Efficiency and Synergy in a Multi-Unit Auction with and without Package Bidding: an Experimental Study

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Abstract

Combinatorial auctions allow bidding for combinations of items (package bids) and are a useful tool when complementarities/synergies among sale objects are present. In such cases, package biding improves economic efficiency by helping bidders win packages that capture the value added due to synergies. The revenue of the seller may also increase as a result. At the same time, when synergies are small or absent, a combinatorial auction rule can be strategically "abused" by the bidders hurting economic efficiency. This paper studies the use of a single-round sealed-bid first-price combinatorial auction in an intrinsically asymmetric setup where some bidders are able to bid for many items while others are restricted (e.g., geographically or financially) to bid for a single item only. Our theoretical model illustrates one of the ways in which a combinatorial auction rule can improve efficiency at a high level of synergies but can hurt it when synergies are small. The principal driving force behind the result is the "free-riding" behavior among local bidders induced by the combinatorial auction rule. The paper reports experimental results that lend strong support to the importance of the "free-riding" phenomenon. The observed changes in efficiency levels are qualitatively consistent with the theoretical predictions. In addition, regardless of the degree of synergies, the combinatorial auction generated significantly lower revenues both theoretically and in the experimental data.

1 Introduction

Recent years have seen a surge of interest in auction procedures that handle multiple objects. Many recently conducted large-stake auctions involved a seller simultaneously auctioning off multiple objects to buyers who are interested in one or more of those objects

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(or similarly a buyer simultaneously procuring several items from a number of sellers). Among the studied examples are spectrum auctions where governments sold licenses for the use of airwave spectrum (e.g. Ausubel et al., 1997, Klemperer, 2002), procurement auctions to select operators of bus routes in London (Cantillon and Pesendorfer, 2007), and procurement auctions to determine providers of school meals in Chile (Epstein et al., 2002).

There are several reasons for selling multiple objects simultaneously, but in this work we focus on the advantages of this approach when the value of a package of several objects is higher than the sum of standalone values of its components. When such complementarities (or "synergies" as they are often called) exist, standard single-object auctions that sell the items separately can allocate the items inefficiently. The synergistic value of a package is lost if a bidder fails to acquire all the components in separate single-object auctions. To improve efficiency an auction procedure should allow bidders to take advantage of value complementarities: in addition to single-item bids it should allow package bids for combinations of items. A package bid is binding only if the bidder acquires all the items in the package, otherwise her bids for smaller packages and/or individual items apply. To determine the winner(s) of such an auction, the auctioneer has to consider all possible allocations of the objects and choose the one that maximizes the revenue. Such procedures are usually referred to as combinatorial, or package, auctions. Apart from benefiting the bidders and having a better chance of allocating the objects efficiently (i.e. to the party that values them the most), such procedures may also increase the seller's revenue. This is because realized synergies increase the overall size of the economic pie in the market, which may benefit the seller as well as the bidders. The usefulness of such auctions has been limited until recently because of the complexities associated with formulating bids as well as determining the winning bid combination. With advances in technology and understanding of the properties of such auctions their use has increased dramatically.

One type of combinatorial auctions builds on iterative ascending procedures. This type of auctions is attractive because it simplifies the problem of winner determination and assists bidders in discovering their valuation structure (Pekec and Rothkopf, 2003). The two major issues that the designers of iterative procedures have to address are the *exposure* and the *threshold* problems. Both of these can have a negative effect on the level of submitted bids, the seller's revenue and economic efficiency. One the one hand, when bids for combinations of objects are restricted or not allowed at all, bidders in trying to acquire a synergistic package of items may end up with only a part of the package and pay more for the acquired items than they are worth to them. This can leave the bidders financially exposed, hence the name for the phenomenon[1]. On the other hand, when

[1]Financial exposure problem is most severe in simultaneous ascending auctions where package bids are not allowed at all. Technically, such auctions are not combinatorial. However, they are sometimes

package bids are allowed, bidders may face the so-called threshold problem. Consider the situation where a package bid competes against a sum of bids for the subsets of the package[2]. In this situation, even though the bidders behind the subset bids may place a higher value on the package (i.e. the sum of their values is higher than the value of the package bidder), absent of explicit coordination each of them may be unable and/or unwilling to submit a bid high enough for the combination of all the subset bids to top the package bid. Thus, by solving one problem (exposure) package bidding introduces another problem (threshold). This creates a trade-off that designers of combinatorial auctions have to take into account. Due to the complexity of models that would capture the important features of iterative combinatorial procedures this trade-off has not received a systematic theoretical treatment.

Another type of combinatorial auctions builds on sealed-bid single-round first-price procedures (Cantillon and Pesendorfer 2007, Epstein et al. 2002). These auctions have a number of attractive features such as their resistance to collusive behavior, encouragement of participation, and their transparency with respect to determination of payments as winners pay their own bids (for a discussion see Crampton, 1998). The main difficulty in such auctions is that bidders need to submit bids for all possible combinations of objects simultaneously and the seller needs to solve a full-fledged problem of determining the combination of bids that maximizes the revenue. However, when the number of objects is small the procedure is quite practical. In this study we show that introducing package bidding in sealed-bid single-round first-price procedures involves a trade-off which parallels the one that package bidding generates in the context of iterative ascending procedures. We show that efficiency of allocation depends on the degree of synergies among the objects. On the one hand, package bidding may improve efficiency when synergies are present. On the other hand, it may hurt the efficiency when the synergies are absent or insignificant. We point out the driving forces behind the trade-off and how its mechanics are influenced by the specificity of first-price rules. In particular, we show the interaction of asymmetries between bidders with respect to their ability to obtain synergies from packages of objects, the degrees of synergies, and whether or not the auction rule allows package bidding.

It is an established fact that asymmetries of value distributions among bidders in first-price auctions result in asymmetries in equilibrium bidding behavior (Lebrun, 1999; Maskin and Riley, 2000). In turn, the asymmetric bidding behavior gives rise to equilib-

considered as a benchmark and a viable alternative to combinatorial auctions (see e.g. Crampton, 2006). Financial exposure problem is completely eliminated when bids on any combination of objects are allowed. Between these two extremes an auctioneer may consider placing some restrictions on the number/type of package bids to alleviate some of the computational burden at the expense of re-introducing the risk of financial exposure for the bidders.

[2] Since in combinatorial auctions the auctioneer considers all the possible allocations of the items to the bidders and chooses the one that brings the highest revenue, a package bid for a certain set of objects can be viewed as competing against sums of bids for subsets that form its partition.

rium inefficiencies because the bidder with the highest value is no longer guaranteed to win the auction. In the context of multi-object auctions with complementarities such asymmetry can arise when some of the bidders can take advantage of complementarities among objects, while others may not be able to. For example, some smaller bidders (henceforth Local) may be financially constrained or geographically restricted so that they are unable to purchase more than one object. At the same time, larger bidders (Global) are likely to be able to acquire packages of objects and enjoy potential synergies. This setup creates asymmetries in value distributions between the Global and the Local bidders. We adopt it as the basis for our theoretical model. Thus, in the model a single item is auctioned off in each of two markets where one Local bidder with a unit demand bids against the Global bidder who demands one unit in each market. If the Global bidder obtains the items in both markets his or her value increases by a multiplicative factor depending on the degree of synergies among the items. Clearly, such an asymmetry is not important when complementarities are absent. In this case the objects can be sold separately with no detrimental effect on efficiency. With synergies, however, selling the objects separately may result in an inefficient allocation. In fact, the equilibrium inefficiency increases with the degree of synergies. Roughly speaking, when synergies are present each auction between a Local and the Global bidder becomes an asymmetric auction in the sense of Maskin and Riley (2000) where the value distribution of the Global bidder is more advantageous than the value distribution of the Local bidder.

Allowing package bidding in this environment introduces a strategic incentive that makes the "geographic" asymmetry relevant even if there are no synergies. When complementarities are present it is natural to expect package bids to be higher than the sum of bids for the individual items (and/or smaller combinations of items) comprising the package[3]. This is exactly the motivation for allowing package bids, and such a pattern is necessary to achieve economic efficiency. However, Cantillon and Pesendorfer (2007) show that even when complementarities are not present a bidder has a strategic incentive to submit a "non-trivial" package bid, i.e. a package bid higher than the sum of his or her bids for individual items. In other words, allowing the use of combinatorial bidding when synergies are absent may be "abused" to gain a strategic advantage. We show that under an auction rule with package bidding the Global bidder finds it optimal to submit only the package bid for all the items while submitting zero bids for individual items. This is the extreme type of "non-trivial" package bidding pointed out by Cantillon and Pesendorfer (2007). Such behavior on the part of the Global bidder introduces a need for cooperation

[3] Cantillon and Pesendorfer (2007) study procurement auctions. In such auctions sellers compete among themselves for the contract to deliver a product or service to a buyer at a certain price. Typically, lower bids have a higher probability of winning. In this context, in the presence of cost synergies one would expect a package bid to be *lower* than the sum of the bids for the individual items.

among Local bidders because for them to win in this situation the sum of their bids needs to be higher than the Global's package bid (the threshold problem). However, as explicit coordination is not possible, the Local bidders are left with an incentive to free ride on each other. Consequently, Locals' bids are depressed. This force works in the opposite direction to the effect of complementarities so that employing the package auction rule when synergies are relatively high improves the efficiency of allocation. As a result, there is a trade-off between the degree of synergies among the objects and the auction rule used. An additional important implication of our model is that package bidding can have a significant *negative* effect on the seller's revenue when compared to the auction rule that sells the objects separately, regardless of the level of synergies.

Solving a general model of combinatorial auctions is notoriously difficult. For the purposes of this investigation we make a number of simplifications in our theoretical analysis in order to solve for the exact bidding functions and use them as benchmarks in evaluating the performance of the theory. (**discuss the CP example and our same value assumption here?**) Despite the simplifications, we view our model as capturing key elements of interest. The model clearly illustrates the mechanics of the trade-off where a first-price sealed-bid auction rule with package bidding can hurt economic efficiency at low levels of synergies among the sale items, but improve it when synergies are high. At the same time, the model is simple enough so that we can obtain numerical and in some cases analytical equilibrium bidding functions.

We test the predictions of our model using experimental approach. The experimental investigation can be viewed as complementary to the empirical approach. Empirical studies that employ structural estimation of auction data typically derive identification conditions based on theoretically predicted equilibrium bidding behavior and proceed to estimate the underlying value/cost structure of the bidders from the observed data. These estimates can then be used to perform counterfactual simulations and assess the appropriateness of particular auction procedures. (**put reference to a seminal empirical auction study**) Cantillon and Pesendorfer (2007) use this approach to study the performance of combinatorial sealed-bid first-price procedures in the context of procurement auctions for bus route licenses in London. Using the data obtained from these auctions they partly recover the underlying cost distribution and conclude that the synergies are typically negative, i.e. the bus routes are substitutes rather than complements. An experimental study proceeds in the manner opposite to such empirical studies. An experimental investigation starts by inducing a particular valuation structure and then tests whether the observed bidding behavior corresponds to the theoretical predictions. In addition, the experimenter has a large degree of control over auction rules, information structure and in particular the degree of synergies. Thus, experiments can help assess the strength of strategic forces and see whether the theory has predictive power. They can point out systematic behavioral patterns and deviations from equilibrium behavior that would help predict the ultimate performance of alternative auction formats.

We conduct several experimental sessions to assess the strength of the aforementioned strategic forces. In four experimental conditions we vary the auction rule (with or without package bidding) and the level of synergies (0% and 50%). By and large, results are consistent with the theoretical predictions. We find that bidders tend to bid more aggressively when synergies are high and less aggressively under the auction rule with package bidding. Particularly impressive is the decrease in the level of Local bids when package biding is introduced. Thus, the Local bidders do seem to respond to the "free-riding" incentive and exacerbate the threshold problem. This behavior also accounts for the significantly lower seller's revenue observed in the package bidding conditions with and without synergies. Without package bidding, economic efficiency is lower when the synergies are at 50%. The efficiency is also lower with package bidding than without it when the synergies are absent. All these results are in line with the theoretical predictions. However, the evidence with respect to the most interesting change in efficiency is mixed. The auction rule with package bidding is supposed to attain a higher efficiency level when synergies are high compared to no synergies. We do not find strong support for this prediction. These results suggest that first-price auctions with package bidding should be used with caution. They do not necessarily improve the efficiency but at the same time they can have a substantial negative impact on the seller's revenue.

In the next session we present the details of our model. We characterize Risk-Neutral Nash equilibria for first-price auction formats with and without package bidding. In Section 3 the experimental design is presented. Section 4 contains an extensive analysis of the experimental data. Finally, the last section contains our conclusions.

2 Theoretical Considerations

In constructing the theoretical model we followed three main principles: tractability, suitability for experimental investigation and ease of comparison between alternative auction rules. As a result, we made a number of simplifying assumptions which - although admittedly restrictive - allow us to explore the key features of the auction rules in question in a tractable manner. In the model there are two categories of bidders: Local bidders and a Global bidder. There are 2 markets with a single item for sale in each of these markets. There is one Local bidder in each market who is interested only in the item in that particular market. The Global bidder is interested in both items in the sense that he or she derives a non-zero value from each of them. Thus, the Global bidder is present in every market where he or she bids against a single Local bidder. Bidders have private values for the objects. These values are distributed independently across bidder categories. The

Local bidders do not know the value of the Global bidder and vice versa. The Local bidders' value $v_l \in [0, 1]$ is distributed uniformly. The value is the same for all Local bidders. Thus, we assume that the value is the same within the Local bidder category but it is distributed independently from that of the Global bidder. This assumption reflects the idea that the values of the Local bidders are likely to be more correlated with each other than with the value of the Global bidder. This is a reasonable assumption in an environment where the category of the bidder, Global or Local, has a strong influence on the bidder's value through various characteristics such as size, cost of capital, etc. In addition, taking the correlation among Local values to the extreme affords significant simplifications. The Global bidder's value $v_g \in [0, 1]$ is also distributed uniformly. If the Global obtains both items the value of the package is $2\beta v_g$, where $\beta \geq 1$ is the degree of synergy from owning all the items. In other words, βv_g is the value of each item in the package, but only if the whole package is acquired. If only one item is acquired its value is v_g.

We study two single-round sealed-bid first-price auction rules. As such, under both rules sealed bids are collected from the bidders and the winner is determined in a single round. No bid information is revealed until the auction outcome is announced. This is in contrast to iterative procedures where several rounds of bidding are held and bidders are allowed to update their bids based on publicly revealed information about the bids in the previous round. Since these are first-price auctions, the winner pays his or her winning bid.

Under the first rule (henceforth Separate rule), outcome in the first market is determined independently from the outcome in the second market. In each market two bids are submitted: a bid from the Global bidder and a bid from the Local bidder in that market. The item is awarded to the higher bidder in a particular market regardless of the bids in the other market. This auction can be interpreted as two simultaneous single-item auctions. The second rule (henceforth Package rule) differs from the Separate rule in two ways. First, in addition to bids for single items in each market (standalone bids), the Global bidder is allowed to submit a cross-market package bid for both items. This bid applies only if the Global bidder acquires both items. If only one item is allocated to the Global bidder, the standalone bid for that item applies. Second, the outcome in each market is determined by considering all the bids in both markets. The auctioneer reviews all possible allocations of the 2 objects to the 3 bidders and chooses the one that brings the highest revenue. There are 4 such allocations in this case. First, both items can be allocated to the Global bidder who would pay the package bid. Second, both items can be allocated to the Local bidders. The other two allocations involve the item in one market going to the Global bidder, who would pay the standalone bid for that item, and the item in the other market going to the Local bidder. For example, suppose that the Global bidder decides to submit zero standalone bids and a positive package bid. In this case the auctioneer would compare the package bid to the sum of Local bids. If the Global package bid is higher, both items go to the Global bidder. Otherwise, the items are allocated to the Local bidders. Note that if in addition to the package bid the Global bidder submits standalone bids, these bids compete against his or her package bid since they can be a part of an alternative winning allocation.

Due to our assumptions about the bidders' valuation structure we can make simplifications to the bidders' objective functions. These simplifications allow us to obtain (numerical and in some cases analytical) Risk-Neutral Nash equilibrium (RNNE) bidding functions. Thus, under the Separate auction rule we show that the Global bidder has no incentives to submit different bids in each market (Proposition 1). When Package rule is used we show that the Global bidder has no incentives to submit non-zero standalone bids (Proposition 2). In what follows, we start by characterizing the equilibrium under the Separate auction rule and then move to the Package rule. In both cases we focus on the Risk-Neutral Nash Equilibria where Local bidders follow the same (symmetric) bidding strategy. We do so because there is no *a priori* reason for any asymmetry across markets to arise in this one-shot game.

2.1 Separate auction rule

In Proposition 1 below we show that under the Separate auction rule a risk-neutral Global bidder has no incentive to submit different bids in different markets if the Local bidders follow a symmetric (i.e. the same for all Locals) bidding strategy. Let $b_l(.)$ denote the strategy of the Local bidders as a function of their value. Then under the Separate auction rule the Global bidder's problem is to choose a bid b to solve the following maximization problem:

$$\max_{b} \ 2\left[\beta v_g - b\right] \Pr\left[b \geq b_l\left(v_l\right)\right] \tag{1}$$

Proposition 1 states the result that allows us to write the Global bidder's problem in this manner.

Proposition 1 Suppose all the Local bidders follow a symmetric strategy $b_l(.)$ and suppose b^* is the unique maximizer of the problem in (1) given $b_l(.)$. Then the Global bidder's best response is to submit the same bid b^* in all the markets even though the Global bidder is not restricted to do so. (Proof is in the Appendix).

The result relies on the fact that the Locals' strategy is symmetric and the Locals share the same value v_l. Given these assumptions the Global bidder is better off submitting the same bid b^* in both markets. Consequently, because the ranking of bids is the same in every market, the Global bidder can win only the whole package or nothing. Whenever the bid of the Global is higher than the bid of the Locals ($b \geq b_l(v_l)$) he or she receives both items and enjoys the synergistic value $2\beta v_g$. Thus, the simplified formulation in (1) is appropriate.

Assuming that $b_l(.)$ is monotone and denoting its inverse $\sigma_l(.)$ the problem in (1) can be rewritten as:

$$\max_b \; 2\left[\beta v_g - b\right] \Pr\left[\sigma_l(b) \geq v_l\right] = \max_b \; 2\left[\beta v_g - b\right] \sigma_l(b)$$

which implies the following First Order Condition (FOC) for maximization:

$$\sigma_l'(b)\left[\beta v_g - b\right] = \sigma_l(b)$$

In equilibrium it should be the case that $v_g = \sigma_g(b)$, where $\sigma_g(.)$ with $\sigma_g'(.) > 0$ is the inverse of the bidding strategy of the Global $b_g(.)$:

$$\sigma_l'(b)\left[\beta \sigma_g(b) - b\right] = \sigma_l(b) \tag{2}$$

Given the fact that the Global follows the same bidding strategy $b_g(.)$ in all the markets, all Local bidders face the same problem:

$$\max_b \left[v_l - b\right] \Pr\left[b \geq b_g(v_g)\right] = \left[v_l - b\right] \sigma_g(b) \tag{3}$$

with the FOC:

$$\sigma_g'(b)\left[v_l - b\right] = \sigma_g(b)$$

and after imposing the equilibrium condition $v_l = \sigma_l(b)$:

$$\sigma_g'(b)\left[\sigma_l(b) - b\right] = \sigma_g(b) \tag{4}$$

The two conditions in (2) and (4) characterize an equilibrium in inverse bidding functions under the Separate auction rule. Additional equilibrium requirements provide boundary conditions for this system of differential equations. In equilibrium it should be the case that both categories of bidders submit bids on the same interval $\left[0, \bar{b}\right]$ where $\bar{b}$ is determined endogenously. Individual rationality dictates that the bidders with the lowest value of 0 should submit zero bids since negative bids are forbidden and a positive bid would yield a strictly negative expected payoff. In terms of the inverse bidding functions this condition can be written as $\sigma_g(0) = \sigma_l(0) = 0$. Additionally, the bidders with the highest value of 1 submit the bid $\bar{b}$ regardless of whether they are Global or Local: $\sigma_g(\bar{b}) = \sigma_l(\bar{b}) = 1$. This is because for bidders in either category there is no reason to submit a bid higher than $\bar{b}$ if this is the highest bid that their opponents ever submit. Thus, the equilibrium can be characterized by the system of differential equations in inverse bidding functions with two boundary conditions:

$$\begin{cases} \sigma_g'(b) = \frac{\sigma_g(b)}{[\sigma_l(b)-b]} & b \in [0,\bar{b}] \\ \sigma_l'(b) = \frac{\sigma_l(b)}{[\beta\sigma_g(b)-b]} \\ \sigma_g(0) = \sigma_l(0) = 0 \\ \sigma_g(\bar{b}) = \sigma_l(\bar{b}) = 1 \end{cases} \tag{5}$$

In the case when $\beta = 1$ (no synergy), the solution is linear $b_g(v_g) = \frac{1}{2}v_g$ and $b_l(v_l) = \frac{1}{2}v_l$. This is the familiar solution from the symmetric auction theory (e.g. Vickrey, 1961). With synergies, there is a known analytical solution to an equivalent boundary value problem (Plum, 1992): $b_l(v_l) = \frac{v_l}{1+\sqrt{1-cv_l}}$ and $b_g(v_g) = \frac{\beta v_g}{1+\sqrt{1+c(\beta v_g)^2}}$ where $c = \frac{\beta^2-1}{\beta^2}$. Note that $\bar{b} = b_l(1) = b_g(1) = \frac{\beta}{1+\beta} < 1$, i.e. the highest bid $\bar{b}$ is less than the highest value 1 and is increasing in the degree of synergies β. It is easy to show[4] that $\forall v \in (0,1)$, $b_g(v) > b_l(v)$. Plum (1992) shows that the system in (5) admits an essentially unique solution. Thus, the assumption of Proposition 1 is satisfied and indeed the solution to (5) is the unique equilibrium under the Separate auction rule where Local bidders follow a symmetric bidding strategy.

2.2 Package auction rule

It turns out that under the Package auction rule the Global bidder's problem also can be written as (1) if certain assumptions are satisfied. Proposition 2 below proves that in a Risk-Neutral Nash equilibrium where Locals follow a symmetric bidding strategy the only non-zero bid that the Global bidder submits is the bid for the package of 2 items. In particular, the Global bidder has no use for bids on individual items. This is not true in general but the simplified valuation structure in our problem produces this convenient simplification. Let the Global package bid be $2b$, so that b is the per-item bid easily comparable to bids under the Separate auction rule. Again, since only the package bid is submitted by the Global bidder, only 2 auction outcomes are possible: either the package goes to the Global bidder ($2b \geq 2b_l(v_l)$) or the items are allocated to the Local bidders.

[4] We will show here that $\forall\beta > 1$ and $b \in (0,\bar{b})$, $\Phi(b) \equiv \sigma_g(b) - \sigma_l(b) < 0$, assuring the result. From the system of differential equations (5) $\Phi'(b) = \sigma_g'(b) - \sigma_l'(b) = \frac{\sigma_g(b)}{[\sigma_l(b)-b]} - \frac{\sigma_l(b)}{[\beta\sigma_g(b)-b]}$. $\sigma_g(\bar{b}) = \sigma_l(\bar{b}) = 1$, implies that $\lim_{b<\bar{b},b\to\bar{b}} \Phi'(\bar{b}) = \frac{1}{[1-\bar{b}]} - \frac{1}{[\beta-\bar{b}]} > 0$. This implies that $\exists\ \epsilon > 0$, such that $\forall b \in (\bar{b}-\epsilon,\bar{b})$, $\Phi(b) \equiv \sigma_g(b) - \sigma_l(b) < 0$. Assume momentary that $\exists b_0 \in (0,\bar{b}-\epsilon]$, such that $\Phi(b_0) \equiv \sigma_g(b_0) - \sigma_l(b_0) = 0$. It results in $\Phi'(b_0) = \sigma_g'(b_0) - \sigma_l'(b_0) = \frac{\sigma_g(b_0)}{[\sigma_l(b_0)-b_0]} - \frac{\sigma_l(b_0)}{[\beta\sigma_g(b_0)-b_0]} > 0$. However, that implies that $\forall b \in (b_0,\bar{b})$, $\Phi(b) \equiv \sigma_g(b) - \sigma_l(b) \geq 0$, a contradiction. We must then conclude that $\forall\beta > 1$ and $b \in (0,\bar{b})$, $\Phi(b) \equiv \sigma_g(b) - \sigma_l(b) < 0$. See Maskin and Riley (2000) for other properties of such auctions.

The problem of the Global bidder is then:

$$\max_b \left[2\beta v_g - 2b\right] \Pr\left[2b \geq 2b_l\left(v_l\right)\right] = 2\left[\beta v_g - b\right] \sigma_l\left(b\right) \tag{6}$$

which has the same form as the maximization problem in (1) and yields the same equilibrium condition (2).

However, the Local bidder's maximization problem is different. Due to the fact that the auction rule compares the sum of the Local bids with the Global bidder's package bid, the other Local bid enter each Local bidder's problem. Suppose the Global bidder follows $b_g\left(.\right)$ and the other Local bidder follows $b_l\left(.\right)$, then the problem of a Local bidder is to chose b that solves the following maximization problem:

$$\max_b \left[v_l - b\right] \Pr\left[b + b_l\left(v_l\right) \geq 2b_g\left(v_g\right)\right] = \left[v_l - b\right] \sigma_g\left(\frac{b + b_l\left(v_l\right)}{2}\right) \tag{7}$$

with FOC:

$$\frac{1}{2}\sigma_g'\left(\frac{b + b_l\left(v_l\right)}{2}\right)\left[v_l - b\right] = \sigma_g\left(\frac{b + b_l\left(v_l\right)}{2}\right) \tag{8}$$

In equilibrium $b_l\left(v_l\right) = b$ and $v_l = \sigma_l(b)$:

$$\frac{1}{2}\sigma_g'(b)\left[\sigma_l(b) - b\right] = \sigma_g\left(b\right) \tag{9}$$

yielding the second equilibrium condition. Condition (9) indicates that there is some *free-riding* on the part of the Local bidders under this auction rule. The left-hand side is the marginal benefit of increasing the bid for the Local bidder. It is twice smaller compared to the independent markets. Thus, in equilibrium the Local bidders are likely to be less aggressive. To see this intuitively suppose the Global bidder follows the strategy $b_g\left(v_g\right) = 0.5v_g$ and the other Local bidder follows the strategy $b_l\left(v_l\right) = 0.5v_l$, i.e. the equilibrium strategies under the Separate auction rule with $\beta = 1$. Then (8) implies $\left[v_l - b\right] = b + 0.5v_l$ or $b = 0.25v_l$. In other words, the Local bidder has an incentive to submit a bid lower than that of the other Local bidder.

The resulting system that characterizes the equilibrium is:

$$\begin{cases} \sigma_g'(b) = \frac{2\sigma_g(b)}{\left[\sigma_l(b) - b\right]} & b \in \left[0, \tilde{b}\right] \\ \sigma_l'(b) = \frac{\sigma_l(b)}{\left[\beta\sigma_g(b) - b\right]} & \\ \sigma_g\left(0\right) = \sigma_l(0) = 0 & \\ \sigma_g(\tilde{b}) = \sigma_l(\tilde{b}) = 1 & \end{cases} \tag{10}$$

Note that the boundary condition $\sigma_g(\tilde{b}) = \sigma_l(\tilde{b}) = 1$ is still applicable because all the Local bidders share the same value. Therefore, a Local bidder with the highest value has no reason to bid above $\tilde{b}$ if the highest Global bid is $2\tilde{b}$[5]. The following Proposition states that the system in (10) indeed characterizes an equilibrium under the Package auction rule.

Proposition 2 Under the Package auction rule with two markets, there is an equilibrium where the Local bidders follow a symmetric bidding strategy and the Global bidder submits only the package bid for both items even though bids for individual items are allowed (the proof is provided in the Appendix).

Proposition 2 is proved through a sequence of results. We first show that the package bid of the Global bidder has to be higher than the sum of his or her bids for individual items. We then show that if the Global bidder were to submit non-zero single-item bids, one such bid would be sufficient. The non-trivial and somewhat surprising part of the proof is showing that, although submitting a non-zero single-item bid is not without benefit, the Global bidder is better off by setting both of his or her single-item bids to zero. A single-item bid can win one item in some situations where the package bid would not have won any. However, there is a cost associated with submitting a non-zero single-item bid. Sometimes the Global bidder's package bid loses only because his or her single-item bid wins together with the Local bid in the other market. We show that the cost outweighs the benefit and that the Global bidder is better off submitting only the package bid. We prove it by relying on the convexity of the Local bidders' bidding function, a result derived from the system in (10).

As with the Separate auction rule we can easily show that the bidding functions characterized by (10) are such that given the same value the Global bidder submits a higher bid than do the Local bidders. However in this case the result holds even if no synergies are present: $\forall \beta \geq 1$, $v \in (0,1)$, $b_g(v) > b_l(v)$[6]. In other words, the Global bidder is more aggressive than the Local bidders.

Although the authors are not aware of an analytical solution to the boundary value problem in (10), we can solve this system of differential equations numerically using a

[5] If the Local bidders can have different values this argument does not work. Even if $2\tilde{b}$ is the highest possible Global bid, the Local bidder with value 1 may have an incentive to bid above $\tilde{b}$ since the other Local bidder may have a lower value and submit a lower bid.

[6] We will show here that $\forall \beta \geq 1$ and $b \in (0, \tilde{b})$, $\Phi(b) \equiv \sigma_g(b) - \sigma_l(b) < 0$, assuring the result. From the system of differential equations (10) $\Phi'(b) = \sigma_g'(b) - \sigma_l'(b) = \frac{2\sigma_g(b)}{[\sigma_l(b)-b]} - \frac{\sigma_l(b)}{[\beta\sigma_g(b)-b]}$. $\sigma_g(\tilde{b}) = \sigma_l(\tilde{b}) = 1$, implies that $\lim_{b<\tilde{b}, b\to\tilde{b}} \Phi'(\tilde{b}) = \frac{2}{[1-\tilde{b}]} - \frac{1}{[\beta-\tilde{b}]} > 0$. This implies that $\exists\ \epsilon > 0$, such that $\forall b \in (\tilde{b}-\epsilon, \tilde{b})$, $\Phi(b) \equiv \sigma_g(b) - \sigma_l(b) < 0$. Assume momentary that $\exists b_0 \in (0, \tilde{b}-\epsilon]$, such that $\Phi(b_0) \equiv \sigma_g(b_0) - \sigma_l(b_0) = 0$. It results in $\Phi'(b_0) = \sigma_g'(b_0) - \sigma_l'(b_0) = \frac{2\sigma_g(b_0)}{[\sigma_l(b_0)-b_0]} - \frac{\sigma_l(b_0)}{[\beta\sigma_g(b_0)-b_0]} \geq \frac{n\sigma_g(b_0)}{[\sigma_l(b_0)-b_0]} - \frac{\sigma_l(b_0)}{[\sigma_g(b_0)-b_0]} = \frac{(n-1)\sigma_g(b_0)}{[\sigma_l(b_0)-b_0]} > 0$. However, that implies that $\forall b \in (b_0, \tilde{b})$, $\Phi(b) \equiv \sigma_g(b) - \sigma_l(b) \geq 0$, a contradiction. We must then conclude that $\forall \beta \geq 1$ and $b \in (0, \tilde{b})$, $\Phi(b) \equiv \sigma_g(b) - \sigma_l(b) < 0$.

	Synergy					
	0%	25%	50%	75%	100%	200%
	$\beta = 1$	$\beta = 1.25$	$\beta = 1.5$	$\beta = 1.75$	$\beta = 2$	$\beta = 3$
Separate	1	.956	.933	.922	.917	.917
Package	.907	.934	.935	.934	.933	.937

Table 1: Expected Efficiency of Allocation

backward-shooting method[7]. The uniqueness of the solution is established in the literature for $\beta = 1$. In this case, the system also characterizes the equilibrium in an asymmetric auction with 2 categories of bidders whose values have different distributions on the same interval[8]. Lebrun (1999) proves uniqueness for such auctions.

2.3 Equilibrium outcomes under the two auction rules

Since the efficiency associated with the two auction rules is a central issue of this study, the first market outcome measure we look at is the allocation efficiency implied by the theoretically predicted equilibrium behavior. The allocation efficiency is simply the frequency with which the items are allocated to the bidders who value them most[9]. Table 1 presents the relationship in our two-market model between the allocation efficiency and the level of synergies (β) under the two auction rules. The results in the table are obtained by performing 1 million simulated auctions using the equilibrium bidding functions[10] and

[7] Backward shooting method involves starting with an initial guess of the free parameter $\tilde{b}$, solving the system backwards from the boundary condition using any standard algorithm (we use 4^{th}-order Runge-Kutta method) and verifying whether the initial condition is satisfied. If not, $\tilde{b}$ is adjusted in the appropriate direction until the initial condition is satisfied within the acceptable margin of error. Backward shooting is necessary due to a singularity at the initial condition. This method is used in BIDCOMP^2 program by Li and Riley (1999).

[8] Suppose there are only 2 bidders in an auction. One bidder is called Local with values distributed uniformly on $[0, 1]$. The other bidder is called Global and has a value distribution characterized by the distribution function $F_{v_g}(v) = v^{\frac{1}{2}}$ on the same interval. Using the terminology from the asymmetric auction literature, the Local bidder is the strong bidder, while the Global bidder is the weak bidder reflecting the stochastic relationship between the corresponding value distributions. The Local bidder's maximization problem is: $\max_b [v_l - b] \Pr[b \geq b_g(v_g)] = \max_b [v_l - b][\sigma_g(b)]^{\frac{1}{2}}$, yielding FOC: $[\sigma_g(b)]^{\frac{1}{2}} = \frac{1}{2}[v_l - b][\sigma_g(b)]^{\frac{1}{2}-1}\sigma_g'(b)$ or equivalently: $\sigma_g'(b) = \frac{2\sigma_g(b)}{[v_l - b]}$, i.e. the same condition as in our Package problem with $\beta = 1$. The second condition arising from the Global bidder's maximization problem is also equivalent.

[9] An alternative measure of efficiency is the ratio of the realized surplus (the value of the items to the winners of the auction) to the highest possible surplus (the highest value attainable by a feasible allocation of the items to the bidders). We use this measure in the analysis of the experimental data to complement our findings on the allocation efficiency.

[10] Those bidding functions obtained numerically were evaluated at 100 equally spaced nodes. Then a 5^{th} order polynomial was fitted to those nodes (least squares regression, $R^2 = 1.0000$) to obtain a continuous approximation to the bidding functions.

	Allocation	Seller's	Bidder's Profit		Winning Probability	
	Efficiency	Revenue	Global	Local	Global	Local
Separate ($\beta = 1$)	100%	.333	.167	.167	0.5	0.5
Separate ($\beta = 1.5$)	93.3%	.405	.324	.123	0.6	0.4
Package ($\beta = 1$)	90.7%	.272	.213	.176	.593	.407
Package ($\beta = 1.5$)	93.5%	.329	.393	.134	.678	.322

Table 2: Expected Auction Outcomes (normalized per market)

taking the sample average[11]. These numbers illustrate the effect of the interaction between the level of synergies and the auction rule. The first column of the Table highlights the effect of the auction rule when synergies are absent. In this case the Separate auction rule is the best as it achieves efficient allocations 100% of the time. In contrast, the Package rule brings to the fore the underlying asymmetry between Local and Global bidders in terms of their ability to purchase multiple items. The rule creates different incentives for the Local bidders (the free-riding component) than for the Global bidder. As a result, the equilibrium bidding behavior is different for the two categories of bidders and inefficient allocations occur with a positive probability. The other columns of Table 1 show what happens at higher levels of synergies. Clearly, the performance of the Separate rule deteriorates because it does not provide the Global bidder with the tools to take advantage of the differences in his or her preferences for the package of items versus the items acquired individually. The Package rule, on the other hand, does allow the Global bidder to submit a package bid and therefore the efficiency improves. At some high level of synergies (around 50%) the Package rule begins to outperform the Separate rule in terms of the allocation efficiency.

Figure 1 depicts the RNNE bidding strategies. The equilibrium bidding functions are provided for the cases of no synergies as well as 50% synergies (i.e. $\beta = 1.5$) under both auction rules. Figure 1 clearly shows that the bidding functions of the Global bidder are above those of the Local bidders in all the cases except for the Separate auction rule with no synergies (upper-left panel). We use these four conditions for our experimental investigation[12].

Table 2 summarizes some other key expected market outcomes obtained from simulations using the bidding functions shown in Figure 1. All the measures are given per market (or equivalently per item). As the numbers in the Table suggest, one implication

[11] Since all the auctions in the simulation are independent and the variance of the measure is finite, the sample average converges almost surely to the expected value (by the strong law of large numbers).

[12] We considered using $\beta = 2$ or even $\beta = 3$. The patterns of interest to this study are more pronounced at such high levels of synergies. However, aside from exacerbating potential fairness issues, higher values of β could have trivialized (behaviorally) the decision for the Global bidder. With a sufficiently large value advantage the Global bidders might have shown a preference for the highest possible Local value as the "safe" bid.

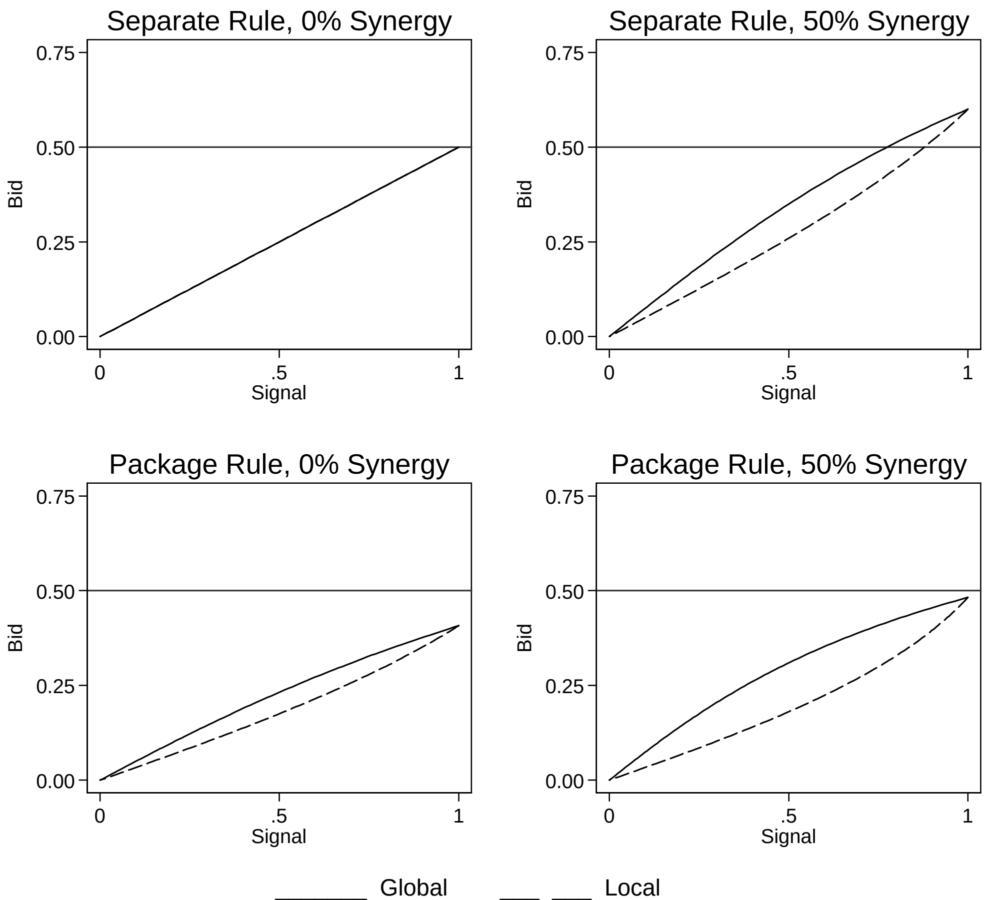

Figure 1: RNNE Bidding Strategies.

of our assumptions is that the Package auction rule has a negative effect on the seller's revenue regardless of the level of synergies. It means that under the Package rule the negative effect on the revenue, due to free-riding among the Local bidders, far outweighs any possible positive effects.

Before describing our experimental design we would like to make an observation that will be used later to interpret behavioral responses to changes in the environment. When there is a change in the environment, either in the auction rule or in the level of synergies, we distinguish between two types of incentives to modify one's behavior: *direct* and *indirect*. Direct incentives exist if a change in the environment directly affects the bidder's payoff. The incentives are indirect if the change in strategy is only a response to bidders with direct incentives to modify strategies. Thus, when synergies are introduced ($\beta > 1$), holding the auction rule constant, the Global bidder faces a direct incentive to alter his/her behavior while the Local bidders face an indirect incentive. When moving from the Separate auction rule to the Package auction rule, holding the synergy level constant, the roles are reversed. As Figure 1 illustrates, changes in bidding behavior in response to direct incentives are predicted to be more substantial than in response to indirect incentives.

3 Experimental Design

To investigate the interaction between the level of synergies and the two auction rules and its effect on various auction outcomes we use a 2x2 factorial design. Four experimental conditions are derived by combining the two auction rules with two levels of synergies: Separate rule with no synergies (condition 1 or C1), Separate rule with 50% synergies (condition 2 or C2), Package rule with no synergies (condition 3 or C3), and Package rule with 50% synergies (condition 4 or C4). Two computer-based experimental sessions were conducted for each of the four conditions (see Table (9) in the Appendix). The software for the experiment was designed using the zTree experimental software toolbox (Fischbacher, 1999). Selected screenshots of the software are provided in the Appendix. Consistent with the interpretation of the theoretical model, the game is framed as an auction taking place simultaneously in 2 markets. In each market there are two bidders: one Local bidder and the Global bidder. Local bidders can bid only in their own markets. The Global bidder can bid for items in both markets. The auction rules and the presence or absence of synergies are explained to the subjects in this context. A sample of instructions is also provided in the Appendix. In our implementation we scale and round the values that subjects receive to be random integers between 0 and 100 (inclusive). The same sequence of random values was used for all experimental conditions. This approach fixes the sample of values to better evaluate the treatment effects.

The procedures followed during the experiments are quite standard. In the beginning of each session written instructions were distributed to the subjects and read aloud by the experimenter. Following the instructions, the subjects went through several trial periods with no monetary implications to learn the functionality of the experimental software. In the beginning of the first trial period, subjects were divided into Global and Local bidders so that for every Global bidder there were 2 Local bidders. These roles were assigned randomly and anonymously and did not change throughout a given session. During a session, subjects participated in a series of 60 periods of auction bidding in groups of 3 bidders[13]. In each group one Global bidder faced 2 Local bidders. Subjects were randomly and anonymously re-assigned to different groups in every period. Sessions lasted for approximately 2 hours. The subjects were paid based on the cumulative earnings during the

[13] The resulting number of groups corresponds to the number of subjects in the Global role and is given in Table (9)

experiment converted into US dollars using the specified conversion factors for their role (see Table 10 in the Appendix). The conversion rates were announced in the beginning of the experiment. The rates are such that a subject following the RNNE strategy is expected to earn approximately $40. In addition to the cumulative earnings the subjects were paid a $6 show-up fee.

In every period subjects received random values prior to bidding. The values were different across groups and roles. In accordance with our theoretical assumptions the Local bidders in the same auction shared the same value and the Global bidder's value was the same in both markets. Every subject was asked to submit bid(s) based on the value they had received. A simple calculator was built into the experimental software that allowed subjects to experiment with different bids and evaluate potential profits. With a specially designated button, the subjects finalized their bids. Limited feedback was provided to the subjects once the outcome had been determined. The subjects were shown all the bids in their group along with only their own value and profit. The history of bidding was available for review by the subjects. Below we describe the details of the experimental procedure that differed across the auction rules.

Separate rule (C1, C2): The Separate auction rule treats the 2 markets independently. In other words, the winner of the auction is determined based on the bids in that market only. Each Local bidder submits a single bid in her market. The Global bidder submits two bids, one bid in each market. We focus on the symmetric (across markets) Nash Equilibrium where the Local bidders follow the same equilibrium strategy and the Global bidder submits the same bid in both markets. Thus, we require that the Global bidder submits the same bid in both markets. This also keeps to the minimum the differences between the two auction rules. At the same time, the Local bidders form their bids independently and therefore their bids may differ. In each market the highest bidder is awarded the item and pays her bid. The profit of the Local bidder is her value less her bid if she obtains the item and 0 otherwise. The profit of the Global bidder is based on how many items she acquires. If she obtains a single item then her profit is her value less her bid. We allow for such an event even though it is not observed in equilibrium. If both items are acquired then her profit is $2(\beta \times v_g - bid)$, i.e. she receives additional payoff determined by the synergy factor β.

Package rule (C3, C4): In contrast to the Separate rule this auction rule assigns items based on bids in both markets. Our theoretical results suggest that in an equilibrium where Local bidders follow a symmetric bidding strategy the Global bidder only submits a bid for the package of both items. Again, to focus on this equilibrium subjects in the role of the Global bidder are allowed to submit only the package bid. No bids for individual items are allowed. Note that this is equivalent to submitting identical bids for both markets as required by the Separate rule. The key difference between the rules is that under the

Package rule the outcome of the auction is determined by comparing the Global bidder's package bid and the *sum* of the 2 Local bids. Although the sum of the Local bids is used to determine the winner, the profit for a Local bidder is the difference between the Local value and her *own* bid if she obtains the item. The profit of the Global bidder is $2\beta \times v_g-$ *package bid* if both items are obtained. The Global bidder cannot acquire a single item under this auction rule.

4 Results

4.1 Bidding

In the following we report the results from 8 experimental sessions: 2 sessions for each condition. Experimental sessions were carried out at the Ohio State University. Participants were undergraduate students who had taken at least one course in economics. They were recruited via bulk E-mail advertisement.

In all our calculations and presentation of the results we use bids per item. Since in the experimental setup we solicit package bids as well as bids for individual items, a straightforward transformation is required to make these quantities comparable. In particular, the Global package bids in conditions 3 and 4 are divided by 2 to obtain the per item bids that are directly comparable to the bids of the Local bidders and Global bids under the Separate auction rule.

4.1.1 Relation to the theoretical benchmark

First, we look at the functional relationship between bids and values (the bidding strategies). We estimate the bidding functions using a random-effects model. The functions are estimated for each of the 4 conditions separately. To accommodate non-linearity of bidding functions a quadratic polynomial of values is fitted in each case[14]. To distinguish between bidder categories a dummy L_i is included along with the interaction terms between the dummy and the polynomial terms. The dummy takes on the value of 1 for Local bidders and 0 otherwise. The full specification is as follows:

$$b_{it} = \beta_0 + \beta_1 v_{it} + \beta_2 v_{it}^2 + \gamma_0 L_i + \gamma_1 L_i v_{it} + \gamma_2 L_i v_{it}^2 + u_i + e_{it} \quad (11)$$

where b_{it} and v_{it} are the bid and the value of the bidder i in period t respectively, and u_i and e_{it} are zero-mean error components.

[14] Higher order polynomials did not significantly improve the fit in most cases. At the same time, quadratic polynomials are able to closely approximate the theoretically predicted strategies. Thus, quadratic polynomials fitted to the RNNE strategies yield R^2 between 0.998 (Local strategy in C4) to 1.000 (all other cases)

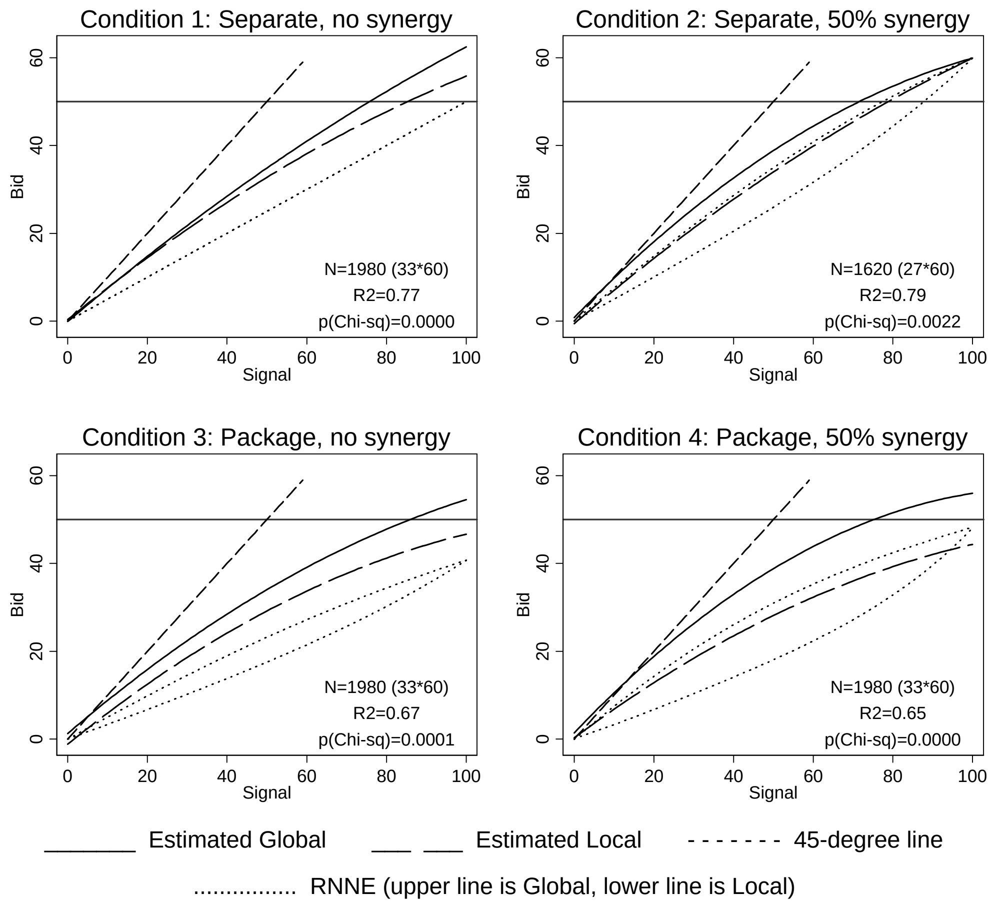

Figure 2: Estimated bidding functions, random effects regressions.

Figure (2) shows the estimated functions based on all the data in each experimental condition. There are four panels each of which contains the estimated bidding functions of the Global bidders (solid line) and Local bidders (long dashed line) along with a 45° line (short dashed line) and the corresponding RNNE (dotted lines). Although the RNNE bidding functions for both roles are depicted using the same line pattern, one can easily distinguish between them by noting that the RNNE function of the Global bidders is above that of the Local bidders for conditions $2-4$. In addition, each panel shows the number of observations used (N = number of cross-sectional groups (subjects) $\times$ number of periods) and R^2 of the regression ($R2$). Also provided is p-value of the Wald χ^2-test ($p(Chi-sq)$) that assesses the joint significance of the Local dummy and all its interaction terms (H_0: $\gamma_0 = 0$; $\gamma_1 = 0$; $\gamma_2 = 0$). A low p-value indicates that the subjects in the two roles behave differently. Figure (3) provides the same information dropping the first

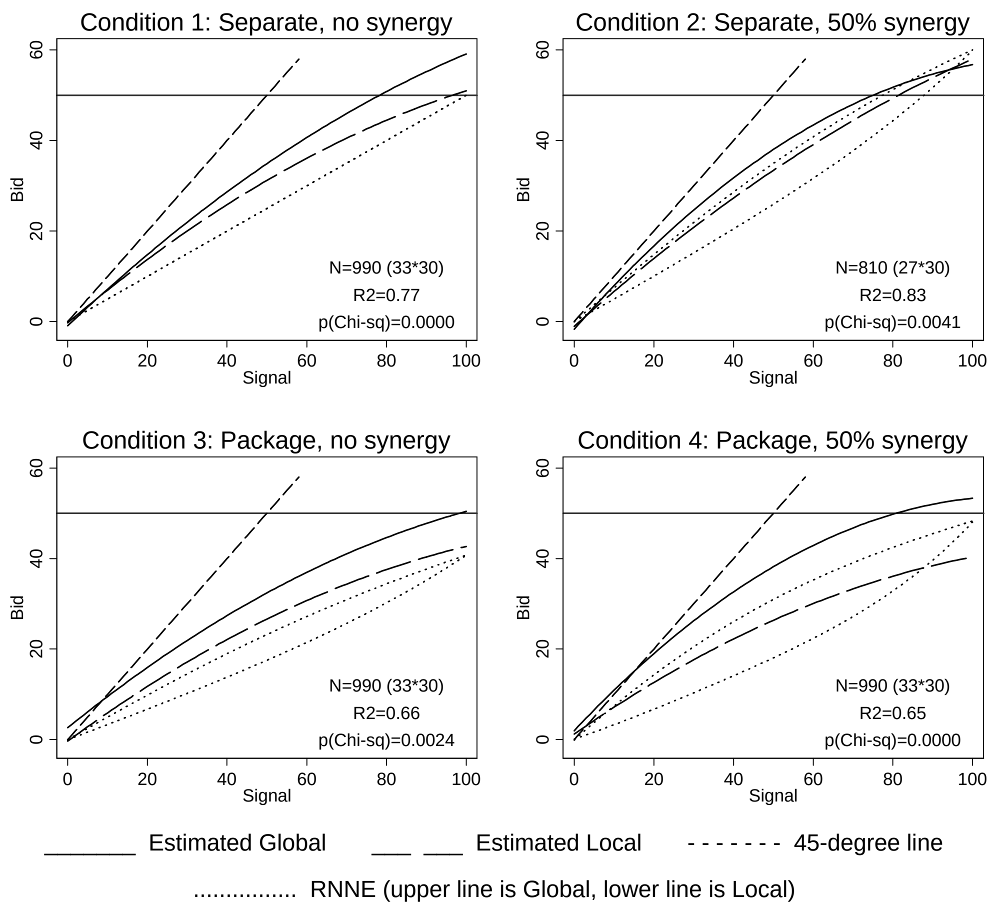

Figure 3: Estimated bidding functions, random effects regressions (last 30 periods).

30 periods. Comparing the two figures does not reveal any obvious differences in bidding patterns except that bids tend to be generally lower in the last 30 periods.

The first unambiguous feature of the data is that Global bidders are more aggressive in all 4 conditions. This is supported by joint significance of the regression coefficients corresponding to the bidding function of the Local bidders (χ^2-test). In all cases the null hypothesis that both bidder categories behave in the same way can be rejected at 1% significance level. According to the RNNE predictions this should be the case in all the environments except for the benchmark condition 1 where there is no synergy and the Separate auction rule is used. The source of the deviation in the benchmark condition is not clear[15]. However, for the purposes of our investigation we focus on changes in bidding

[15] We suspect that framing of the problem and the specifics of the experimental procedure may have contributed to this result. We plan to investigate this aspect in our future work.

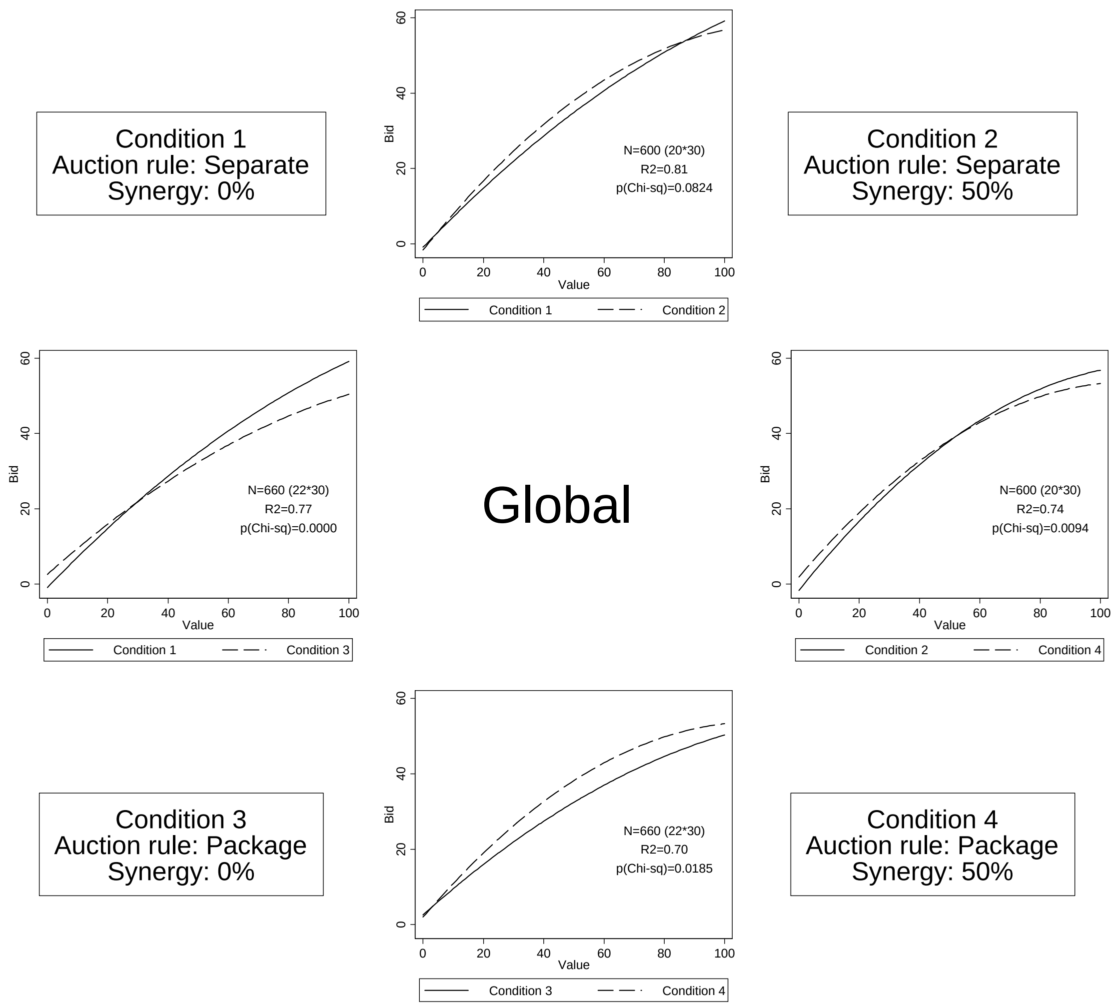

Figure 4: Comparing estimated bidding functions across conditions (last 30 periods), Global bidders

behavior *relative* to the benchmark which we discuss in more details when comparing behavior between conditions. Overall, the differences in behavior between Global and Local bidders can be summarized in the following observation:

Observation 1 *The estimated bidding functions of the Global bidders tend to be above the estimated bidding functions of the Local bidders in all 4 conditions.*

4.1.2 Changes in bidding across conditions

By examining Figures (2) and (3) it is not immediately obvious whether the differences in bidding patterns across conditions correspond to the theoretically predicted comparative statics. Specifically, does bidding tend to be relatively more aggressive in the presence of synergies and relatively less aggressive under the Package auction rule? To test these hypotheses we estimate random effects regressions for each bidder category using data from two "adjacent" conditions. We focus on the second half of the experiments (last 30 periods). The specification is identical to (11) except that the dummy and its interaction terms are used to distinguish between the bidding functions under the two conditions. The Wald χ^2-test can be used to determine whether the two functions are different. The 4 panels of Figure (4) show the pairs of estimated bidding functions for the Global bidders. The top panel compares the Global bidding functions in conditions 1 and 2. Similarly, the other three panels compare conditions 1 and 3, 2 and 4, and 3 and 4. Each panel contains information about the number of observations (N), R^2 ($R2$), and the p-value of the χ^2-test ($p(Chi - sq)$). The results are in part consistent with the previously stated hypotheses. In the markets with synergies Global bidders tend to submit higher bids (top and bottom panels). Under the Package rule Global bidders tend to submit lower bids but only when values are high (left and right panels). For a range of low values the estimated bidding function under the Separate rule is above that under the Package rule (C1 vs C3 and C2 vs C4) contrary to the theoretical prediction. This is particularly true when synergies are present (right panel). We can summarize this in the following observation:

Observation 2 *Global bidders tend to submit higher bids in markets with synergies. They also tend to submit lower bids under the Package auction rule but only when values are high.*

Figure (5) provides similar information about the observed bidding behavior of the Local bidders. From this figure it is clear that Package auction rule is the strongest factor affecting the bidding behavior of the Local bidders (the left and the right panels). Consistent with the theoretical prediction, Package auction rule induces less aggressive bidding among the Local bidders which is likely to be driven by the free-riding effect. The effect of synergy is not nearly as uniform. Under the Separate auction rule, Local bidders bid higher when synergies are present (top panel). However, the effect of synergies is not statistically significant under the Package auction rule (bottom panel). One explanation is that the effect is masked by the overwhelming response to the change in auction format. The latter is likely to introduce sizeable variability in bidding.

Observation 3 *Local bidders tend to submit dramatically lower bids under the Package auction rule. The effect of synergies is not uniform: under the Separate auction rule Local bidders are more aggressive when synergies are present, while no clear effect is observed under the Package auction rule*

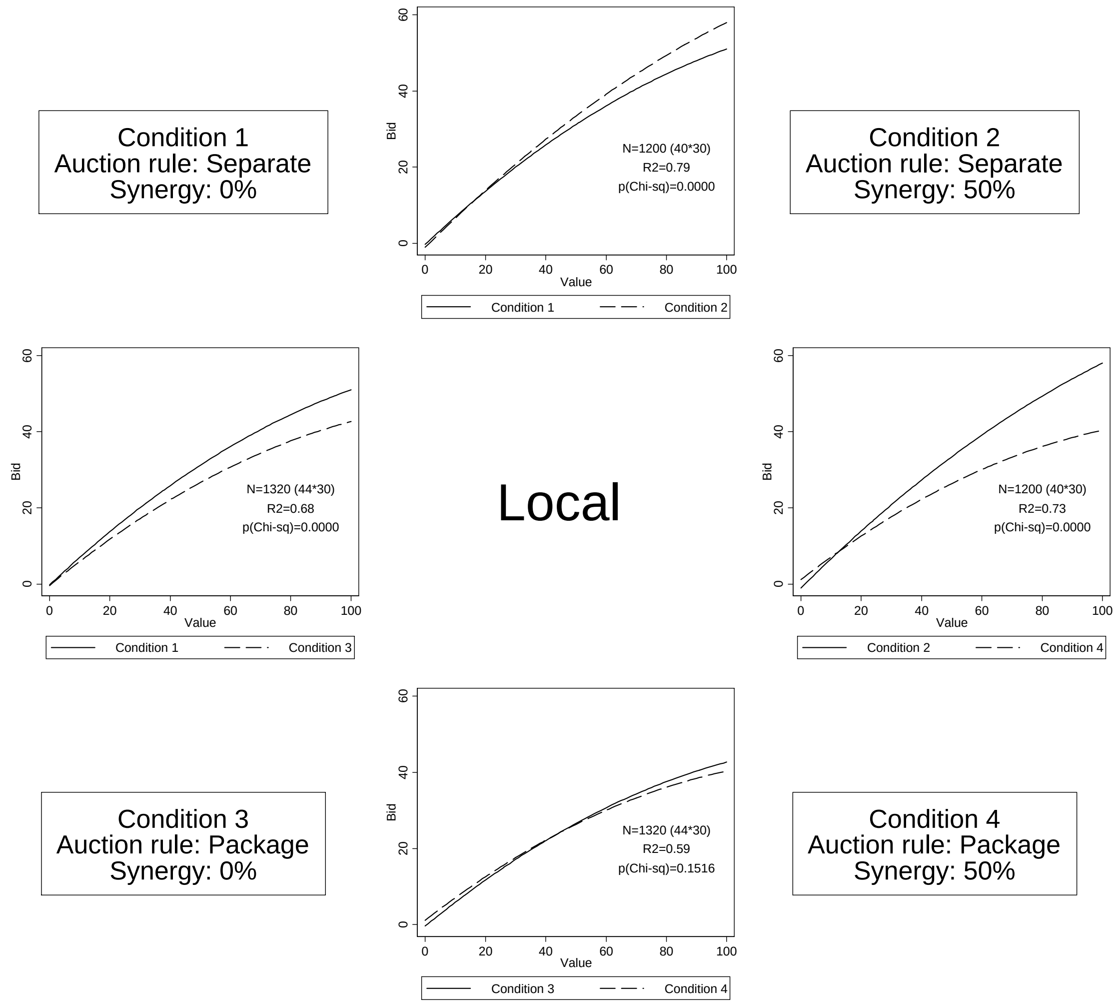

Figure 5: Comparing estimated bidding functions across conditions (last 30 periods), Local bidders

The last two observations are consistent with the idea that subjects tend to respond stronger to the "direct" incentives. In our theoretical discussion we noted that Local bidders' maximization problem under either auction rule does not change with the level of synergies. The level of synergy β does not appear in either equation (3) or equation (7). Thus, *if* the Global bidder were to follow exactly the same strategy when synergies are 0% and 50%, the best response of the Local bidders would have been to keep their bidding behavior unchanged. More aggressive bidding on the part of the Local bidders at the higher level of synergies is a result of more subtle "indirect" incentives. It can be viewed as a response to more aggressive bidding by the Global bidder. On the other hand, Local bidders' maximization problem differ significantly depending on the auction rule.

Comparison of the Locals' decision-making problem under the Separate auction rule (3) to that under the Package auction rule (7) confirms that bidding less aggressively under the Package rule is a "direct" incentive (free-riding) for the Locals. Thus, it is not surprising that Local bidders respond strongly to the auction rule used, while the degree of synergies affect their behavior much less uniformly.

For the Global bidder the opposite is true: more aggressive bidding in response to a higher level of synergies is the "direct" incentive. Even if Locals use the same strategy when $\beta = 1.5$ as when $\beta = 1$ it is clear from equations (1) and (6) that the Global bidder has a strong incentive to adjust his/her bidding behavior as the level of synergies changes. On the other hand, a change in the auction rule does not change the decision problem for the Global bidder. The prediction that the Global bidder should bid less aggressively under the Package auction rule is a result of a more subtle equilibrium adjustment. We summarize these findings in the following observation:

Observation 4 *The Local bidders are more responsive to the change in auction rule, while the Global bidders are more responsive to the introduction of synergies.*

4.1.3 Other features of the data

A quick look at Figure (2) suggests that subjects submit bids that are on average higher than the RNNE. Additionally, comparing Figure (2) and Figure (3) suggests that this overbidding declines as experiment progresses. In order to assess the statistical significance of these observations we run random effects regressions with the following specification:

$$d_{it} = \beta_0 + \beta_1 p + u_i + e_{it}$$

where $d_{it} = b_{it}^{observed} - b_{it}^{RNNE}$ is the difference between the observed bids and the predicted bids (overbidding margin) for bidder i in period t, and p is the experimental period. Estimation is performed separately for each condition and each bidder category. This specification allows us to examine how the average deviation from the RNNE behaves over time. The fitted values are depicted on Figure (6). All estimated coefficients are statistically significant at 1%. Estimation results confirm that mean overbidding margin is positive in all cases and has a statistically significant downward time trend. Despite the trend, bidding remains above RNNE in all cases but one. Bidding above the RNNE is a common finding in auction experiments (Kagel and Levin, 1993; see Kagel, 1995 for a review). Risk aversion as a possible explanation has been a subject of a debate for many years. Risk averse bidders in first-price auctions are predicted to be more aggressive in the sense that their bids are higher as they discount their values less than a risk-neutral bidder would. In many cases risk-aversion can be ruled out as the main driving force for

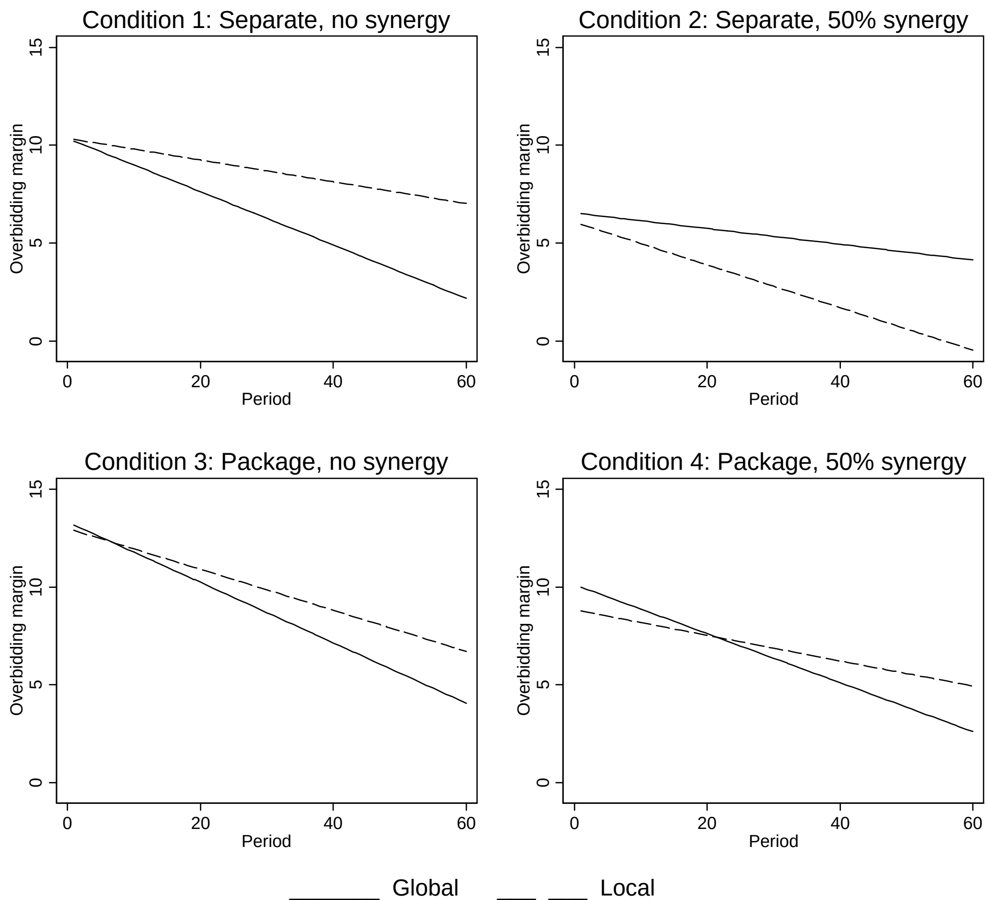

Figure 6: Overbidding relative to RNNE over time.

overbidding (**need a good reference here**). However, this behavioral regularity cannot be ignored and underscores the importance of using a benchmark condition such as C1 and focusing on bidders' behavior relative to the benchmark rather than examining the absolute numbers.

Observation 5 *The observed bids tend to be above the RNNE. There is a downward time trend in the general level of bidding aggressiveness. Despite the time trend, the level of aggressiveness fails to reach the RNNE in almost all the cases.*

4.2 Market outcomes

In this section we examine how the bidding patterns described above translate into auction outcomes. In particular, we are interested in allocation efficiency and other measures

		No Synergy	50% Synergy	Difference	p-value
Separate	Observed	C1: 0.838	C2: 0.830	-0.004	0.4331
	(RNNE)	(1.000)	(0.896)		
Package	Observed	C3: 0.809	C4: 0.879	0.070	0.0039
	(RNNE)	(0.894)	(0.930)		
Difference		-0.029	0.063		
p-value		0.093	0.008		

Table 3: Allocation Efficiency (last 30 periods)

including the seller's revenue and bidders' profits. The results presented below are averages over the two markets. Although in the symmetric RNNE (i.e. where the Local bidders follow the same bidding strategy) outcomes in both markets are exactly the same, this is not the case in the data. As a result, each measure is computed for both markets separately, then the average is taken. In addition, profits of the Global bidder are reported per item, i.e. if a Global bidder won two items and received 40 units in profit, the reported profit is 20. Profits of the Local bidders are again averaged across the two markets. These modifications are performed in order to align the results with the theoretical model and to make them comparable across conditions.

4.2.1 Allocation efficiency

One of the key predictions of the theoretical model is the efficiency implications of the relationship between the auction rules and the level of synergy between objects. An efficient allocation is an allocation where the item goes to the bidder with the highest value. Theoretically, the Separate auction rule works perfectly when there is no additional benefit to the Global bidder from acquiring items in both markets. In this case the efficiency is 100%. However, when the items are complements the efficiency deteriorates. The opposite is true under the Package auction rule.

Efficiency can be measured in different ways. The straightforward way to calculate efficiency is to compute the fraction of allocations that are efficient. We refer to this measure as *eff1*. Table (3) reports *eff1* for each of the 4 conditions. Rows correspond to the auction rule used (Separate or Package) while columns correspond to the degree of synergy (No Synergy, 50% Synergy). We report sample averages using the data from the last 30 periods. The outcomes implied by the RNNE strategies and the realized set of values are also provided (in brackets). The third row and the third column report average differences in the observed levels of efficiency between adjacent conditions. These differences are calculated as follows. As mentioned above the same sequence of random values is used in all conditions. Furthermore, the same random matching of those values into groups is used across conditions. Thus, Global and Local bidders in group j in period t in condition C1 have the same values as the bidders in the corresponding roles in group j in period t in the other three conditions. Due to this data structure, the differences between outcomes in corresponding groups are attributable exclusively to differences in bidding behavior rather than differences in values of the bidders. We compute these differences and perform a t-test under the null hypothesis that the mean difference is zero and the alternative hypotheses prescribed by the comparative statics predictions for each case. The last row and the last column report one tail p-values of this test. Due to the fact that we have data only for 9 groups in condition C2, the reported average differences and the tests are based on these 9 groups whenever C2 is used in comparison. For average differences in the other cases as well as for the average observed efficiency levels we report results based on the data from all 11 groups[16].

As expected, the observed efficiency is lower than the RNNE predictions. This is a common finding in auction experiments. Thus, Cox et al. (1982) report that in their data from several first-price auction experiments with various numbers of participants (auction sizes from 3 to 9), the average fraction of efficient allocations was 0.88 (ranging from 0.83 to 0.95). The values of *eff1* reported in Table (3) are of a similar magnitude. Although the smallest auction size in Cox et al. (1982) is 3 bidders, the comparison is relevant as they find no clear monotonic relationship in their data between the level of efficiency and the number of auction participants.

Comparing the observed efficiency between conditions we find support for most of the theoretical predictions. Efficiency falls somewhat as the level of synergy increases under the Separate auction rule (C1 vs C2). The change is not statistically significant in the sample. Efficiency is also lower when Package rule is used in the absence of synergies (C1 vs C3). The decrease is significant at 10% level. A rather dramatic increase in efficiency is observed if Package rule is used in the environment with synergies compared to no synergies (C3 vs C4). The increase is highly statistically significant ($p<0.01$). As a result, the efficiency is indeed higher under the Package rule compared to the Separate rule when synergies are at 50% (C2 vs C4). The observed difference is comparable in magnitude to the predicted (see the difference between the RNNE values in brackets).

Observation 6 *Consistent with the RNNE, synergies are associated with a lower frequency of efficient allocations under the Separate auction rule and a higher frequency of efficient allocations under the Package auction rule. Switch from Separate to Package auction rule in the absence of synergies has a negative impact on efficiency while it has a*

[16] As a result, the reported average difference may not correspond to the difference between the reported efficiency levels whenever C2 is involved in a comparison.

		No Synergy	50% Synergy	Difference	p-value
Separate	Observed	C1: 0.971	C2: 0.940	-0.030	0.000
	(RNNE)	(1.000)	(0.987)		
Package	Observed	C3: 0.961	C4: 0.967	0.006	0.238
	(RNNE)	(0.989)	(0.991)		
Difference		-0.010	0.028		
p-value		0.076	0.002		

Table 4: Fraction of captured surplus (last 30 periods)

positive effect when synergies are present.

4.2.2 Fraction of captured surplus

The fraction of captured surplus, or *eff2*, is another way to assess the efficiency of an auction procedure. In our simple model it is calculated as the ratio of the value of the winner to the highest value among the auction participants, which may or may not be the same. More generally, it measures what percentage of the value of the most efficient (highest-value) allocation is captured by the observed allocation. As such, it does not distinguish between the identities of the bidders. In other words, as long as the value of the bidder who received the object is close enough to the value of the bidder who was supposed to receive it as part of the efficient allocation, the penalty to *eff2* is small. As a result, *eff2* is at least as high as *eff1* and is usually much higher. Also, unlike the frequency of efficient allocations, *eff2* is sensitive to such manipulations as adding a constant to all the values. Table (4) reports the average *eff2* in each condition. The results are similar to those with *eff1* except that for *eff2* the statistically insignificant difference is between conditions with different synergy levels under the Package rule (C3 vs C4) rather than under the Separate rule. Despite this observation, *eff2* just like *eff1* is substantially higher under the Package rule compared to the Separate rule when synergies are present (C2 vs C4). The differences between the results from *eff1* and *eff2* are due in part to the fact that the most severe penalty to *eff1* results from any misallocation while the most severe penalty to *eff2* occurs from the Global bidder obtaining just one object and losing the synergistic value component under the Separate auction rule. While the former is reflected in a generally lower level of *eff1* compared to *eff2*, the latter explains the fact why, under the Separate auction rule, synergies have a stronger negative effect on efficiency as measured by *eff2* compared to *eff1*.

Observation 7 *Comparative statics results for the fraction of captured surplus are consistent with that of allocation efficiency and the RNNE in all cases. They are mostly*

		No Synergy	50% Synergy	Difference	p-value
Separate	Observed	C1: 42.6	C2: 45.0	2.9	0.000
	(RNNE)	(34.0)	(41.3)		
Package	Observed	C3: 37.1	C4: 40.3	3.2	0.000
	(RNNE)	(27.8)	(33.5)		
Difference		-5.5	-4.6		
p-value		0.000	0.000		

Table 5: Seller's revenue (last 30 periods)

statistically significant except for the increase in efficiency under the Package rule due to higher synergies.

4.2.3 Seller's revenue

The seller's revenue is another important measure of auction performance. It is directly related to the magnitude of bids. As a result, more aggressive bidding leads to higher revenues for the seller. Thus, by inducing more aggressive bidding, synergies are expected to bring about a higher level of seller's revenue. This is not surprising since synergies increase the overall economic surplus (i.e. the total value realizable by an efficient allocation). A part of the increase is transferred to the seller. The degree of synergies among objects, however, is an exogenous characteristic of the bidders' preferences and is not something that can be easily controlled by the auction designer. More interestingly, Package bidding rule is predicted to have a strong negative effect on the seller's revenue since the free-riding incentives it introduces depress the general level of bids. Thus, using package bidding in an environment without synergies may reduce not only the efficiency but, due to less aggressive bidding, the seller's revenue as well. Although at higher levels of synergies the Package rule seems to be superior as far as the efficiency is concerned, its RNNE performance in terms of the seller's revenue is consistently worse when compared to the Separate rule.

Table (5) reports the average seller's revenue. As usual, the averages provided are per-item. As one can see from the table, the theoretical predictions with respect to the *direction* of the changes in the seller's revenue between conditions are mimicked perfectly by the data. First, the average revenue is higher in the presence of synergies regardless of the auction rule (C1 vs. C2, $p = 0.000$; and C3 vs. C4, $p = 0.000$). Under the Package auction rule this result obtains in spite of the fact that subjects in the role of Local bidders do not bid higher when synergies are present (see bottom panel of Figure (5)). Thus, the main driving force behind this observation is more aggressive bidding on the part of the

		No Synergy	50% Synergy	Difference	p-value
Separate	Observed	C1: 12.4	C2: 28.1	15.1	0.000
	(RNNE)	(16.8)	(34.5)		
Package	Observed	C3: 15.5	C4: 36.1	20.6	0.000
	(RNNE)	(22.3)	(41.5)		
Difference		3.1	9.0		
p-value		0.000	0.000		

Table 6: Global bidders' profits

		No Synergy	50% Synergy	Difference	p-value
Separate	Observed	C1: 11.0	C2: 10.1	-0.7	0.131
	(RNNE)	(17.2)	(12.0)		
Package	Observed	C3: 12.8	C4: 9.6	-3.2	0.000
	(RNNE)	(17.4)	(12.9)		
Difference		1.8	-1.1		
p-value		0.013	0.095		

Table 7: Local bidders' profits

Global bidders. Second, Package rule results in lower seller's revenue regardless of the level of synergies (C1 vs. C3, $p = 0.000$; and C2 vs. C4, $p = 0.000$). In this case, the result is driven primarily by the appropriate response of the Local bidders. This is another confirmation of the observation that the response to the change in the auction rule (threshold/free-riding problem) is a very robust phenomenon.

Parameterization of our model is such that in theory the effects of synergies and auction rule on the revenue are roughly of equal magnitude. Thus, the negative effect of the Package rule is canceled out by the positive effect of the 50% synergy so that the revenue is almost the same in conditions 1 and 4. The observed revenue is noticeably smaller in condition 4 than in condition 1 (difference of -2.4, $p = 0.001$). Thus, we conclude that the negative effect of package bidding on the seller's revenue is relatively larger in magnitude than the positive effect of the synergies between items.

Observation 8 *Changes in the seller's revenue are consistent with the RNNE. It is higher in the presence of synergies and lower under the Package rule. The negative effect of the Package rule is stronger than the positive effect of synergies.*

4.2.4 Buyers' Profits

Tables (6) and (7) report the per-item average profits of the bidders. There are several interesting features of the data. As expected, the biggest observed differences between the two bidder categories are in the conditions with 50% synergy (C2 and C4). The presence of synergy between objects unambiguously benefits Global bidders. Synergies lead to significant increases in Global bidders' profits under both the Separate rule (+15.1, C1 vs. C2, $p = 0.000$) and the Package rule (+20.6, C3 vs. C4, $p = 0.000$). They also obtain more than twice the profits of their Local opponents in conditions C2 and C4. Such a disparity in earnings is consistent with a more advantageous position of the Global bidders.

Local bidders' profits are predicted to decrease under either auction rule as the synergy is introduced. This is observed in data under the Package auction rule (-3.2, C3 vs. C4, $p = 0.000$). However, under the Separate auction rule the change is small and not statistically significant (-0.7, C1 vs. C2, $p = 0.131$).

Observation 9 *(Synergy effect) Consistent with the RNNE Global bidders benefit from synergies. On the other hand, the predicted decline in the profits of the Local bidders is small; it is also not statistically significant under the Separate rule.*

Package auction rule is primarily beneficial to Global bidders. Increase in profits is statistically significant for Global bidders without synergies (+3.1, C1 vs. C3, $p = 0.000$) as well as when $\beta = 1.5$ (+9.0, C2 vs. C4, $p = 0.000$). Despite the dramatic changes in bidding behavior (see the right and left panels in Figure 5), the profits of the Local bidders do not change much. Without synergies, the profits increase slightly (+1.8, C1 vs. C3, $p = 0.013$). When the synergy level is 50%, then the change is negative and borderline statistically significant (-1.1, C1 vs. C3, $p = 0.095$). These findings point at an unusual phenomenon. It is often found (e.g. Chernomaz, 2006) that in first-price auctions bidders have a tendency to be excessively aggressive even though less aggressive bidding would have brought significantly higher profits. Here, we find that the Local bidders under the Package rule are dramatically less aggressive when compared to the Separate auction rule, even though the benefits of doing so are very small or even negative. Thus, the free-riding incentives of the Package rule appear to have an effect comparable to and clearly distinguishable from the usual tendency of the bidders to bid too high.

Observation 10 *(Auction rule effect) Consistent with the RNNE Global bidders benefit from the Package auction rule. Dramatically less aggressive bidding under the Package rule results in slightly higher profits for the Local bidders when synergies are absent and in lower profits when synergies are present.*

Table (8), where we report the frequency of winning for Global bidders (results for Local bidders are a mirror image and are not reported), reveals several other interesting

		No Synergy	50% Synergy	Difference	p-value
Separate	Observed	C1: 0.564	C2: 0.552	-0.020	0.147
	(RNNE)	(0.491)	(0.604)		
Package	Observed	C3: 0.618	C4: 0.712	0.094	0.000
	(RNNE)	(0.597)	(0.697)		
Difference		0.055	0.174		
p-value		0.006	0.000		

Table 8: Global bidders' frequency of winning

features of the data. First of all, the asymmetry in bidding behavior across types is more pronounced under the Package bidding rule. Since the values are distributed uniformly on the same interval for all bidders, the differences in winning probabilities between bidder categories reflect directly the differences in their bidding functions. The probability of winning is higher for the Global bidders in all the conditions as a result of more aggressive bidding. However, under the Package rule the discrepancy is higher, an observation consistent with much less aggressive bidding on the part of the Local bidders as a result of free-riding. Finally, the winning probabilities are much closer to the RNNE prediction under the Package auction rule than under the Separate auction rule.

Observation 11 *The differences in probabilities of winning between the two bidder categories are larger and more aligned with the RNNE prediction under the Package auction rule than under the Separate auction rule.*

5 Conclusion

This paper is a contribution to the growing literature on the use of package bidding in multi-object auctions. We take a close look at the trade-offs associated with the use of a sealed-bid first-price procedures that allow package bidding. At the cost of ignoring some features of combinatorial bidding which make it particularly difficult to analyze, we focus on a simple model that illustrates how the effect of package bidding on the economic efficiency and the seller's revenue varies depending on the degree of synergies between the sale objects. A number of simplifications allow us to obtain Risk-Neutral Nash equilibrium bidding functions as well as some performance measures for auctions with and without package bidding. With this important theoretical benchmark at hand, we evaluate the strength of the major strategic forces involved using laboratory experiments with undergraduate students.

A major reason to contemplate the use of package bidding is the presence of synergies (complementarities) among the objects to be auctioned off. Single-item auctions are ill-equipped to handle such situations resulting in potential penalties to economic efficiency and seller's revenue. The bidders for whom the objects are complementary may bid cautiously to avoid acquiring only a part of the package and losing the synergistic component of the value. Multi-item auctions with package bidding solve this issue, but introduce a type of free-riding incentive which in turn may cause some bidders to bid too low hurting efficiency and the revenue.

In a simple model with two types of bidders we show how these forces interact with each other in the context of sealed-bid first-price single-round auctions. In the model we analyze there are two markets with one item for sale. A single Local bidder in each market bids against the Global bidder who is able to compete in both markets. The values for the sale objects are distributed independently across the two bidder categories. The major assumption that helps us with the tractability of the model is that Local bidders have the same value for the objects. We view it as applicable to the settings where the values of the Local bidders tend to be more correlated with each other than with the value of the Global bidder. Using this model we analyze the performance of two auction rules: the Separate auction rule which is simply 2 simultaneous single-item auctions, and the Package auction rule which, in addition to standalone bids, allows the Global bidder to submit a package bid for both items. The predictions of the model confirm the intuition that the higher the degree of synergies is, the lower is the allocation efficiency attained by the Separate auction rule. The opposite pattern is true for the Package auction rule. Package bidding is used strategically by the Global bidder causing the threshold problem among the Local bidders. When synergies are absent this results in a higher probability of misallocations. When synergies are high this effect counteracts the exposure effect improving the efficiency of allocation.

In this setting we prove an interesting result which suggests that under the Package auction rule the Global bidder only needs to use the package bid and sets the standalone bids for individual items to zero. As a result, the Package rule essentially compares the Global package bid to the sum of the Local bids. Intuitively, this feature of the Package rule creates the threshold problem, an incentive for the Local bidder to free-ride on each other and bid less aggressively. The direct implication of this phenomenon is a decrease in the seller's revenue. In fact, the decrease is so dramatic that the seller using the Package rule is expected to fair much worse than the seller using the Separate rule regardless of the level of synergies.

To investigate the aforementioned trade-offs we use a 2x2 factorial experimental design with the two auction rules and two levels of synergies (0% and 50%). In line with results from previously conducted studies, subjects tend to bid above the Risk-Neutral Nash equilibrium predictions. The differences in behavior between the Local and the Global bidders are largely consistent with the theory. Global bidders are more aggressive than the Local bidders in all the settings. This should be the case in all but one of the conditions. More importantly, the comparative statics of our model are mostly supported by the experimental evidence. Bidders tend to be more aggressive when synergies are present than when there are no synergies. They also tend to be less aggressive under the Package rule than under the Separate rule. From the analysis of these effects we conclude that subjects are more responsive to direct incentives. As such, the response to the higher level of synergies is more pronounced among the Global bidders, while the change in behavior under the Package rule is much more dramatic among the Local bidders. We find that the latter is a very robust behavioral phenomenon and is likely to be driven by strong free-riding incentives. Less aggressive bidding among subjects in the Local role under the Package rule brings them only a marginal increase in profits when compared to the Separate rule. This observation should be contrasted to the findings in other studies where bidders tend to bid more aggressively at great penalties to their profits.

The experimental evidence with respect to the efficiency or the two auction formats supports the presence of the aforementioned trade-off. When synergies are absent we find that Separate auction rule achieves a higher level of efficiency compared to the Package rule. On the other hand, at 50% synergies Package rule performs better than the Separate rule. The latter result is significant even though the theoretically predicted effect is small. The implications of the bidding behavior for the seller's revenue are also directionally consistent with the theory. More aggressive bidding when synergies are high brings higher revenues. Less aggressive bidding under the Package rule has a substantial negative effect. In fact, Package rule is consistently worse with respect to the amount of money it is able to raise. Despite the superior performance with respect to efficiency, the negative effect of the Package rule on the seller's revenue together with the robustness of the free-riding effect found in the experimental data point at a potentially significant downside of package bidding.

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7 Appendix

7.1 Appendix: Proof of Proposition 1

Denote by $b_g^H(x)$ and $b_g^L(x)$ the solution to the Global's maximization problem and by $\sigma(.)$ the strictly monotonic inverse function of the Locals' bid function in equilibrium. Assume that $b_g^H(x) > b_g^L(x) \geq 0$. We show first that $b_g^H(x) > b_g^L(x) \geq 0$ implies $x > b_g^H(x)$. Subsequently, we show that if $b_g^H(x) > b_g^L(x) \geq 0$, then either the pair $(b_g^H(x), b_g^H(x))$ and/or the pair $(b_g^L(x), b_g^L(x))$ are also profit maximizers, a contradiction to the uniqueness result.

The Profit of the Global at the optimal choice is:

$$\Pi_g(b_g^H(x), b_g^L(x)) \equiv \sigma(b_g^L(x))[2\beta x - (b_g^H(x) + b_g^L(x))] + [\sigma(b_g^H(x)) - \sigma(b_g^L(x))][x - b_g^H(x)]. \tag{12}$$

If instead the Global uses $b_g^L(x)$ in both market profit will be:

$$\Pi_g(b_g^L(x), b_g^L(x)) \equiv \sigma(b_g^L(x))[2\beta x - (b_g^L(x) + b_g^L(x))]. \tag{13}$$

Denote by DL the difference between (12) and (13), thus:

$$\begin{aligned} DL &\equiv [\Pi_g(b_g^L(x), b_g^L(x)) - \Pi_g(b_g^H(x), b_g^L(x))] \\ &= \sigma(b_g^L(x))[b_g^H(x) - b_g^L(x))] - [\sigma(b_g^H(x)) - \sigma(b_g^L(x))][x - b_g^H(x)]. \end{aligned} \tag{14}$$

However, $x \leq b_g^H(x)$ implies that $DL \geq \sigma(b_g^L(x)[b_g^H(x) - b_g^L(x))] > 0$, a contradiction, thus, $b_g^H(x) > b_g^L(x) \geq 0$ implies $x > b_g^H(x)$.

If instead the Global uses $b_g^H(x)$ in both market profit will be:

$$\begin{aligned} \Pi_g(b_g^H(x), b_g^H(x)) &= \sigma(b_g^H(x))[2\beta x - (b_g^H(x) + b_g^H(x))] \\ &= \sigma(b_g^H(x))[2\beta x - (b_g^H(x) + b_g^L(x))] - \sigma(b_g^H(x))[b_g^H(x) - b_g^L(x)] \end{aligned} \tag{15}$$

Denote by DH the difference between (12) and (15), thus:

$$\begin{aligned} DH &\equiv [\Pi_g(b_g^H(x), b_g^H(x)) - \Pi_g(b_g^H(x), b_g^L(x))] \\ &= [\sigma(b_g^H(x)) - \sigma(b_g^L(x))][2\beta x - (b_g^H(x) + b_g^L(x))] - \sigma(b_g^H(x))[b_g^H(x) - b_g^L(x)] \\ &- [\sigma(b_g^H(x)) - \sigma(b_g^L(x))][x - b_g^H(x)]. \end{aligned} \tag{16}$$

Since the pair $b_g^H(x)$ and $b_g^L(x)$ are profit maximizing it should be the case that $DL < 0$ and $DH < 0$, otherwise either the pair $(b_g^H(x), b_g^H(x))$ and/or the pair $(b_g^L(x), b_g^L(x))$ are also profit maximizers. Thus, $[DL + DH] < 0$. However,

$$\begin{aligned}[D1 + D2] = &[\sigma(b_g^H(x)) - \sigma(b_g^L(x))][2\beta x - b_g^H(x) - b_g^L(x)] \\ &- [\sigma(b_g^H(x)) - \sigma(b_g^L(x))][b_g^H(x) - b_g^L(x)] - 2[\sigma(b_g^H(x)) - \sigma(b_g^L(x))][x - b_g^H(x)] \\ &= [\sigma(b_g^H(x)) - \sigma(b_g^L(x))][2\beta x - b_g^H(x) - b_g^L(x) - b_g^H(x) + b_g^L(x) - 2x + 2b_g^H(x)] \\ &= [\sigma(b_g^H(x)) - \sigma(b_g^L(x))][2(\beta - 1)x] \geq 0, \end{aligned} \tag{17}$$

and strict when $\beta > 1$.

7.2 Appendix: Proof of Proposition 2

Let us denote market 1 as "left" and market 2 as "right" for notational convenience. Let $b_l^{left}(v_l)$ be the bidding function of the Local bidder in the left market and $b_l^{right}(v_l)$ be the bidding function of the Local bidder in the right market. Also, let $\lambda(v_g) \equiv 2b_g^{pack}(v_g)$, $b_g^{left}(v_g)$, and $b_g^{right}(v_g)$ denote the Global's bidding functions for the package, left item alone and right item alone respectively. Without loss of generality, assume that $b_g^{left}(v_g) \geq b_g^{right}(v_g)$.

Result 1 In equilibrium, $b_g^{left}(v_g) + b_g^{right}(v_g) \leq \lambda(v_g)\ \forall \beta \geq 1$, i.e. the Global bidder's package bid is at least as large as the sum of the bids for individual items. By implication, using a package bid is at least as beneficial as not using it at all.

Proof. Suppose $b_g^{left}(v_g) + b_g^{right}(v_g) > \lambda(v_g)$ then the package bid $\lambda(v_g)$ never wins and therefore the Global bidder is effectively using only standalone bids. It is easy to show that adjusting $\lambda(v_g)$ upward so that $\lambda(v_g) = b_g^{left}(v_g) + b_g^{right}(v_g)$ is beneficial for the Global bidder. On standalone bids the Global bidder wins 2 items if $b_g^{right}(v_g) > b_l^{left}(v_l) = b_l^{right}(v_l)$ and consequently $b_g^{left}(v_g) + b_g^{right}(v_g) > b_l^{left}(v_l) + b_l^{right}(v_l)$. He or she wins nothing if $b_g^{left}(v_g) < b_l^{left}(v_l) = b_l^{right}(v_l)$ and consequently $b_g^{left}(v_g) + b_g^{right}(v_g) < b_l^{left}(v_l) + b_l^{right}(v_l)$. Suppose $b_g^{left}(v_g) > b_g^{right}(v_g)$. Then he or she wins only one item if $b_g^{left}(v_g) > b_l^{left}(v_l) = b_l^{right}(v_l) > b_g^{right}(v_g)$, i.e Local bids exceed the Global bid for the right item but fall short of the Global bid for the left item. In this situation $b_g^{left}(v_g)$ and $b_l^{right}(v_l)$ win one item each. We can break down this event even further: $b_g^{left}(v_g) > b_l^{left}(v_l) = b_l^{right}(v_l) \geq \frac{b_g^{left}(v_g)+b_g^{right}(v_g)}{2}$ and $\frac{b_g^{left}(v_g)+b_g^{right}(v_g)}{2} > b_l^{left}(v_l) = b_l^{right}(v_l) > b_g^{right}(v_g)$. By setting $\lambda(v_g) = b_g^{left}(v_g) + b_g^{right}(v_g)$ the Global bidder can increase his or her expected profits because doing so would not affect the profits except in the last case. In the last case the package bid would win the auction and would bring both items to the Global bidder. Without the package bid, the Global would only win one item despite the fact that the sum of Global bids is greater than the sum of Local bids. Note that this argument holds for $\beta = 1$. In the case when $b_g^{left}(v_g) = b_g^{right}(v_g)$, the Global bidder is no worse off by setting $\lambda(v_g) = b_g^{left}(v_g) + b_g^{right}(v_g)$. ■

Result 2 The Global bidder never needs to use all three bids in equilibrium.

Proof. Since $b_g^{left}(v_g) \geq b_g^{right}(v_g)$ and $b_l^{left}(v_l) = b_l^{right}(v_l)$ it follows that $b_g^{left}(v_g) + b_l^{right}(v_l) \geq b_g^{right}(v_g) + b_l^{left}(v_l)$. In the presence of a package bid $\lambda(v_g)$, the Global cannot win both items on stand-alone bids and since $b_g^{left}(v_g) + b_l^{right}(v_l) \geq b_g^{right}(v_g) + b_l^{left}(v_l)$ the Global can at most win one item. This is when $b_g^{left}(v_g) + b_l^{right}(v_l) \geq 2b_g^{pack}(v_g)$ and $b_g^{left}(v_g) \geq b_l^{left}(v_l) = b_l^{right}(v_l)$. In this case the Global wins the left item if $b_g^{left}(v_g) + b_l^{right}(v_l) > b_g^{right}(v_g) + b_l^{left}(v_l)$ or the right item if $b_g^{left}(v_g) + b_l^{right}(v_l) = b_g^{right}(v_g) + b_l^{left}(v_l)$ and the tie break goes this way. However, in the last event, winning the right item is the same as winning the left item. Therefore, having two stand-alone bids is not helpful. ■

Thus assume from now on that when the Global is using a package bid $\lambda(v_g) \equiv 2b_g^{pack}(v_g)$, she uses at most one stand-alone bid, say, $b_g^{left}(v_g)$.

Result 3 Whenever the Global use $\lambda(v_g) \equiv 2b_g^{pack}(v_g)$ and $b_g^{left}(v_g)$, $b_g^{left}(v_g) > b_g^{pack}(v_g)$, i.e. the stand-alone bid is greater than half of the package bid.

Proof. Suppose that the Global wins the left item alone namely, $(b_g^{left}(v_g) + b_l^{right}(v_l)) \geq 2b_g^{pack}(v_g)$ and assume first that $b_g^{left}(v_g) < b_g^{pack}(v_g)$. However, this implies that $b_l^{right}(v_l) > b_g^{pack}(v_g) > b_g^{left}(v_g)$, so that $b_l^{left}(v_l) = b_l^{right}(v_l) > b_g^{left}(v_g)$. But then, $b_l^{left}(v_l) + b_l^{right}(v_l) > b_g^{left}(v_g) + b_l^{right}(v_l)$, and $b_g^{left}(v_g)$ is not part of the winning allocation. For the case where, $b_g^{left}(v_g) = b_g^{pack}(v_g)$, replace by $b_l^{right}(v_l) > b_g^{pack}(v_g)$ by $b_l^{right}(v_l) \geq b_g^{pack}(v_g)$, so that $b_l^{left}(v_l) = b_l^{right}(v_l) \geq b_g^{left}(v_g)$. If the inequality is strict, then apply the same proof as before. Thus, assume instead that $b_l^{left}(v_l) = b_l^{right}(v_l) = b_g^{left}(v_g)$. But than $b_l^{left}(v_l) + b_l^{right}(v_l) = b_g^{left}(v_g) + b_l^{right}(v_l) = 2b_g^{pack}(v_g)$. In this event the allocation is a three way tie and the Global expected payoffs are: $L \equiv \frac{1}{3}[v_g - b_g^{left}(v_g)] + \frac{1}{3}[2\beta v_g - 2b_g^{pack}(v_g)]$. This is, with a probability of 1/3 the global gets the left item and earns $[v_g - b_g^{left}(v_g)]$, and with a probability of 1/3 the global gets the package and earns, $[2\beta v_g - 2b_g^{pack}(v_g)]$. In this event, not submitting $b_g^{left}(v_g)$ results in a two-way tie, $b_l^{left}(v_l) + b_l^{right}(v_l) = 2b_g^{pack}(v_g)$ with the Global getting the package with a probability of 1/2 for a payoff: $R \equiv \frac{1}{2}[2\beta v_g - 2b_g^{pack}(v_g)]$. However, $R - L = \frac{1}{6}[2\beta v_g - 2b_g^{pack}(v_g)] - \frac{2}{6}[v_g - b_g^{left}(v_g)] = \frac{2}{6}[(\beta - 1)v_g + (b_g^{left}(v_g) - b_g^{pack}(v_g))] \geq \frac{2}{6}(\beta - 1)v_g \geq 0$. ■

Thus, assume from now on that whenever the Global use $\lambda(v_g) \equiv 2b_g^{pack}(v_g)$ in combination with a stand-alone bid $b_g^{left}(v_g)$, $b_g^{left}(v_g) > b_g^{pack}(v_g)$ and let $d \equiv [b_g^{left}(v_g) - b_g^{pack}(v_g)] > 0$. For $v_g \in (0, 1)$, denote by $v_{l0} \in (0, 1)$ the Local value such that $b_l^{left}(v_{l0}) +$

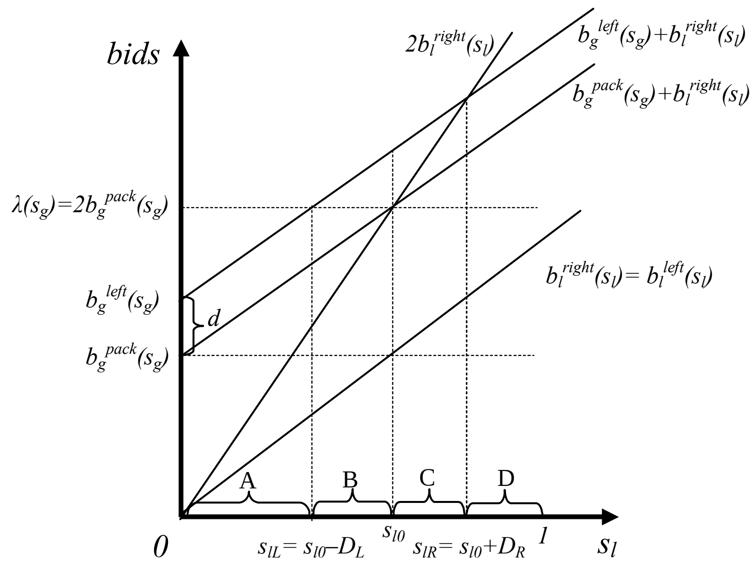


Figure 7: Relevant Regions

$b_l^{right}(v_{l0}) = 2b_l^{right}(v_{l0}) = 2b_g^{pack}(v_g)$ (The obvious reason why such $v_{l0} \in (0, 1)$ exists is left out). Note that since $b_g^{left}(v_g) + b_l^{right}(v_{l0}) \equiv d + 2b_l^{right}(v_{l0}) > 2b_l^{right}(v_{l0})$. Thus, denote by $v_{lR} \equiv v_{l0} + D_R$, that v_l such that $b_g^{left}(v_g) + b_l^{right}(v_{lR}) = 2b_l^{right}(v_{lR})$, (It is easy to see why it can never be optimal to have such a $b_g^{left}(v_g) \geq b_l^{right}(1)$ so this possibility is ignored.) In words, v_{lR} is where the Global's stand-alone bid is equal to $b_l^{right}(v_{lR})$. Since $b_g^{left}(v_g) > b_g^{pack}(v_g) = b_l^{right}(v_{l0})$, it implies that $D_R > 0$. Denote by $v_{lL} \equiv v_{l0} - D_L$ that v_l such that $b_g^{left}(v_g) + b_l^{right}(v_{lL}) = 2b_g^{pack}(v_g)$. (We ignore here, the possibility that $b_g^{left}(v_g) \geq 2b_g^{pack}(v_g)$, as it is easy to show that it can never be optimal to have such a $b_g^{left}(v_g)$.) In words, v_{lL}, is where the combination of the Global's stand-alone bid plus the Local's $b_l^{right}(v_{lL})$, is equal to the Global's package bid $2b_g^{pack}(v_g)$. $b_l^{right}(v_{lL}) = 2b_g^{pack}(v_g) - b_g^{left}(v_g) = b_g^{pack}(v_g) - d < b_g^{pack}(v_g) = b_l^{right}(v_{l0})$, implies that $D_L > 0$. Figure 7 illustrates these definitions with linear bidding functions.

There are four possible events to consider: $A : v_l \in [0, v_{l0} - D_L]$; $B : v_l \in (v_{l0} - D_L, v_{l0}]$; $C : v_l \in (v_{l0}, v_{l0} + D_R)$; and $D : v_l \in [v_{l0} + D_R, 1]$. In A, $b_l^{right}(v_l) + b_l^{left}(v_l) < b_g^{left}(v_g) + b_l^{right}(v_l) \leq 2b_g^{pack}(v_g)$, so that the Global's package bid wins and $b_g^{left}(v_g)$ is not relevant. In D, $b_l^{right}(v_l) + b_l^{left}(v_l) \geq Maxv_l[2b_g^{pack}(v_g), b_g^{left}(v_g) + b_l^{right}(v_l)]$ so that both items go to the Locals and the Global's stand-alone bid $b_g^{left}(v_g)$ is also not relevant. In C, $2b_g^{pack}(v_g) < b_l^{left}(v_l) + b_l^{right}(v_l) < b_g^{left}(v_g) + b_l^{right}(v_l)$. Here the Global's package would lose to the Locals' bids but the combination of the Global's stand-alone bid and the (right) Local's bid wins. Therefore, in C having the stand-alone bid results in a net gain of $[v_g - b_g^{left}(v_g)] > 0$. In B, $b_l^{left}(v_l) + b_l^{right}(v_l) < 2b_g^{pack}(v_g) < b_g^{left}(v_g) + b_l^{right}(v_l)$.

Thus, the Global (as in C) wins the left item for a net gain of $[v_g - b_g^{left}(v_g)]$. However, had the Global refrained from using the stand-alone bid, she would win the package for a net gain of $[2\beta v_g - 2b_g^{pack}(v_g)]$. The (opportunity) loss associated with submitting the stand-alone bid is: $\{[2\beta v_g - 2b_g^{pack}(v_g)] - [v_g - b_g^{left}(v_g)]\} = \{[(2\beta - 1)v_g - b_g^{left}(v_g)] + 2d\} \geq \{[v_g - b_g^{left}(v_g)] + 2d\} > [v_g - b_g^{left}(v_g)]$. Thus, the loss associated with using the stand-alone bid in B (and replacing otherwise the win by the package bid) is larger than the gain associated with using the stand-alone bid in C and winning where the package bid does not win. Of course the above demonstration is neither necessary nor sufficient to conclude whether using a stand-alone bid is desirable or not since we have not considered the probabilities of these two events which we do next. Since we assumed that v_l is distributed *uniformly,* it follows that conditional on $v_l \notin [A \cup D]$, v_l is still distributed *uniformly* on $B \cup C$. Thus, $Prob[v_l \in C = (v_{l0}, v_{l0} + D_R)] \gtreqless Prob[v_l \in B = (v_{l0} - D_L, v_{l0})]$ AS $D_R \gtreqless D_L$. Next, note that (it is easy to see it from figure 1), $2b_g^{pack}(v_g) + d + \int_{v_{l0}}^{v_{l0}+D_R} b_l'^{right}(v_l)dv_l = 2b_g^{pack}(v_g) + \int_{v_{l0}}^{v_{l0}+D_R} 2b_l'^{right}(v_l)dv_l$, OR, $d = \int_{v_{l0}}^{v_{l0}+D_R} b_l'^{right}(v_l)dv_l = b(v_{l0} + D_R) - b(v_{l0})$. Also note that, $2b_g^{pack}(v_g) + \int_{v_{l0}-D_L}^{v_{l0}} b_l'^{right}(v_l)dv_l = 2b_g^{pack}(v_g) + d$, OR, $d = \int_{v_{l0}-D_R}^{v_{l0}} b_l'^{right}(v_l)dv_l = b_l^{right}(v_{l0}) - b_l^{right}(v_{l0} - D_L)$. It follows that $\frac{b_l^{right}(v_{l0}+D_R) - b_l^{right}(v_{l0})}{D_R} = \frac{b_l^{right}(v_{l0}) - b_l^{right}(v_{l0}-D_L)}{D_L}\frac{D_L}{D_R}$.

Result 4 A convex $b_l^{left}(v_l) = b_l^{right}(v_l)$ is sufficient to rule out the use of the combination $\lambda(v_g) \equiv 2b_g^{pack}(v_g)$ and $b_g^{left}(v_g)$.

Proof. A convex $b_l^{left}(v_l) = b_l^{right}(v_l)$ implies that $\frac{b_l^{right}(v_{l0}+D_R) - b_l^{right}(v_{l0})}{D_R} > \frac{b_l^{right}(v_{l0}) - b_l^{right}(v_{l0}-D_L)}{D_L}$ and since $\frac{b_l^{right}(v_{l0}+D_R) - b_l^{right}(v_{l0})}{D_R} = \frac{b_l^{right}(v_{l0}) - b_l^{right}(v_{l0}-D_L)}{D_L}\frac{D_L}{D_R}$, it must be that $\frac{D_L}{D_R} > 1$, namely, $D_L > D_R$. Thus, not only the loss associated with using the stand-alone bid in B is strictly larger that the gains from using the stand-alone bid in C, but also the probability of B is strictly larger than that of C. ■

Next we show that indeed $b_l^{left}(v_l) = b_l^{right}(v_l)$ IS convex.

Result 5 $b_l^{left}(v_l) = b_l^{right}(v_l)$ is convex.

Proof. Rather than showing that $b_l^{left}(v_l) = b_l^{right}(v_l)$ is convex on $v_l \in [0,1]$, we will show that $\sigma_l(b)$ is strictly concave on $[0, \bar{b}]$. In equation (10) we have, $\sigma_l'(b) = [\sigma_l(b)/(\beta\sigma_g(b) - b)]$ and $\sigma_g'(b) = \frac{2\sigma_g(b)}{\sigma_l(b)-b}$. Thus,

$$\sigma_g(b) = \frac{1}{\beta}[\frac{\sigma_l(b)}{\sigma_l'(b)} + b] \tag{18}$$

It follows from (18) that,

$$\sigma_g'(b) = \frac{1}{\beta}\{2 - \frac{\sigma_l''(b)\sigma_l(b)}{[\sigma_l'(b)]^2}\} \tag{19}$$

Again from equation (10) and using (18), $\sigma_g'(b) = \frac{2\sigma_g(b)}{\sigma_l(b)-b} = \frac{1}{\beta}[\frac{\sigma_l(b)}{\sigma_l'(b)} + b]\frac{2}{\sigma_l(b)-b}$. Upon equating the two expressions that we derived for $\sigma_g'(b)$ and with some straightforward manipulations we get:

$$\frac{\sigma_l''(b)\sigma_l(b)}{[\sigma_l'(b)]^2} = [\frac{2b}{\sigma_l(b)-b}]\{\frac{\sigma_l(b)}{b}[\frac{\sigma_l'(b)-1}{\sigma_l'(b)}] - 2\} \tag{20}$$

It is easy to verify that for $b \in [0, \bar{b}]$, $sign[\sigma_l''(b)] = sign\{\frac{\sigma_l(b)}{b}[\frac{\sigma_l'(b)-1}{\sigma_l'(b)}]-2\}$. Using l'Hospital on equation (9) in the text one can show that, $\sigma_l'(0) = 2$ (interesting, as this is independent of β): $\lim_{b=>0} \frac{\sigma_l(b)}{b} = \lim_{b=>0} \frac{\sigma_l(b)-\sigma_l(0)}{b} = \sigma_l'(0) = 2$. Thus, $\lim_{b=>0}\{\frac{\sigma_l(b)}{b}[\frac{\sigma_l'(b)-1}{\sigma_l'(b)}] - 2\} = \{2 \times [\frac{2-1}{2}] - 2\} = -1$. Since $\sigma_l(b) - b > 0$ $\forall b \in (0, \bar{b}]$, it follows that there exists $b_0 \in (0, \bar{b}]$ such that $\forall b \leq b_0$, $\sigma_l''(b) < 0$. Assume that $b_1 \in (b_0, \bar{b})$ is the first time that $\sigma_l''(b) = 0$. It follows that $\{\frac{\sigma_l(b_1)}{b_1}[\frac{\sigma_l'(b_1)-1}{\sigma_l'(b_1)}] - 2\} = 0$, and also that $\sigma_l'(b_1) - 1 > 0$. But then, $\sigma_l'''(b_1) = \{\frac{\sigma_l(b_1)}{b_1}[\frac{\sigma_l'(b_1)-1}{\sigma_l'(b_1)}] - 2\} \times \frac{d}{db}\{\frac{[\sigma_l'(b_1)]^2}{\sigma_l(b_1)}[\frac{2b_1}{\sigma_l(b_1)-b_1}]\} + \{\frac{[\sigma_l'(b_1)]^2}{\sigma_l(b_1)}[\frac{2b_1}{\sigma_l(b_1)-b_1}]\} \times \frac{d}{db}\{\frac{\sigma_l(b_1)}{b_1}[\frac{\sigma_l'(b_1)-1}{\sigma_l'(b_1)}] - 2\} = \{\frac{[\sigma_l'(b_1)]^2}{\sigma_l(b_1)}[\frac{2b_1}{\sigma_l(b_1)-b_1}]\} \times \frac{d}{db}\{\frac{\sigma_l(b_1)}{b_1}[\frac{\sigma_l'(b_1)-1}{\sigma_l'(b_1)}] - 2\}$. However, $\frac{d}{db}\{\frac{\sigma_l(b_1)}{b_1}[\frac{\sigma_l'(b_1)-1}{\sigma_l'(b_1)}] - 2\} = \frac{\{[\sigma_l'(b_1)(\sigma_l'(b_1)-1)][b_1\sigma_l'(b_1)]\} - \{[\sigma_l'(b_1)\sigma_l(b_1)(\sigma_l'(b_1)-1)]\}}{[b_1\sigma_l'(b_1)]^2} = \frac{\{[\sigma_l'(b_1)(\sigma_l'(b_1)-1)][b_1\sigma_l'(b_1)]-\sigma_l(b_1)]\}}{[b_1\sigma_l'(b_1)]^2}$, where the last simplification is due to the fact that $\sigma_l''(b_1) = 0$. Since $\forall b \in (0, b_1)$, $\sigma_l''(b) < 0$ and $\sigma_l(0) = 0$, it follows that $[b_1\sigma_l'(b_1)] < \sigma_l(b_1)$ and thus, (recall $\sigma_l'(b_1) - 1 > 0$), $\frac{\{[\sigma_l'(b_1)(\sigma_l'(b_1)-1)][b_1\sigma_l'(b_1)]-\sigma_l(b_1)]\}}{[b_1\sigma_l'(b_1)]^2} < 0$. We conclude that $\sigma_l''(b)$ starts being negative for small enough b's and cannot become positive since as $\sigma_l''(b) = 0$ yields, $\sigma_l'''(b) < 0$. ■

Summarizing all the results so far we conclude:

Proposition 2 The equilibrium that we characterized for the package auction rule assuming that the Global bidder is not allowed to also use stand-alone bids is an equilibrium even if we do allow the Global to use stand-alone bids.

7.3 Appendix: Other Tables and Figures

	Session	# of subjects			# of observations			# of auctions
		Global	Local	Total	Global	Local	Total	
Separate	1	5	10	15	300	600	900	300
($\beta = 1$)	2	6	12	18	360	720	1080	360
Separate	1	5	10	15	300	600	900	300
($\beta = 1.5$)	2	4	8	12	240	480	720	240
Package	1	5	10	15	300	600	900	300
($\beta = 1$)	2	6	12	18	360	720	1080	360
Package	1	5	10	15	300	600	900	300
($\beta = 1.5$)	2	6	12	18	360	720	1080	360
Total		42	84	126	2520	5040	7560	2520

Table 9: Experimental Sessions

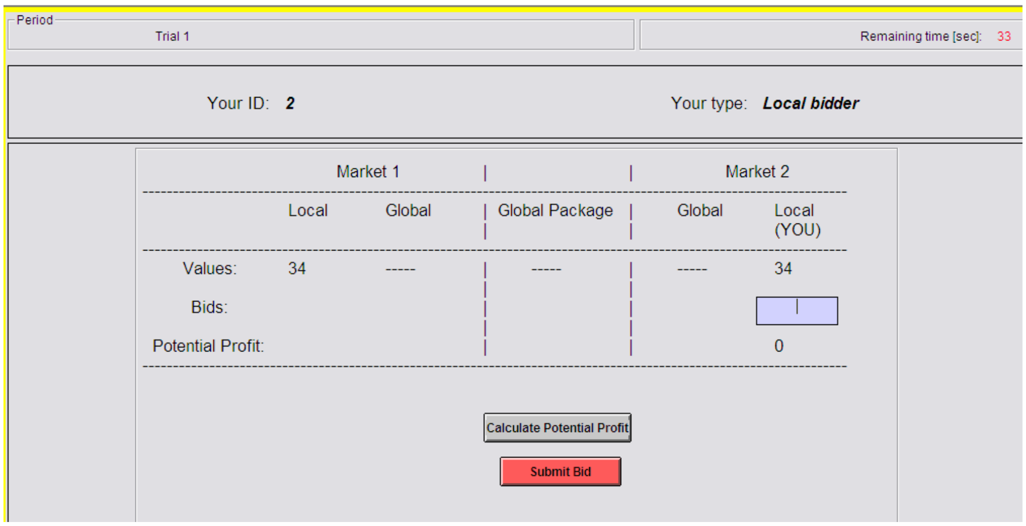
	Separate $\beta = 1$	Separate $\beta = 1.5$	Package $\beta = 1$	Package $\beta = 1.5$
Global	\$0.02/1ECU	\$0.01/1ECU	\$0.02/1ECU	\$0.01/1ECU
Local	\$0.04/1ECU	\$0.04/1ECU	\$0.04/1ECU	\$0.04/1ECU

Table 10: Conversion Rates

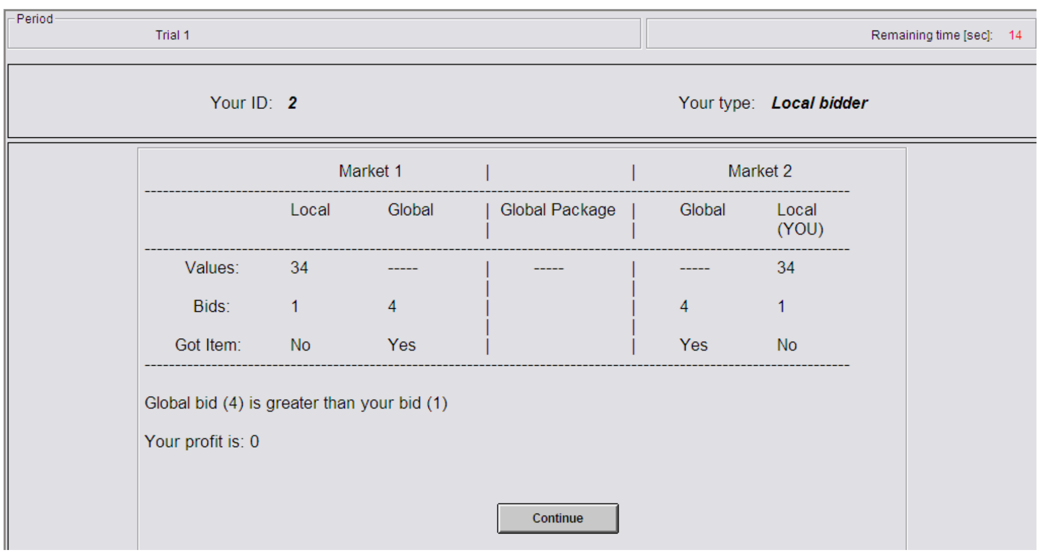
7.4 Appendix: Software Screenshots

7.4.1 Separate Auction rule, Local Bidder Screens

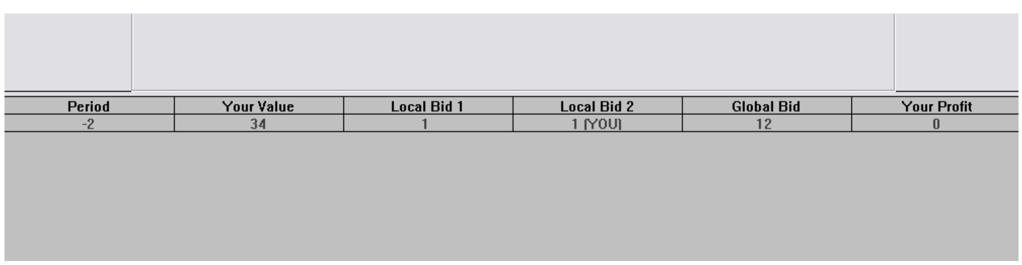
Decision screen of a Local bidder:



Outcome screen of a Local bidder:

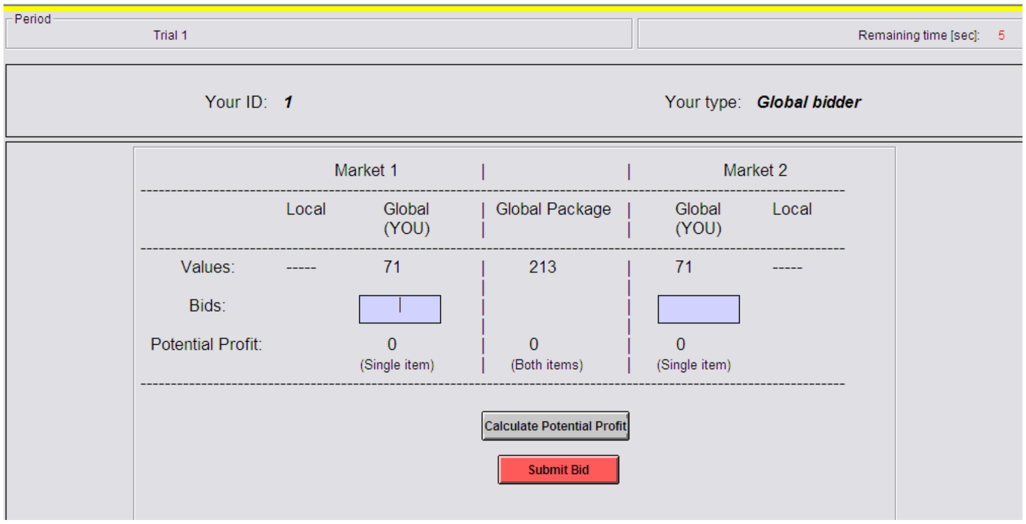


History of a Local bidder (always at the bottom of a screen):

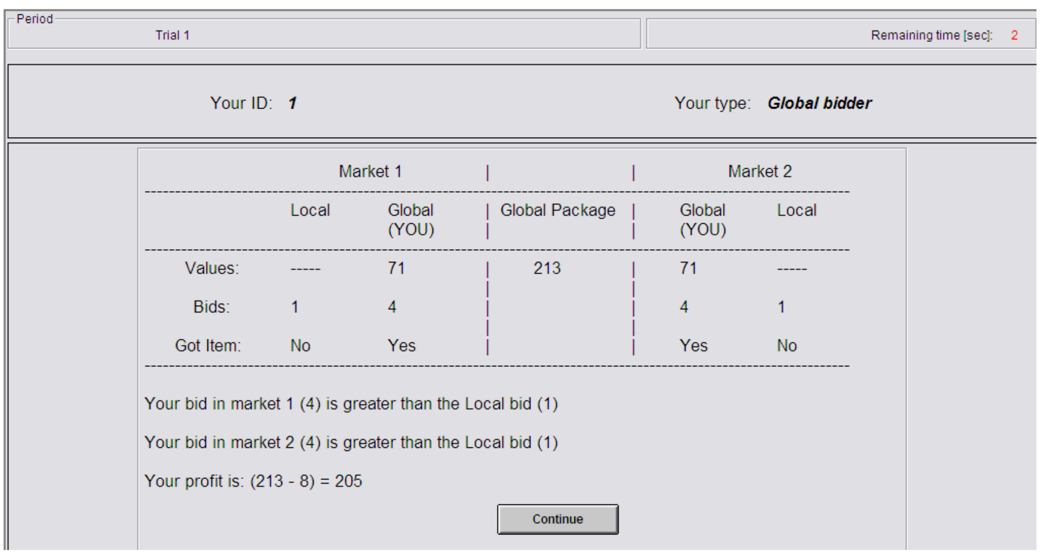


7.4.2 Separate Auction rule, Global Bidder Screens

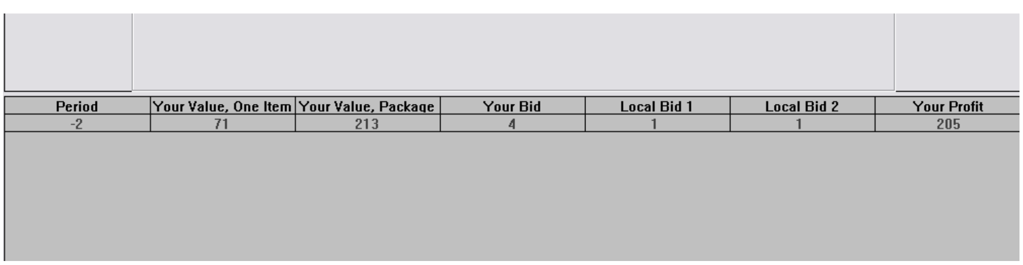
Decision screen of a Global bidder:



Outcome screen of a Global bidder:

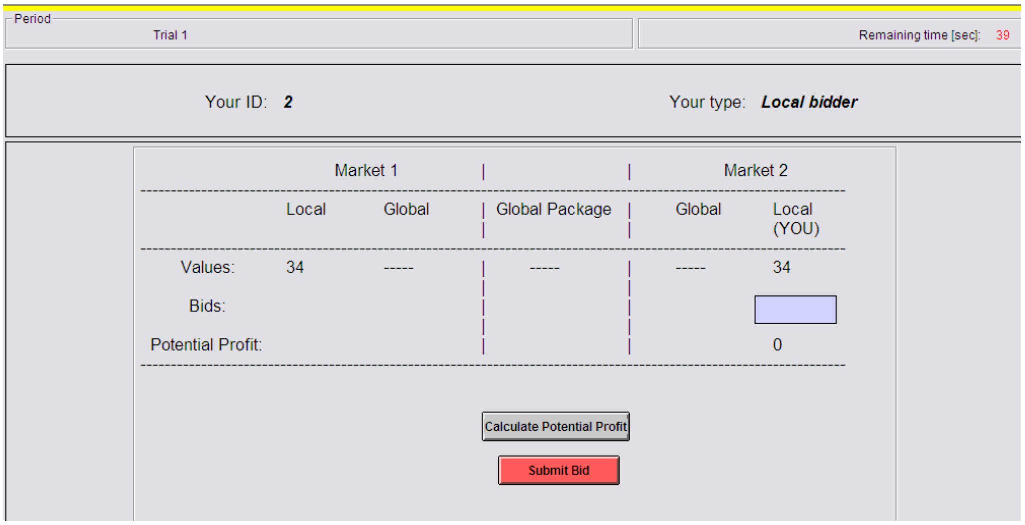


History of a Global bidder (always at the bottom of a screen):

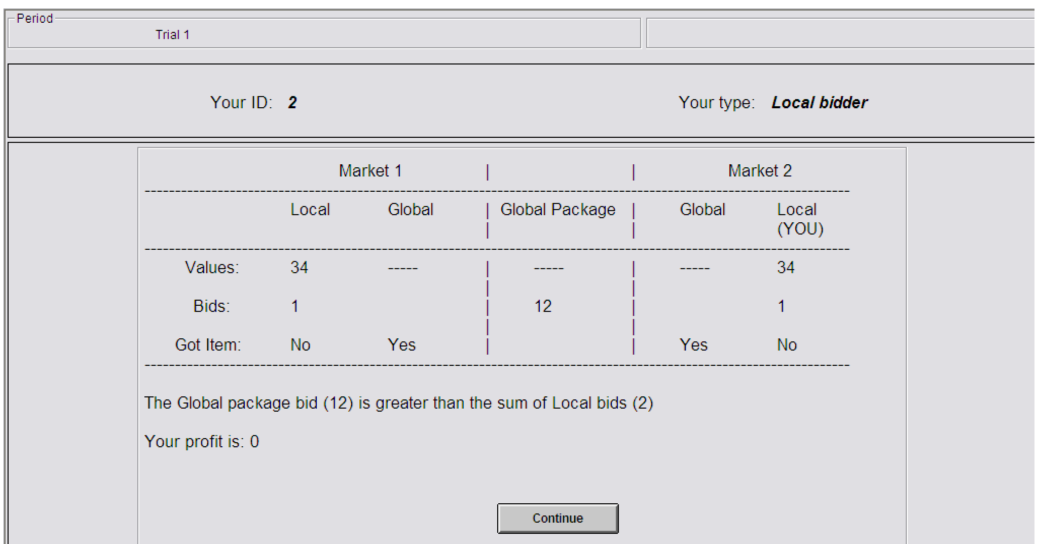


7.4.3 Package Auction rule, Local Bidder Screens

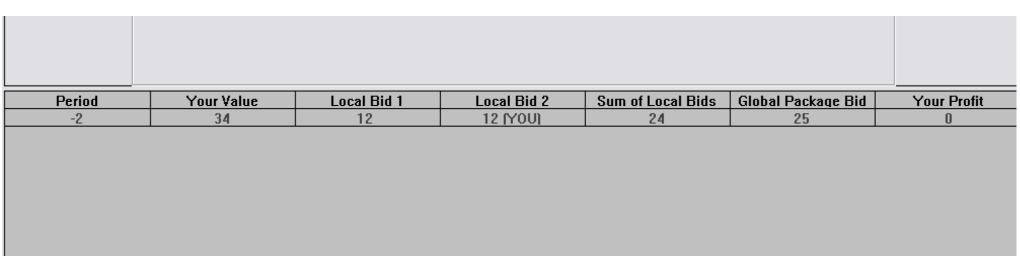
Decision screen of a Local bidder:



Outcome screen of a Local bidder:

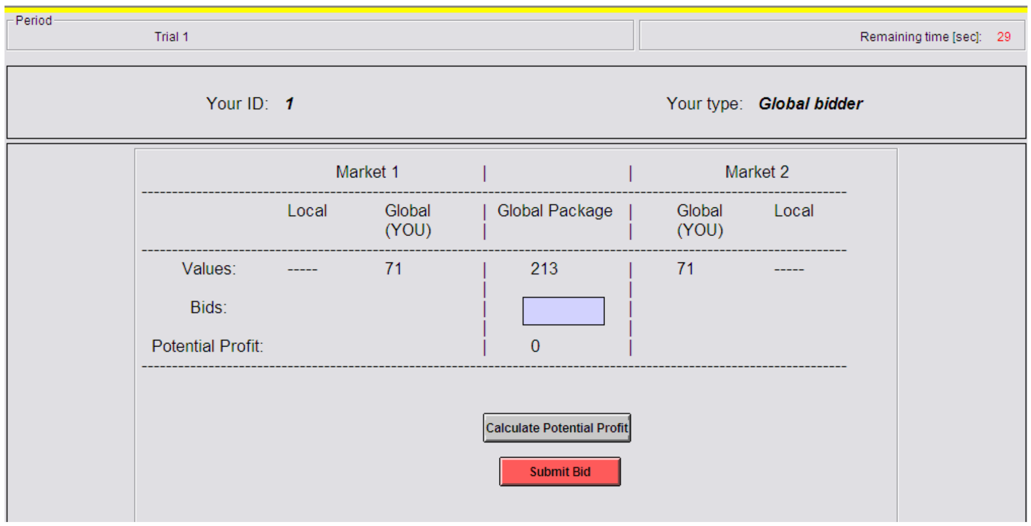


History of a Local bidder (always at the bottom of a screen):

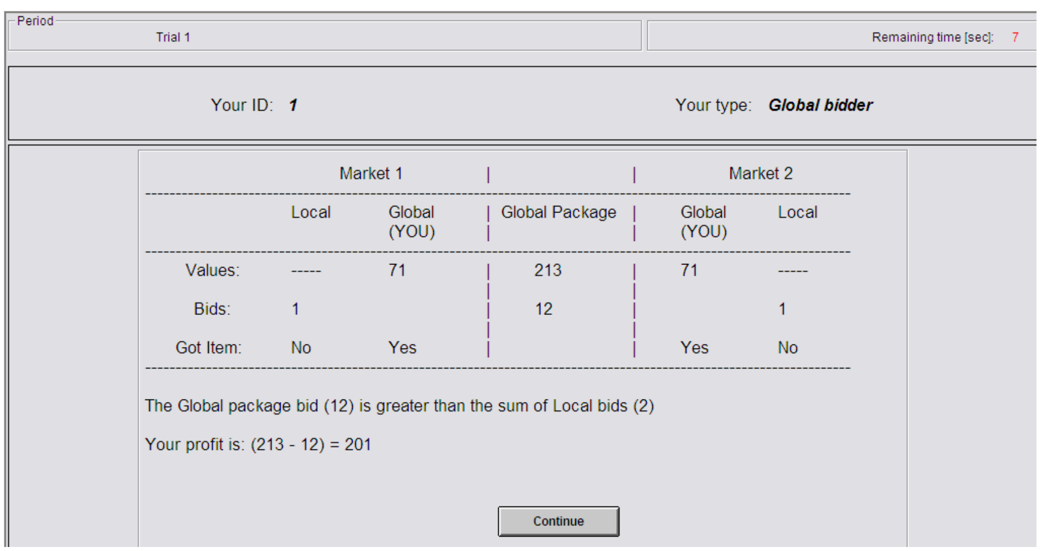


7.4.4 Package Auction rule, Global Bidder Screens

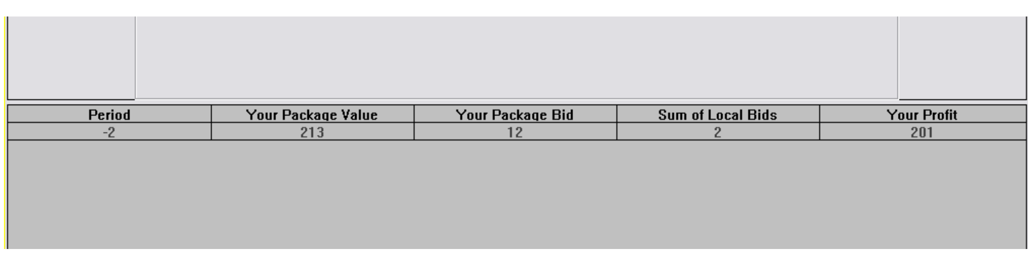
Decision screen of a Global bidder:



Outcome screen of a Global bidder:



History of a Global bidder (always at the bottom of a screen):



7.5 Appendix: Instructions

7.5.1 Separate Auction rule (with 50% synergies)

This is an experiment in the economics of decision-making. The National Science Foundation has provided funds for conducting this research. The instructions are simple, and if you follow them carefully and make good decisions, you may earn a CONSIDERABLE AMOUNT OF MONEY, which will be PAID TO YOU IN CASH at the end of the experiment.

You will participate in a series of approximately 50 auction periods.

In every period you will be randomly matched into __ groups of 3 people

Bidding:

In each group of 3, bidders will bid for items in 2 markets.

There is one item for sale in each of the 2 markets.

In each market there is one Local bidder who is interested only in that market.

A single Global bidder has interest in both markets.

Each Local bidder will submit a bid for the item in her market.

The Global bidder will submit bids for items in both markets. Global bids in both markets must be the same.

If the Global bid is greater than the Local bid in a specific market, the item in that market goes to the Global bidder.

If the Local bid is greater than the Global bid, the item goes to the Local bidder.

Values and Profit:

In each period, prior to bidding, Global and Local bidders are assigned values for the items: a Global value and a Local value.

Values, bids and profits are in Experimental Currency Units (ECU).

If you are a Global bidder you do not know the value of the Local bidders. Similarly, Local bidders do not know the value of the Global bidder.

The values are random numbers between 0 and 100. Each number is equally likely.

The Local value and the Global value are drawn independently (and likely to be different).

If a Local bidder gets the item, her profit is (Local Value – Her Bid).

If the Global bidder gets a single item, her profit is (Global Value – Her Bid).

If the Global bidder gets a package of both items, her profit is (3×Global Value – 2×Her Bid). In other words, she receives a Global value for each item, plus one additional Global value.

Those who do not get the item, do not earn any profit and do not have to pay their bid

Example (see Table below): Suppose, by chance, Global value is 60 and Local value is 60 (just for illustration, as it is very unlikely). Then the Global bidder's value for the package is 3×60 =180. Case 1: Suppose each Local bidder bids 30 for her item and the Global bidder bids 31 for each item. Then, the Global bid, 31, is larger than the Local bid in each market, 30, and both items go to the Global bidder. The Global bidder earns a profit of ECU118 (180 – 62 =118) and each Local bidder earns Zero. Case 2: Suppose that the Global bids were 29 (rather than 31). In this case the Global bid, 29, is lower than the Local bid in each market, 30. Therefore, each of the Local bidders receives one item and earns ECU30 (60 – 30 = 30). The Global bidder earns zero. Case 3: Suppose that the Global bids were 29, Local bid in Market 1 was 30 but the Local bid in Market 2 was 28. In this case the Global bid, 29, is lower than the Local bid in Market 1, 30, but higher than the Local bid in Market 2, 28. Therefore the Local bidder in Market 1 earns ECU30 (60 – 30 = 30) and the Global bidder earns ECU31 (60 – 29 = 31). The Local bidder in Market 2 earns zero.

		Market 1		Global Package	Market 2	
		Local	Global		Global	Local
	Values:	60	60	(180)	60	60
Case 1:	Bids:	30	31	(62)	31	30
Global gets the items	Profits:	0		118		0
Case 2:	Bids:	30	29		29	30
Locals get the items	Profits:	30	0		0	30
Case 3:	Bids:	30	29		29	28
Global gets one item	Profits:	30	0		31	0

Other things:

A coin is flipped (by the computer) to break any possible ties.

You will be assigned your role (Global or Local) randomly.

Your role will remain the same throughout the experiment.

Your earnings (in ECU) will be the sum of your profits in all periods.

ECU earnings will be converted into dollars using the following conversion rates: Global bidders will get $1 per ECU100 and Local bidders will get $1 per ECU25.

In addition you will receive $6 for showing up on time.

Your goal should be to maximize your profits.

Note that higher bids increase your chance of getting an item BUT decrease your profit if you get it.

Summary:

You will participate in 50 auction periods.

You will be randomly matched into groups of 3 in every auction period.

2 Local bidders will be assigned a random Local value between 0 and 100.

1 Global bidder will be assigned a random Global value between 0 and 100.

Local bidders will each submit a bid for the item in their own market.

Global bidder will submit bids in both markets. Global bids in both markets must be the same.

Auction outcome is determined by comparing the Global bid and the Local bid in each market.

If the Local bid is higher, the item goes to the Local bidder, and that Local bidder's profit is (Local value – Her Bid).

If the Global bid is higher, the item goes to the Global bidder.

If the Global bidder gets a single item her profit is (Global value – Her Bid).

If the Global bidder gets a package of both items her profit is (3×Global value – 2×Her Bid).

Your cumulative earnings will be converted into dollars and paid to you in cash at the end of the experiment.

7.5.2 Package Auction rule (with 50% synergies)

This is an experiment in the economics of decision-making. The National Science Foundation has provided funds for conducting this research. The instructions are simple, and if you follow them carefully and make good decisions, you may earn a CONSIDERABLE AMOUNT OF MONEY, which will be PAID TO YOU IN CASH at the end of the experiment.

You will participate in a series of approximately 50 auction periods.

In every period you will be randomly matched into __ groups of 3 people

Bidding:

In each group of 3, bidders will bid for items in 2 markets.

There is one item for sale in each of the 2 markets.

In each market there is one Local bidder who is interested only in that market.

A single Global bidder has interest in both markets.

Each Local bidder will submit a bid for the item in her market.

The Global bidder will submit a package bid for both items together.

If the Global package bid is greater than the sum of the Local bids, both items go to the Global bidder.

If the sum of the Local bids is greater than the Global package bid, the items go to the Local bidders: each Local bidder gets the item in her market.

Values and Profit:

In each period, prior to bidding, Global and Local bidders are assigned values for the items: a Global value and a Local value.

Values, bids and profits are in Experimental Currency Units (ECU).

If you are a Global bidder you do not know the value of the Local bidders. Similarly, Local bidders do not know the value of the Global bidder.

The values are random numbers between 0 and 100. Each number is equally likely.

The Local value and the Global value are drawn independently (and likely to be different).

If a Local bidder gets the item, her profit is (Local Value – Her Bid).

If the Global bidder gets the package, her profit is (3×Global Value – Her Package Bid). In other words, she receives a Global value for each item, plus one additional Global value.

Those who do not get the item, do not earn any profit and do not have to pay their bid

Example (see Table below): Suppose, by chance, Global value is 60 and Local value is 60 (just for illustration, as it is very unlikely). Then the Global bidder's value for the package is 3×60 =180. Case 1: Suppose each Local bidder bids 30 for her item and the Global bidder bids 61 for the package. Then, the Global bid, 61, is larger than the sum of the Local bids (60 = 30 + 30), and both items go to the Global bidder. The Global bidder earns a profit of ECU119 (180 – 61 =119) and each Local bidder earns Zero. Case 2: Suppose that the Global bid was 59 (rather than 61). In this case the Global bid, 59, is lower than the sum of the Local bids, 60. Therefore, each of the Local bidders receives one item and earns ECU30 (60 – 30 = 30). The Global bidder earns zero.

		Market 1		Global Package	Market 2	
		Local	Global		Global	Local
	Values:	60	60	(180)	60	60
Case 1:	Bids:	30		61		30
Global gets the items	Profits:	0		119		0
Case 2:	Bids:	30		59		30
Locals get the items	Profits:	30		0		30

Other things:

A coin is flipped (by the computer) to break any possible ties.

You will be assigned your role (Global or Local) randomly.

Your role will remain the same throughout the experiment.

Your earnings (in ECU) will be the sum of your profits in all periods.

ECU earnings will be converted into dollars using the following conversion rates: Global bidders will get $1 per ECU100 and Local bidders will get $1 per ECU25.

In addition you will receive $6 for showing up on time.

Your goal should be to maximize your profits.

Note that higher bids increase your chance of getting an item BUT decrease your profit if you get it.

Summary:

You will participate in 50 auction periods.

You will be randomly matched into groups of 3 in every auction period.

2 Local bidders will be assigned a random Local value between 0 and 100.

1 Global bidder will be assigned a random Global value between 0 and 100.

Local bidders will each submit a bid for the item in their own market.

Global bidder will submit a package bid for both items.

Auction outcome is determined by comparing the Global package bid and the sum of the two Local bids.

If the sum of the Local bids is higher, the items go to the Local bidders, and each Local bidder's profit is (Local value – Her Bid).

If the Global package bid is higher, the items go to the Global bidder, and her profit is (3×Global value – Her Package Bid).

Your cumulative earnings will be converted into dollars and paid to you in cash at the end of the experiment.