P_j^1) > pr(P_j^1)pr(A \cap P_j^2) \).

Summing over (disjoint) \( P_j^1 \subseteq M \), we have \( pr(P_j^2)pr(A \cap M) > pr(P_j^1)pr(A \cap M) \).

Summing over (disjoint) \( P_j^2 \subseteq M \), we have \( pr(M)pr(A \cap M) > pr(M)pr(A \cap M) \), a contradiction.

3. O-R, exercise 103.2.

**Answer:** The game is defined by

\( N = \{1, 2\}, \ H = \{\text{stop}, \text{continue}\} \cup \{(\text{continue}, y) : y \in Z \times Z\} \), where \( Z \) is the set of nonnegative integers.

\( P(\emptyset) = 1 \) and \( P(\text{continue}) = \{1, 2\} \).

To find the subgame perfect equilibria, first consider the subgame following “continue.” If one of the players chooses a positive integer, then the other player can increase her payoff by choosing a larger integer, so this is not consistent with equilibrium. However, the subgame is in equilibrium if both players choose zero, \( y = (0, 0) \). Given that the only equilibrium of the subgame is \( (0, 0) \), player 1 receives a payoff of 1 by choosing “stop,” and a payoff of 0 by choosing “continue.” Therefore, the unique subgame perfect equilibrium is given by \( ((\text{stop}, 0), 0) \).