Large Market Games with Demand Uncertainty

James Peck[1]

Department of Economics

The Ohio State University

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[1] 1945 North High Street, Columbus, Ohio 43210. Phone: 614-292-0182, Fax: 614-292-3906, peck.33@osu.edu. I would like to thank Matt Jackson and David Schmeidler for helpful conversations. Any errors are my responsibility.

Abstract

We consider a market game with a continuum of consumers, where the measure of each type is stochastic. Nature selects the set of active consumers, who make bids and offers on $\ell - 1$ spot market trading posts. Existence of type-symmetric Nash equilibrium is proven. When facing price uncertainty, best responses are unique, and a Nash equilibrium to the sell-all game is typically not a Nash equilibrium to the original game. Under plausible circumstances, consumers strictly prefer to be on one side of the market. *Journal of Economic Literature* Classification Numbers: C7, D4, D8.

1 Introduction

The Shapley-Shubik market game model has been and will continue to be an important tool for understanding competitive markets. When information is symmetric and the number of consumers is large, open-market Nash equilibrium approximates a competitive equilibrium.[2] Because prices are determined explicitly from the bids and offers of consumers, there is a potential distinction between a player being too small to influence the price and a player being a price-taker. With asymmetric information, a player could be uncertain of the bids and offers of others, and therefore not know the price, while being unable to influence the price with her own bids and offers.[3] As a result, the model avoids the paradoxical conclusions of competitive rational expectations models when information is asymmetric. Thus, the market game model with asymmetric information might be useful in areas, such as finance and macroeconomics, where the price-taking assumption has led to paradox.

The purpose of this paper is to study the existence of equilibrium, and to characterize the equilibrium strategies as far as possible, in a market game with a continuum of consumers, and where the measure of each type of active consumer is stochastic. Think of nature as selecting the set of active consumers out of a larger set of potentially active consumers. Then the active consumers choose how much of commodities 1 to $\ell - 1$ to offer on the $\ell - 1$ trading posts; bids are made with the numeraire commodity, ℓ. The price of commodity j is a spot market price, specifying noncontingent delivery of commodity j. The realization of spot market prices depends on the realization of demand.[4] The usual formulation specifies uncertainty about endowments, preferences, or technology, rather than uncertainty about who will show up at the market. We specify uncertainty about the measure of active consumers, because it is a major concern to participants on real world markets, and because it arises naturally if we take seriously the notion that traders must arrive at a "trading post" in order to be counted.[5]

[2] See Dubey and Shubik [3], Jaynes, Okuno, and Schmeidler [9], and Postlewaite and Schmeidler [14]. Peck and Shell [11] show that the Nash equilibrium allocation can be arbitrarily close to a competitive equilibrium allocation when short sales are allowed, even with a small number of consumers. Ghosal and Morelli [6] look at myopic retrading, and show convergence to a Pareto optimum.

[3] See [1] and [7]. In [10], players are small but do not know the price, because the endowments supplied to the market are not known at the time players bid.

[4] There are missing markets, in the sense that consumers have no opportunity to insure themselves against the risk of being on the long side of the (spot) market. Demand uncertainty implies asymmetric information, because an active consumer knows that she is active, but the rest of the market does not know that she is active.

[5] However, uncertainty about endowments or preferences can be accomodated in the present model, by interpreting a type to include the realization of one's endowment or preferences. See the concluding remarks for a discussion.

The existence of type-symmetric Nash equilibrium is shown in Proposition (4.2). Although existence of equilibrium, with a continuum of consumers, is hardly surprising, a number of obstacles must be overcome. The approach adopted by Peck, Shell, and Spear [13], for the model without uncertainty, does not work when consumers face price uncertainty. When prices are known, there is a continuum of best responses for a given player yielding the same consumption bundle, characterized by the amount of "wash sales." It follows that an equilibrium to the game in which all endowments are offered for sale, for which existence is relatively easy to prove, is also an equilibrium to the original game. With demand uncertainty, however, an equilibrium to the sell-all game is not an equilibrium to the original game, in general. The approach adopted here is to put ε bids and offers on each market, and take limits as ε approaches zero, as in Dubey and Shubik [3]. Their analysis must be adapted to the current model and extended, in order to show that the limiting bids and offers cannot be converging to zero.

We derive the following results about best responses and equilibrium strategies. Unless the equilibrium distribution of prices is degenerate, a consumer's best response is uniquely determined. Thus, the multiplicity of best responses in the certainty model is not robust to the introduction of (even) small amounts of demand uncertainty. Under very plausible circumstances, consumers respond to the equilibrium distribution of prices by restricting themselves to one side of the market. When there are only two commodities, Proposition (5.1) provides a simple sufficient condition to guarantee that a consumer will be on one side of the market. This condition is satisfied for any equilibrium if the consumer is sufficiently risk averse, the demand uncertainty is nontrivial, and the initial allocation is not Pareto optimal.

2 Demand Uncertainty

We consider an exchange economy with n types of consumers and ℓ commodities. Consumers are competitive, in the sense of being unable to influence prices. However, as we shall see, consumers are not price takers, because consumers do not know the aggregate bids and offers of others when they submit their own bids and offers. To capture these ideas, we represent the set of consumers of each type as a continuum. For $i = 1, ..., n$, let the set of "potentially active" consumers of type i be represented by the interval, $[i-1, i]$. From the set of potentially active consumers, $[0, n]$, nature selects a Lebesgue measurable subset of active consumers, who play the market game. Since our analysis centers exclusively on type-symmetric Nash equilibrium, and potential deviations therefrom, the only relevant uncertainty concerns the aggregate measure of each type of consumer. Let α_i denote the Lebesgue measure of active type-i consumers, and let α be given by $\alpha = (\alpha_1, ..., \alpha_n)$. We assume the existence of density functions for α, conditional on being an active consumer of type i, denoted by $f_i(\cdot)$, which are assumed to be strictly positive over the support, $\{\alpha : \underline{\alpha}_i \leq \alpha_i \leq \overline{\alpha}_i$ for $i = 1, ..., n\}$. We assume that $\overline{\alpha}_i > \underline{\alpha}_i > 0$ holds for $i = 1, ..., n$.[6]

For $t \in [0, 1]$, let $x_{i,t}(\alpha) \in \Re^{\ell}_{+}$ denote the consumption vector of consumer t of type i in aggregate demand state α. The consumption of commodity j by a consumer (i, t) in state α is denoted as $x^j_{i,t}(\alpha)$. The endowment vector of each consumer of type i is denoted by $\omega_i \in \Re^{\ell}_{++}$. The Bernoulli utility function of a type-i consumer is denoted by u_i, and each consumer is a von Neumann-Morgenstern expected utility maximizer. We make the following assumptions on u_i.

Assumption 1: For each i, the Bernoulli utility function u_i is strictly increasing, strictly concave, and twice continuously differentiable.[7] For all $j = 1, ..., \ell$ and all $\overline{x} > 0$, there exists $\underline{U} > 0$ such that

$$\frac{\partial u_i(x_i)}{\partial x^j_i} \geq \underline{U} \tag{1}$$

holds for all x_i satisfying $x^j_i \leq \overline{x}$. For all $j = 1, ..., \ell$ and all $\underline{x} > 0$, there exists $\overline{U} < \infty$ such that

$$\frac{\partial u_i(x_i)}{\partial x^j_i} \leq \overline{U} \tag{2}$$

holds for all x_i satisfying $x^j_i \geq \underline{x}$.

Notice that Assumption 1 is trivially satisfied if the utility function is separable.

3 The Market Game, Γ

There are $\ell - 1$ trading posts. On trading post j, active consumers offer to sell commodity j and bid with commodity ℓ. Inactive consumers are assumed to offer and bid zero. We denote consumer (i, t)'s offer of commodity j and bid for commodity j, *conditional on being active*, as $q^j_{i,t}$ and $b^j_{i,t}$, respectively.

[6] Because there is aggregate uncertainty, being active conveys information, so different types might have different beliefs about α, even if they started with common priors. We have in mind the following process. First, nature draws α from an unconditional density, f. Next, for $i = 1, ..., n$, nature chooses a set of active consumers, whose measure is α_i. This selection is done such that each "potential" type-i consumer is equally likely to be chosen.

[7] Our results continue to hold for utility functions defined only on $\Re^{\ell}_{+}$, where utility converges to negative infinity as some x^j_i approaches zero (for example, log utility). We can always truncate the utility function at a sufficiently low enough level that the truncation does not affect the equilibrium.

Also, let $q_{i,t} = (q_{i,t}^1, ..., q_{i,t}^{\ell-1})$ and $b_{i,t} = (b_{i,t}^1, ..., b_{i,t}^{\ell-1})$ denote consumer (i,t)'s offers and bids, respectively. A consumer's offer cannot exceed her endowment of the commodity in question, and the total amount of commodity ℓ bid on all posts cannot exceed her endowment of commodity ℓ. The strategy set of consumer (i,t) is then given by

$$S_{i,t} = \left\{ (q_{i,t}, b_{i,t}) \in \Re_+^{2(\ell-1)} : q_{i,t}^j \leq \omega_i^j \text{ for } j < \ell \text{ and } \sum_{j=1}^{\ell-1} b_{i,t}^j \leq \omega_i^\ell \right\}.$$

We will sometimes denote the strategy of consumer (i,t) by $\sigma_{i,t}$.

When strategies are type-symmetric, the total amount of commodity j offered on trading post j $(j \neq \ell)$ and the total amount of commodity ℓ bid on trading post j in state α, $Q^j(\alpha)$ and $B^j(\alpha)$, are given by

$$Q^j(\alpha) = \sum_{i=1}^{n} \alpha_i q_i^j \quad \text{and} \tag{3}$$

$$B^j(\alpha) = \sum_{i=1}^{n} \alpha_i b_i^j. \tag{4}$$

We define the price of commodity j in state α to be

$$p^j(\alpha) = \frac{B^j(\alpha)}{Q^j(\alpha)}. \tag{5}$$

Thus, the price $p^j(\alpha)$ is the spot market price of commodity j, in terms of commodity ℓ.[8]

The outcome function or allocation rule is determined as follows. The allocation for *active* consumer (i,t) in state α is given by

$$x_{i,t}^j(\alpha) = \omega_i^j - q_{i,t}^j + \frac{b_{i,t}^j}{p^j(\alpha)} \quad \text{for } j < \ell \tag{6}$$

$$x_{i,t}^\ell(\alpha) = \omega_i^\ell - \sum_{j=1}^{\ell-1} b_{i,t}^j + \sum_{j=1}^{\ell-1} p^j(\alpha) q_{i,t}^j\ , \tag{7}$$

with the convention that $0/0 = 0$.[9]

Expected utility, while concave in consumption, is not necessarily jointly concave in bids and offers.

[8] Considering strategies that are not type-symmetric might allow the price to depend on the identities of the active traders, and not just the aggregate demand state, α. We would need a sigma-algebra over the subsets of $[0, n]$. The simplest way to ensure that the game is well defined would be to cancel trade if the bids or offers are not integrable. Introducing this machinery would not affect our analysis of type-symmetric Nash equilibrium. See [2] for a discussion and analysis of a market game with a continuum of players.

[9] See [13] for an interpretation.

Dubey and Shubik [3] and Peck, Shell, and Spear [13] solve this problem by observing that the consumption possibilities set is convex, which implies a unique consumption bundle that is a best-response to the bids and offers of other consumers. Convexity of the consumption possibilities set, for the present model with demand uncertainty, does not follow immediately from previous papers. Here, a bid or offer is placed on many spot-market trading posts simultaneously. For example, a bid $b_{i,t}^j$ is made for all state contingent commodities (j, α) such that consumer (i, t) is active in state α. However, due to the fact that an individual cannot affect prices, the consumption possibilities set is indeed convex.

Lemma 3.1: *Fix the (type-symmetric) offers and bids of all consumers except (i, t), which determines $p^j(\alpha)$ for each j and α. Assume that $p^j(\alpha)$ is bounded above zero and below infinity for each j and α. Suppose $\widehat{\sigma}_{i,t} \equiv (\widehat{q}_{i,t}, \widehat{b}_{i,t}) \in S_{i,t}$ yields consumption $\widehat{x}_{i,t}(\alpha)$ in state α, and $\widetilde{\sigma}_{i,t} \equiv (\widetilde{q}_{i,t}, \widetilde{b}_{i,t}) \in S_{i,t}$ yields consumption $\widetilde{x}_{i,t}(\alpha)$ in state α. Then for all λ, $0 \leq \lambda \leq 1$, the strategy $\lambda\widehat{\sigma}_{i,t} + (1-\lambda)\,\widetilde{\sigma}_{i,t}$ is contained in $S_{i,t}$ and yields consumption $\lambda\widehat{x}_{i,t}(\alpha) + (1-\lambda)\widetilde{x}_{i,t}(\alpha)$ in state α.*

Proof. The fact that $\lambda\widehat{\sigma}_{i,t} + (1-\lambda)\,\widetilde{\sigma}_{i,t}$ is contained in $S_{i,t}$ is obvious. From (6), the resulting consumption for $j < \ell$ is

$$\begin{aligned} & \omega_i^j - [\lambda\widehat{q}_{i,t}^j + (1-\lambda)\widetilde{q}_{i,t}] + \frac{\lambda\widehat{b}_{i,t}^j + (1-\lambda)\widetilde{b}_{i,t}}{p^j(\alpha)} \\ = \;& \lambda\left[\omega_i^j - \widehat{q}_{i,t}^j + \frac{\widehat{b}_{i,t}^j}{p^j(\alpha)}\right] + (1-\lambda)\left[\omega_i^j - \widetilde{q}_{i,t}^j + \frac{\widetilde{b}_{i,t}^j}{p^j(\alpha)}\right] \\ = \;& \lambda\widehat{x}_{i,t}^j(\alpha) + (1-\lambda)\widetilde{x}_{i,t}^j(\alpha). \end{aligned}$$

From (7), the resulting consumption of commodity (ℓ, α) is given by

$$\begin{aligned} & \omega_i^\ell - \sum_{j=1}^{\ell-1}[\lambda\widehat{b}_{i,t}^j + (1-\lambda)\widetilde{b}_{i,t}] + \sum_{j=1}^{\ell-1}p^j(\alpha)[\lambda\widehat{q}_{i,t}^j + (1-\lambda)\widetilde{q}_{i,t}] \\ = \;& \lambda\left[\omega_i^\ell - \sum_{j=1}^{\ell-1}\widehat{b}_{i,t}^j + \sum_{j=1}^{\ell-1}p^j(\alpha)\widehat{q}_{i,t}^j\right] + (1-\lambda)\left[\omega_i^\ell - \sum_{j=1}^{\ell-1}\widetilde{b}_{i,t}^j + \sum_{j=1}^{\ell-1}p^j(\alpha)\widetilde{q}_{i,t}^j\right] \\ = \;& \lambda\widehat{x}_{i,t}^\ell(\alpha) + (1-\lambda)\widetilde{x}_{i,t}^\ell(\alpha). \qquad \blacksquare \end{aligned}$$

Proposition 3.2: *Fix the (type-symmetric) offers and bids of all consumers except (i, t), which determines $p^j(\alpha)$ for each j and α. Assume that $p^j(\alpha)$ is bounded above zero and below infinity for each j and α.*

Then the set of best responses for consumer (i, t) is convex. Furthermore, suppose that for each $j < \ell$, $p^j(\alpha)$ depends nontrivially on α. Then the best response is unique.

Proof. Lemma (3.1) shows that the consumption possibilities set for consumer (i, t) is convex. Since the utility function,

$$V_{i,t}(x_{i,t}) = \int_{\alpha} u_{i,t}\left(x_{i,t}(\alpha)\right) f_i(\alpha) d\alpha,$$

is strictly concave in state contingent consumption, it follows that the expected utility maximizing state-contingent consumption is uniquely determined for almost all states, α. Let $\widehat{\sigma}_{i,t} \equiv (\widehat{q}_{i,t}, \widehat{b}_{i,t}) \in S_{i,t}$ and $\widetilde{\sigma}_{i,t} \equiv (\widetilde{q}_{i,t}, \widetilde{b}_{i,t}) \in S_{i,t}$ be arbitrary best responses for consumer (i, t). Lemma (3.1) shows that a convex combination of $\widehat{\sigma}_{i,t}$ and $\widetilde{\sigma}_{i,t}$ yields a convex combination of the consumption bundles, $\widehat{x}_{i,t}$ and $\widetilde{x}_{i,t}$, and therefore is weakly preferred to at least one of the bundles. Therefore, the convex combination of $\widehat{\sigma}_{i,t}$ and $\widetilde{\sigma}_{i,t}$ must also be a best response.

For almost all states, α, the optimal consumption bundle is the same for $\widehat{\sigma}_{i,t}$ and $\widetilde{\sigma}_{i,t}$. From (6), we have

$$\frac{\widehat{b}^j_{i,t}}{p^j(\alpha)} - \widehat{q}^j_{i,t} = \frac{\widetilde{b}^j_{i,t}}{p^j(\alpha)} - \widetilde{q}^j_{i,t} \tag{8}$$

for all $j < \ell$. Rearranging (8), we have for almost all states, α, and all $j < \ell$,

$$p^j(\alpha)[\widehat{q}^j_{i,t} - \widetilde{q}^j_{i,t}] = [\widehat{b}^j_{i,t} - \widetilde{b}^j_{i,t}]. \tag{9}$$

From (9), we see that there are two possibilities. One possibility is that $\widehat{q}^j_{i,t} = \widetilde{q}^j_{i,t}$ and $\widehat{b}^j_{i,t} = \widetilde{b}^j_{i,t}$ hold for all $j < \ell$, which implies $\widehat{\sigma}_{i,t} = \widetilde{\sigma}_{i,t}$, so that the best response is unique. The other possibility is that, for some $j < \ell$, $p^j(\alpha)$ depends trivially on α. That is, for almost all states, α, $p^j(\alpha)$ is constant. ■

Proposition (3.2) indicates that, unless consumers face no uncertainty about prices, best responses are uniquely determined. On the other hand, for the case of certainty, consumers can choose from a continuum of best responses by augmenting net trades with "wash sales." Thus, the indeterminacy of best responses is not robust to the introduction of demand uncertainty, even in small amounts. The intuition is that consumers face a distribution of spot-market prices for a particular physical commodity, so that a wash sale that maintains consumption for one price realization changes consumption for other price realizations.

Without knowing the price, it is impossible to augment bids and offers so that they always cancel each other out.

4 Existence of Type-Symmetric Bayes-Nash Equilibrium

Our symmetry assumptions imply that all type-i consumers have the same beliefs about aggregate demand, conditional on being active. It is natural to ask whether a nontrivial type-symmetric Bayes-Nash equilibrium necessarily exists. One problem to be overcome is establishing that the appropriate best-response mapping is well-defined, and that the fixed point is nontrivial (with positive bids and offers). Peck, Shell, and Spear [13] first show existence for the "sell-all" game in which consumers are forced to offer their entire endowments, and then show that an equilibrium of the sell-all game is also an equilibrium of the original game. To guarantee that best responses are well-defined, the mapping is adjusted to require bids to be positive, but this constraint is shown not to bind at the fixed point. We cannot use this approach here, because a best response that is constrained to offer one's entire endowment is not necessarily a best response when the consumer is unconstrained, as indicated by Proposition (3.2) above.

The existence argument adopted here is similar to the one of Dubey and Shubik [3]. Artificial bids and offers of ε are placed on each market, and a type-symmetric Bayes-Nash equilibrium is shown to exist for the game, Γ^{ε}. Corresponding prices are shown to be uniformly bounded above zero and below infinity, for a sequence of equilibria as ε approaches zero. From the maximum theorem, the limiting strategies form a type-symmetric Bayes-Nash equilibrium for Γ. It is then shown that the limiting bids and offers cannot all be zero, unless autarky is Pareto optimal.[10]

Definition 4.1: The game, Γ^{ε}, is defined to be exactly the game, Γ, with the exception that an offer and bid of $\varepsilon > 0$ are exogenously placed on each trading post. That is, equation (5) is replaced with

$$p^{j,\varepsilon}(\alpha) = \frac{B^j(\alpha) + \varepsilon}{Q^j(\alpha) + \varepsilon}. \tag{10}$$

[10] Dubey and Shubik [3] omit the argument that the limiting equilibrium is not the no-trade equilibrium. Our argument relies on the fact that consumers have no effect on prices, so that any positive ε is large relative to the size of a consumer. See [8] for a different context in which this argument can be applied.

We insert the superscript, ε, into the notation for offers, bids, prices, and consumptions to clarify that the game under consideration is Γ^ε rather than Γ. Our focus is on type-symmetric Bayes-Nash equilibrium. When, for $i = 1, ..., n$, all consumers of type i choose the strategy $\sigma_i^\varepsilon = (q_i^\varepsilon, b_i^\varepsilon)$, only the aggregate realizations of demand in each state are important, and equation (10) can be rewritten as

$$p^{j,\varepsilon}(\alpha) = \frac{\sum_{i=1}^n \alpha_i b_i^{j,\varepsilon} + \varepsilon}{\sum_{i=1}^n \alpha_i q_i^{j,\varepsilon} + \varepsilon}. \tag{11}$$

The expected utility of consumer (i,t), as a function of the type-symmetric strategies chosen by everyone else, σ^ε, and her own strategy, $\overline{\sigma}_{i,t}^\varepsilon$, is denoted by $V_{i,t}^\varepsilon(\overline{\sigma}_{i,t}^\varepsilon, \sigma^\varepsilon)$, given by

$$V_{i,t}^\varepsilon(\overline{\sigma}_{i,t}^\varepsilon, \sigma^\varepsilon) = \int_\alpha u_{i,t}\left(..., \omega_i^j - \overline{q}_{i,t}^{j,\varepsilon} + \frac{\overline{b}_{i,t}^{j,\varepsilon}}{p^{j,\varepsilon}(\alpha)}, ..., \omega_i^\ell - \sum_{j=1}^{\ell-1} \overline{b}_{i,t}^{j,\varepsilon} + \sum_{j=1}^{\ell-1} p^{j,\varepsilon}(\alpha)\overline{q}_{i,t}^{j,\varepsilon}\right) f_i(\alpha) d\alpha. \tag{12}$$

Notice that Lemma (3.1) and Proposition (3.2) continue to hold for Γ^ε, where prices are now written as a function of α, given by equation (11). In particular, we know that the best response correspondence is convex valued.

Proposition 4.2: *Under assumption 1 and our other maintained assumptions, there exists a nontrivial type-symmetric Bayes-Nash equilibrium for* Γ.

Proof. We first show that there exists a nontrivial type-symmetric Bayes-Nash equilibrium for Γ^ε. Denote the best response correspondence for consumer (i,t) as $\gamma_{i,t}^\varepsilon(\sigma^\varepsilon)$, given by

$$\gamma_{i,t}^\varepsilon(\sigma^\varepsilon) = \arg\max V_{i,t}^\varepsilon(\overline{\sigma}_{i,t}^\varepsilon, \sigma^\varepsilon).$$

Given our symmetry assumptions, $V_{i,t}^\varepsilon(\overline{\sigma}_{i,t}^\varepsilon, \sigma^\varepsilon)$ and $\gamma_{i,t}^\varepsilon(\sigma^\varepsilon)$ are independent of t, so we denote the best response correspondence of type-i consumers by $\gamma_i^\varepsilon(\sigma^\varepsilon)$. For each positive ε, prices are bounded above zero and below infinity, so $V_i^\varepsilon(\overline{\sigma}_i^\varepsilon, \sigma^\varepsilon)$ is continuous. Since the strategy sets are compact, it follows that $\gamma_i^\varepsilon(\sigma^\varepsilon)$ is well-defined for each i, ε. It follows from Proposition (3.2) that $\gamma_i^\varepsilon(\sigma^\varepsilon)$ is convex valued. Let $\gamma^\varepsilon(\sigma^\varepsilon) = (\gamma_1^\varepsilon(\sigma^\varepsilon), ..., \gamma_n^\varepsilon(\sigma^\varepsilon))$ hold.

We know that the strategy sets are compact and convex, and that γ^ε is convex-valued. The conditions for Kakutani's fixed point theorem will be satisfied if we can demonstrate that γ^ε is upper hemicontinuous. Suppose that we have a sequence of type-symmetric strategies, $\sigma^{\varepsilon,m} \to \sigma^{\varepsilon,*}$ and (for some i) a sequence of best responses, $\overline{\sigma}_i^{\varepsilon,m} \to \overline{\sigma}_i^{\varepsilon,*}$, such that we have

$$\begin{aligned} \overline{\sigma}_i^{\varepsilon,m} &\in \gamma_i^{\varepsilon}(\sigma^{\varepsilon,m}) \quad \text{for all } m\text{, and} \\ \overline{\sigma}_i^{\varepsilon,*} &\notin \gamma_i^{\varepsilon}(\sigma^{\varepsilon,*}). \end{aligned}$$

Clearly, $\overline{\sigma}_i^{\varepsilon,*}$ is a feasible strategy for type i. Since it is not a best response to $\sigma^{\varepsilon,*}$, we have, for some feasible $\widehat{\sigma}_i$,

$$V_i^{\varepsilon}(\widehat{\sigma}_i, \sigma^{\varepsilon,*}) > V_i^{\varepsilon}(\overline{\sigma}_i^{\varepsilon,*}, \sigma^{\varepsilon,*}).$$

By continuity of V_i^{ε}, we have, for sufficiently large m,

$$V_i^{\varepsilon}(\widehat{\sigma}_i, \sigma^{\varepsilon,m}) > V_i^{\varepsilon}(\overline{\sigma}_i^{\varepsilon,m}, \sigma^{\varepsilon,m}),$$

contradicting the fact that $\overline{\sigma}_i^{\varepsilon,m} \in \gamma_i^{\varepsilon}(\sigma^{\varepsilon,m})$ must hold. This establishes upper hemicontinuity.

Applying Kakutani's fixed point theorem, there exists $\sigma^{\varepsilon} = (\sigma_1^{\varepsilon}, ..., \sigma_n^{\varepsilon})$ such that $\sigma_i^{\varepsilon} \in \gamma_i^{\varepsilon}(\sigma^{\varepsilon})$ for $i = 1, ..., n$. It follows that σ^{ε} is a Nash equilibrium for Γ^{ε}.

We now consider a convergent subsequence of equilibria, $\{\sigma^{\varepsilon}\}$, where σ^{ε} is a Nash equilibrium of Γ^{ε} and $\varepsilon \to 0$.[11] Let us show that all limiting prices are uniformly bounded above zero and below infinity.

Case 1: Suppose we have $p^{j,\varepsilon}(\alpha) \to 0$ for some (j, α).

Then it follows from (11), and the fact that the lower support for α is strictly positive, that $b_i^{j,\varepsilon} \to 0$ for all i and $p^{j,\varepsilon}(\alpha) \to 0$ for all α. Fix a type, i.

Case 1a: For some α, we have $x_i^{\ell,\varepsilon}(\alpha) \to 0$.

From the allocation rule, (7), we have

$$\begin{aligned} \sum_{j' \neq j, \ell} b_i^{j',\varepsilon} &\to \omega_i^{\ell} \quad \text{and} \\ \sum_{j' \neq \ell} p^{j',\varepsilon}(\alpha) q_i^{j',\varepsilon} &\to 0. \end{aligned} \tag{13}$$

For $q_i^{j',\varepsilon}$ to be a best response, we must have $q_i^{j',\varepsilon} \to 0$ for all $j' \neq \ell$. Therefore, for sufficiently small ε and

[11] This is a slight abuse of notation. We should define a sequence indexed by ν, $\{\sigma^{\varepsilon_\nu}\}$, where $\varepsilon_\nu \to 0$ as $\nu \to \infty$.

all $j' \neq \ell$, we have

$$x_i^{j',\varepsilon}(\alpha) > \frac{\omega_i^{j'}}{2}. \tag{14}$$

From (13), there must be some commodity j'' such that $b_i^{j'',\varepsilon} > \frac{\omega_i^\ell}{\ell-2}$ holds. Thus, we have $p^{j'',\varepsilon}(\alpha)$ is uniformly bounded above zero for all α. Differentiating $V_i^\varepsilon(\overline{\sigma}_i^\varepsilon, \sigma^\varepsilon)$ with respect to $b_i^{j,\varepsilon}$ and $b_i^{j'',\varepsilon}$, we have

$$\begin{aligned} \frac{\partial V_i^\varepsilon(\overline{\sigma}_i^\varepsilon, \sigma^\varepsilon)}{\partial b_i^{j,\varepsilon}} &= \int_\alpha \left[\frac{\partial u_i(x_i^\varepsilon(\alpha))}{\partial x_i^j} \frac{1}{p^{j,\varepsilon}(\alpha)} - \frac{\partial u_i(x_i^\varepsilon(\alpha))}{\partial x_i^\ell} \right] f_i(\alpha) d\alpha \quad \text{and} & (15) \\ \frac{\partial V_i^\varepsilon(\overline{\sigma}_i^\varepsilon, \sigma^\varepsilon)}{\partial b_i^{j'',\varepsilon}} &= \int_\alpha \left[\frac{\partial u_i(x_i^\varepsilon(\alpha))}{\partial x_i^{j''}} \frac{1}{p^{j'',\varepsilon}(\alpha)} - \frac{\partial u_i(x_i^\varepsilon(\alpha))}{\partial x_i^\ell} \right] f_i(\alpha) d\alpha \; . & (16) \end{aligned}$$

Since (14) holds for j'', from assumption 1 and equations (15) and (16), it follows that a type-i consumer could increase utility by reducing $b_i^{j'',\varepsilon}$ and increasing $b_i^{j,\varepsilon}$ by the same small amount. Thus, case 1a is impossible.

Case 1b: There exists $\underline{x}$ such that $\lim_{\varepsilon \to 0} x_i^{\ell,\varepsilon}(\alpha) \geq \underline{x}$ holds for all α.

If the liquidity constraint is not binding, so increasing $b_i^{j,\varepsilon}$ is feasible, then assumption 1 and equation (15) imply that utility would be increased. If the liquidity constraint is binding, then there must be some commodity j'' such that $b_i^{j'',\varepsilon} > \frac{\omega_i^\ell}{\ell-2}$ holds, and a type-i consumer could increase utility by reducing $b_i^{j'',\varepsilon}$ and increasing $b_i^{j,\varepsilon}$ by the same small amount. Thus, case 1b is impossible.

Case 2: Suppose we have $p^{j,\varepsilon}(\alpha) \to \infty$ for some (j, α).

Since bids are bounded, we must have $q_i^{j,\varepsilon} \to 0$. Because the lower support for α is strictly positive, we have $p^{j,\varepsilon}(\alpha) \to \infty$ for all α. Differentiating $V_i^\varepsilon(\overline{\sigma}_i^\varepsilon, \sigma^\varepsilon)$ with respect to $q_i^{j,\varepsilon}$, we have

$$\frac{\partial V_i^\varepsilon(\overline{\sigma}_i^\varepsilon, \sigma^\varepsilon)}{\partial q_i^{j,\varepsilon}} = \int_\alpha \left[p^{j,\varepsilon}(\alpha) \frac{\partial u_i(x_i^\varepsilon(\alpha))}{\partial x_i^\ell} - \frac{\partial u_i(x_i^\varepsilon(\alpha))}{\partial x_i^j} \right] f_i(\alpha) d\alpha. \tag{17}$$

For sufficiently small ε, $x_i^{j,\varepsilon}(\alpha) > \frac{\omega_i^j}{2}$ must hold for all α. Feasibility imposes a uniform upper bound on $x_i^{\ell,\varepsilon}(\alpha)$, based on allocating all of commodity ℓ to type-i consumers. By equation (17) and assumption 1, it follows that utility can be increased by increasing $q_i^{j,\varepsilon}$. Thus, case 2 is impossible.

Since prices can be bounded above zero and below infinity, uniformly in α, payoffs are continuous in the type-symmetric strategies σ^ε. From the maximum theorem, it follows that the limiting strategies, σ^* form a Nash equilibrium to the limiting game, which is Γ.

To complete the proof, we must show that the limiting equilibrium is nontrivial. Suppose that all bids and offers are zero at the equilibrium to Γ, so that we have $\sigma^* = 0$. For $(b_i^*, q_i^*) = 0$ to be the limit of the sequence of type i consumers' Nash equilibrium strategies to Γ^ε, (b_i^*, q_i^*) must be a best response to the limiting distribution of prices, $p^*(\alpha)$. The reason is that for any positive ε, no matter how small, an individual consumer cannot affect the distribution of prices, and the distribution of prices is converging to $p^*(\alpha)$. Thus, we have

$$\frac{\partial V_i(0,0)}{\partial b_i^j} = \int_\alpha \left[\frac{\partial u_i(\omega_i^j)}{\partial x_i^j} \frac{1}{p^{j,*}(\alpha)} - \frac{\partial u_i(\omega_i^\ell)}{\partial x_i^\ell} \right] f_i(\alpha) d\alpha \leq 0 \quad \text{and} \tag{18}$$

$$\frac{\partial V_i(0,0)}{\partial q_i^j} = \int_\alpha \left[p^{j,*}(\alpha) \frac{\partial u_i(\omega_i^\ell)}{\partial x_i^\ell} - \frac{\partial u_i(\omega_i^j)}{\partial x_i^j} \right] f_i(\alpha) d\alpha \leq 0. \tag{19}$$

Rewriting (18) and (19), we have

$$\frac{\frac{\partial u_i(\omega_i^j)}{\partial x_i^j}}{\frac{\partial u_i(\omega_i^\ell)}{\partial x_i^\ell}} \leq \frac{1}{\int_\alpha \left[\frac{1}{p^{j,*}(\alpha)}\right] f_i(\alpha) d\alpha} \quad \text{and} \tag{20}$$

$$\frac{\frac{\partial u_i(\omega_i^j)}{\partial x_i^j}}{\frac{\partial u_i(\omega_i^\ell)}{\partial x_i^\ell}} \geq \int_\alpha p^{j,*}(\alpha) f_i(\alpha) d\alpha. \tag{21}$$

Combining (20) and (21) yields

$$\int_\alpha \left[\frac{1}{p^{j,*}(\alpha)}\right] f_i(\alpha) d\alpha \int_\alpha p^{j,*}(\alpha) f_i(\alpha) d\alpha \leq 1. \tag{22}$$

However, a strict version of Jensen's inequality implies that the left side of (22) is strictly greater than one unless the price distribution is degenerate. Therefore, there is a constant, $p^{j,*}$, such that $p^{j,*}(\alpha) = p^{j,*}$, for almost all α. Since this argument holds for all types i and commodities j, it follows from (20) and (21) that all consumers have the same marginal rate of substitution at their endowments, so the initial allocation is Pareto optimal.

We have established the following. If the initial allocation is not Pareto optimal, the supposition that $\sigma^* = 0$ is contradicted, so σ^* is nontrivial. If the initial allocation is Pareto optimal, then $\sigma^* = 0$ is possible, but we also know that there are other nontrivial equilibria to Γ. Simply let everyone choose positive bids and offers on each market, such that we have

$$\frac{b_i^j}{q_i^j} = \frac{\frac{\partial u_i(\omega_i^j)}{\partial x_i^j}}{\frac{\partial u_i(\omega_i^\ell)}{\partial x_i^\ell}}.$$

Clearly, offers can be chosen to be positive, but small enough so that a type-i consumer's bids do not exceed ω_i^ℓ. ■

Proposition 4.3: *In a type-symmetric Nash equilibrium with positive aggregate bids and offers on each market, the distribution of prices is degenerate (i.e., for each j, there is a constant, $p^{j,*}$, such that $p^{j,*}(\alpha) = p^{j,*}$, for almost all α) if and only if the initial endowments are Pareto optimal.*

Proof: Let the initial endowments be Pareto optimal. Since each consumer receives at least as much utility at the Nash equilibrium allocation as the utility of her endowment, concavity of the utility functions implies that the Nash equilibrium allocation must be the endowment allocation. Otherwise, the expected allocation is feasible and Pareto dominates the initial allocation. From the allocation equation, (6), it follows that

$$q_i^{j,*} = \frac{b_i^{j,*}}{p^{j,*}(\alpha)}$$

holds for all i, j, α. Since aggregate bids and offers are positive there must be some type i for which we have $q_i^{j,*} > 0$. Therefore,

$$p^{j,*}(\alpha) = \frac{b_i^{j,*}}{q_i^{j,*}}$$

holds for all α. Thus, the distribution of prices is degenerate.

Let the initial endowments not be Pareto optimal, and suppose that the distribution of prices is degenerate. Then, in equilibrium, each consumer receives a consumption bundle that is independent of α. There must be at least one type of consumer, i, and commodity, j, for which consumer i is a net seller of commodity j. [If, instead, each consumer's N.E. bundle equals her endowment, then her liquidity constraint is not binding. Therefore, all utility gradients at the endowment are proportional to the degenerate price vector, contradicting the fact that endowments are not Pareto optimal.] Since consumer i is a net seller of commodity j, we have

$$p^{j,*}(\alpha) > \frac{b_i^{j,*}}{q_i^{j,*}} \quad \text{for all } \alpha.$$

Given α_{-i}, straightforward algebra yields $p^{j,*}(\overline{\alpha}_i, \alpha_{-i}) < p^{j,*}(\underline{\alpha}_i, \alpha_{-i})$. Because $f(\cdot)$ is strictly positive over its support, this contradicts the supposition that the distribution of prices is degenerate. ■

5 Properties of the Equilibrium

Proposition (3.2) establishes that, when the underlying demand uncertainty creates a nondegenerate distribution of equilibrium prices for each commodity, then consumers' optimal strategies are uniquely determined. The optimal strategy for consumer (i, t) could be strictly in the interior of her strategy set, $S_{i,t}$, or it could be on the boundary. Since it is unusual to find consumers on both sides of a given real-world market simultaneously,[12] of particular interest is the case in which consumers strictly prefer to trade on one side of the market. To be sure, in the real world, consumers often have an endowment of zero for the commodities that they wish to buy, which would require them to be on one side of the market in Γ. However, it is interesting to ask whether the risk of price fluctuations, by itself, is enough to partition the traders into buyers and sellers.

Modifying our previous notation, denote type-i consumption as a function of the price vector, p, and the consumer's strategy (which may or may not be an equilibrium strategy), σ_i, as $x_i(\sigma_i, p) = (x_i^1(\sigma_i, p), ..., x_i^\ell(\sigma_i, p))$. Also, for $j = 1, ..., \ell - 1$, let $H_i^j(p; \sigma_i)$ be given by

$$H_i^j(p; \sigma_i) = \frac{\partial u_i(x_i(\sigma_i, p))}{\partial x_i^j} - p^j \frac{\partial u_i(x_i(\sigma_i, p))}{\partial x_i^\ell}. \tag{23}$$

We can interpret $H_i^j(p; \sigma_i)$ as a type-i consumer's marginal utility of commodity j, given that increasing consumption of j will occur at the expense of reducing consumption of ℓ. There are two effects of p^j on $H_i^j(p; \sigma_i)$, making the overall effect ambiguous. The direct effect of an increase in p^j (holding consumption constant) is to increase the opportunity cost of commodity j. The second term in (23) increases, lowering $H_i^j(p; \sigma_i)$. The net-trade effect of an increase in p^j reduces x_i^j and increases x_i^ℓ. The reason is that the fixed

[12] This statement excludes financial intermediaries and market makers, who attempt to buy low and sell high. In the current model, buying and selling must take place at the same price.

b_i^j purchases less of commodity j, and the fixed q_i^j is sold for more of commodity ℓ. Thus, the net-trade effect of an increase in p^j tends to increase the marginal utility of j and reduce the marginal utility of ℓ, raising $H_i^j(p;\sigma_i)$. Sharp results are available for the case of two commodities, and therefore, one trading post.

From the type-symmetric equilibrium strategies, σ^*, and the density function for active consumers of each type, $f_i(\alpha)$, a distribution of prices is determined, whose density function (when it exists) we denote by $h_i(p)$.

Proposition 5.1: *Assume that there are two commodities, $\ell = 2$, and that the initial allocation is not Pareto optimal. If we have*[13]

$$\frac{\partial H_i(p;\sigma_i^*)}{\partial p} > 0 \tag{24}$$

for all p in the support of h_i (i.e., for all prices consistent with equilibrium), then consumer i will be on one side of market one. That is, we have either $b_i^* = 0$ or $q_i^* = 0$.

Proof. Expected utility of a type-i consumer can be written as a function of her strategy, σ_i, and the density of prices (if such a density function exists), given by

$$V_i(\sigma_i, h_i) = \int_p u_i\left(\omega_i^1 - q_i + \frac{b_i}{p}, \omega_i^2 - b_i + pq_i\right) h_i(p)dp. \tag{25}$$

The proof of Proposition (4.2) demonstrates that equilibrium prices must be bounded away from zero and infinity, so the best response correspondence is well-defined. Furthermore, since the initial allocation is not Pareto optimal, there must exist a strictly positive density function, $h_i(p)$, defined over a nondegenerate support, $[\underline{p}, \overline{p}]$. From Proposition (3.2), the best response, σ_i^*, is uniquely determined. Suppose that $b_i^* > 0$ and $q_i^* > 0$ hold.

We claim that there exists $\widehat{p}$, such that $\underline{p} < \widehat{p} < \overline{p}$ and $H_i(\widehat{p};\sigma_i^*) = 0$ hold. If we have $H_i(p;\sigma_i^*) > 0$ for all $p \in [\underline{p}, \overline{p}]$, this implies

$$\frac{\partial V_i(\sigma_i^*, h_i)}{\partial q_i} = -\int_p H_i(p;\sigma_i^*)h_i(p)dp < 0.$$

[13] Since there are two commodities, all bids and offers are on market one, so we eliminate the commodity superscript.

It follows that utility can be increased by reducing q_i^*, contradicting the fact that σ_i^* is the best response of type-i consumers. If we have $H_i(p;\sigma_i^*) < 0$ for all $p \in [\underline{p}, \overline{p}]$, this implies

$$\frac{\partial V_i(\sigma_i^*, h_i)}{\partial b_i} = \int_p \frac{H_i(p;\sigma_i^*)}{p} h_i(p)dp < 0.$$

It follows that utility can be increased by reducing b_i^*, contradicting the fact that σ_i^* is the best response of type-i consumers. This establishes the claim that there exists $\widehat{p}$, such that $\underline{p} < \widehat{p} < \overline{p}$ and $H_i(\widehat{p};\sigma_i^*) = 0$ hold.

Consider the following deviation for a type-i consumer:

$$\begin{aligned} \overline{q}_i &= q_i^* - \delta \\ \overline{b}_i &= b_i^* - \delta\widehat{p}. \end{aligned}$$

Evaluated at $\delta = 0$, we have

$$\frac{\partial V_i(\sigma_i^*, h_i)}{\partial \delta} = \int_p H_i(p;\sigma_i^*) \frac{p - \widehat{p}}{p} h_i(p)dp. \tag{26}$$

Inequality (24), and the fact that we have $H_i(\widehat{p};\sigma_i^*) = 0$, imply

$$\begin{aligned} H_i(p;\sigma_i^*) &< 0 \quad \text{for } p < \widehat{p}, \text{ and} \\ H_i(p;\sigma_i^*) &> 0 \quad \text{for } p > \widehat{p}. \end{aligned}$$

Therefore, the right side of (26) is strictly positive, so reducing bids and offers with $\delta > 0$ increases utility. This contradicts the fact that σ_i^* is a best response, so our supposition that $b_i^* > 0$ and $q_i^* > 0$ hold is false.

■

Remark 5.2: The conclusion of Proposition (5.1), that a type-i consumer will be on one side of the market, continues to hold when the other consumers choose strategies that are out of equilibrium. We must interpret σ_i^* as the type-i consumer's best response to the strategies played by everyone else, and replace the condition about Pareto optimality with the condition that the consumer faces a strictly positive density function, $h_i(p)$, defined over a nondegenerate support, $[\underline{p}, \overline{p}]$.

Here is some intuition for why a consumer strictly prefers to be on one side of the market. Suppose that the consumer has strictly positive bids and offers. There must be a price, $\widehat{p}$, at which her marginal rate of substitution equals $\widehat{p}$. Reducing bids and offers at the ratio $\widehat{p}$ *reduces* her net purchase of commodity 1 when $p < \widehat{p}$ holds. Under inequality (24), we have $H_i(p;\sigma_i^*) < 0$ for $p < \widehat{p}$, so the net marginal utility of commodity 1 is negative in these states, and the consumer is better off reducing her net purchase of commodity 1. Reducing bids and offers at the ratio $\widehat{p}$ *increases* her net purchase of commodity 1 when $p > \widehat{p}$ holds. Under inequality (24), we have $H_i(p;\sigma_i^*) > 0$ for $p > \widehat{p}$, so the net marginal utility of commodity 1 is positive in these states, and the consumer is better off increasing her net purchase of commodity 1. Thus, when the consumer reduces her bids and offers at the ratio $\widehat{p}$, she is happy to have her net purchase of commodity 1 reduced when the actual price is below $\widehat{p}$ (when she is already consuming a lot), and she is happy to have her net purchase of commodity 1 increased when the actual price is above $\widehat{p}$ (when she is not consuming very much). The only way to avoid this beneficial deviation is to have either her bid or offer be zero, so that the reduction in bids and offers is not feasible.

Remark 5.3: Other things equal, a more risk averse consumer is more likely to be on one side of the market. To see this, we can compute

$$\frac{\partial H_i(p;\sigma_i^*)}{\partial p} = -\frac{\partial^2 u_i(x_i(\sigma_i^*,p))}{\partial (x_i^1)^2}\frac{b_i^*}{(p)^2} - \frac{\partial^2 u_i(x_i(\sigma_i^*,p))}{\partial (x_i^2)^2} p q_i^* - \frac{\partial u_i(x_i(\sigma_i^*,p))}{\partial (x_i^2)} \tag{27}$$

The first two terms on the right side of (27) are positive, and the last term is negative. Let a particular consumer (i,t) become more and more risk averse, holding constant the rest of the economy with a non-degenerate equilibrium price distribution. The entire expression for $\frac{\partial H_i(p;\sigma_i^*)}{\partial p}$ eventually becomes positive, as long as (b_i^*, q_i^*) remains bounded away from $(0,0)$. This occurs if consumer (i,t) has a marginal rate of substitution (at her endowment) that is either always less than or always greater than any equilibrium price realization.

When there is more than one trading post, $\ell > 2$, similar forces are at work, but interaction effects complicate the condition for strictly preferring to be on one side of the market. Even if the utility function is separable in the ℓ commodities, $H_i^j(p;\sigma_i)$ depends on prices other than p^j, because all prices affect x_i^ℓ. In general, it will be impossible to find $\widehat{p}^j$, such that $\underline{p}^j < \widehat{p}^j < \overline{p}^j$ and $H_i^j(\widehat{p}^j, p^{-j};\sigma_i^*) = 0$ hold for all prices of other commodities, p^{-j}.

The intuition for being on one side of the market does not depend on the specification of a continuum of consumers, and the result that the consumer is unable to affect the price. My conjecture is that, for economies with a finite number of consumers and nontrivial demand uncertainty, we have (1) best responses are uniquely determined when the price distribution is nondegenerate, and (2) plausible parameter specifications (involving sufficiently high risk aversion) will have consumers on one side of the market in all Nash equilibria.

6 Concluding Remarks

Our specification of uncertainty can be interpreted as uncertainty over endowments. Obviously, the case in which the endowment of consumer (i, t) is either 0 or ω_i fits directly into our framework. Because the number of types is arbitrary, the model also includes general endowment uncertainty and preference uncertainty as special cases, at least when the number of realizations is finite. Consumers take no actions until they learn their endowments, so the endowment realization can be considered to be part of what determines one's type. For example, suppose that we want to model n "initial" types of consumers, with k nonzero realizations of endowments for each initial type. This is equivalent to nk types of consumers in our specification.[14]

Our game, Γ, is a variant of the Shapley-Shubik market game with commodity money, since bids are made with commodity ℓ. This variant has been studied by [15], [3], [6], and many others. We have avoided the variant studied by [14], [12], [13], [9], and others. In those models, bids are made with a fiat money, and consumers must satisfy a budget constraint or else face punishment. In this literature, severe punishments are imposed to ensure that the budget constraints hold in equilibrium. The problem with severe punishments when there is demand uncertainty is that, to ensure that her budget constraint holds in all states of nature, a consumer will typically allow slack in some states. However, if one consumer's budget constraint holds as a strict inequality, it follows that another consumer's budget constraint must be violated.[15] To focus on the issue of demand uncertainty, rather than bankruptcy and punishments, we have adopted a model without budget constraints.[16]

Because bids and offers cannot be made contingent on price, there is no mechanism in place to ensure

[14] The assumption that $f_i(\alpha)$ is strictly positive over its support might be violated if we model uncertainty over (strictly positive) endowments, but certainty over the measure of active consumers. However, this assumption is only used to ensure that there is actual demand uncertainty in Proposition (4.3) and Proposition (5.1). The existence argument and characterization of best responses under price uncertainty does not depend on this full-support assumption.

[15] See the discussion in [12, section 5].

[16] The problem is not fiat money, per se, but the specification that consumers can create arbitrary amounts of "inside" fiat money, limited only by the threat of punishments if they violate their budget constraint. In a dynamic setting, the acquisition of fiat money can be modeled more carefully, as in [4], [5], and [10].

that marginal rates of substitution are equated across consumers. Only the most extreme coincidence would allow the Nash equililibrium to be ex post efficient. Other mechanisms involving the submission of demand schedules or limit orders could be more efficient.[17] On the other hand, prices in Γ can be computed easily by a clearing house. Perhaps more importantly, consumers can transmit their strategies easily in Γ, while submission of demand schedules can be tedious and time consuming. It would be interesting to explore these tradeoffs. One might expect to observe Shapley-Shubik trading posts on markets where traders face relatively little price uncertainty, at least at the moment a trader places a bid or offer.

[17] A strategy in Γ is similar to a market order, and can be interpreted as the submission of a special two-parameter demand function. See [11]. Submission of arbitrary demand schedules would not allow ex ante efficiency, in general, because there is no way to insure fully against price risk with spot markets only.

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