Coordination on List Prices
and Collusion in Negotiated Prices*

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28 September 2016

Abstract

A collusive practice in some intermediate goods markets is for sellers to coordinate on list prices but not on discounts. We put forth a theory to explain how coordination on list prices can raise final transaction prices even when no customers pay list price, and to identify market conditions conducive to firms profitably engaging in this form of collusive practice.

*The comments of participants at the 2016 Hal White Antitrust Conference (Washington, D.C.) and the 2016 UBC Industrial Organization Conference (Kelowna, British Columbia) are gratefully acknowledged, as is the extremely able research assistance of Ben Rosa and Xingtan (Ken) Zhang. The first author recognizes the financial support of the National Science Foundation (SES-1148129).
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1 Introduction

Collusion entails firms coordinating on the prices that they charge customers. In the context of retail markets, firms agree to supracOMPetitive posted prices and then monitor each other to ensure that those are the prices charged in stores (or online). With intermediate goods markets, it is more complicated because firms set list prices and routinely offer discounts to buyers. While they may agree on list prices, it is really the coordination on final transaction prices that is essential, as noted for the thread cartel.1

[A cartel member] explained that list prices have more of a political importance than a competitive one. Only very small clients pay the prices contained in the lists. As the official price lists issued by each competitor are based on large profit margins, customers regularly negotiate rebates, but no clear or fixed amount of rebates is granted. ... Therefore, the list prices are essentially "fictitious" prices ... while [rebates] were discussed and agreed during the meetings.

For example, the citric acid cartel agreed to list prices and a discount of up to 3% for a company’s five major consumers. Once having coordinated on list prices and discounts, the challenge is then monitoring for compliance. While list prices are public information, discounts are privately negotiated between a buyer and seller which makes it difficult to determine whether cartel members charged the agreed-upon final transaction prices. The solution pursued by many cartels - including those in citric acid, lysine, and vitamins - was to agree to an allocation of sales quotas along with final transaction prices, and then monitor for compliance by comparing realized sales to those quotas.2

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2Harrington (2006), Connor (2008), and Marshall and Marx (2012) provide details on these and other relevant cartels. For an analysis of this collusive practice and related ones, see Harrington and
Contrary to the rather standard collusive practices just described, there are a few intermediate goods cartels that coordinated exclusively on list prices and left firms unconstrained in the discounts that they offered. Furthermore, there was no evidence of sales monitoring to ensure that firms did not gain market share through discounts and, in fact, discounts were regularly given to customers. This pattern occurred in several private litigation cases for which plaintiffs, defendants, and the court often came to different conclusions regarding the efficacy of firms coordinating on list prices.\(^3\)

In *Reserve Supply v. Owens-Corning Fiberglas* (1992), plaintiffs and defendants put forth conflicting claims regarding the collusive role of list prices.\(^4\)

Reserve points to Owens-Corning and CertainTeed’s practices of maintaining price lists for products and ... asserts that these lists have no independent value because no buyer in the industry pays list price for insulation. Instead, it claims that the price lists are an easy means for producers to communicate and monitor the price activity of rivals by providing a common starting point for the application of percentage discounts. ... Owens-Corning and CertainTeed counter by arguing that the use of list prices to monitor pricing would not be possible because the widespread use of discounts in the industry ensures that list prices do not reflect the actual price that a purchaser pays.

The Seventh Circuit Court expressed skepticism with regards to the plaintiffs’ claim.\(^5\)

We agree that the industry practice of maintaining price lists and announcing price increases in advance does not necessarily lead to an in-

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\(^3\)In addition to the cases mentioned below is *Lum v. Bank of America* 361 F.3d, 217, 231 (3d Cir., 2004). For that case, also see the discussion in Holmes (2004).

\(^4\) *Reserve Supply v. Owens-Corning Fiberglas* 971 F. 2d 37 (7th Cir. 1992), para 61.

\(^5\)Ibid, para. 62.
ference of price fixing. ... [T]his pricing system would be, to put it mildly, an awkward facilitator of price collusion because the industry practice of providing discounts to individual customers ensured that list price did not reflect the actual transaction price.

In a case involving the market for urethane, plaintiffs claimed: \(^6\)

[T]hroughout the alleged conspiracy period, the alleged conspirators announced identical price increases simultaneously or within a very short time period. ... [P]urchasers could negotiate down from the increased price. But the increase formed the baseline for negotiations. ... [T]he announced increases caused prices to rise or prevented prices from falling as fast as they otherwise would have.

The Tenth Circuit Court quoted the District Court in supporting this assessment: \(^7\)

The court reasoned that the industry’s standardized pricing structure—reflected in product price lists and parallel price-increase announcements—“presumably established an artificially inflated baseline” for negotiations. Consequently, any impact resulting from a price-fixing conspiracy would have permeated all polyurethane transactions, causing market-wide impact despite individualized negotiations.

In sum, plaintiffs claimed that coordination on announced list prices can raise final negotiated prices. Defendants disputed that claim and questioned whether coordination on anything less than transaction prices could be effective. Our objective is to explore the possibility that firms could successfully collude by coordinating only on list prices while leaving firms with complete discretion in setting final transaction...

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\(^6\) Class Plaintiffs’ Response Brief (February 14, 2014), \textit{In Re: Urethane Antitrust Litigation}, No. 13-3215, 10th Cir.; pp. 8-9.

\(^7\) \textit{In Re: Urethane Antitrust Litigation}, No. 13-3215 (10th Cir. Sep. 29, 2014); p. 7.
prices. The two questions addressed here are: 1) How can coordination in list prices result in supracompetitive transaction prices?; and 2) Having identified a mechanism whereby list prices impact negotiated prices, what market conditions are conducive to firms profitably engaging in this form of collusive practice?

There is a small literature (reviewed in Section 3) that models firms as setting list prices when some customers buy at list and others buy at a discount. In that setting, list prices are then actual transaction prices for some consumers in which case it is easy to see that they can affect the price of at least some transactions. However, if very few buyers pay list price then it is also true that coordination on list prices do not have much of an impact in those models. Here, we want to investigate whether coordination on list prices could have a substantive effect even when very few, if any, consumers pay list prices. For this purpose, a theory is developed that has firms selling at supracompetitive transaction prices by agreeing on list prices even though no customers pay list price. As a modelling device, it is assumed the list price is a cheap-talk message so that it only serves as an announcement about prices. As in the thread cartel, list prices are then “fictitious” but, as we’ll show, can still be informative and impactful.

There are two steps to developing the theoretical argument. First is the provision of a theory that establishes an endogenous connection between announced list prices and final transaction prices. Here, we adapt the insight of Menzio (2007), which was developed in the context of a competitive labor market setting, to an imperfectly competitive product market setting. Suppose buyers negotiate with only a subset of sellers and believe that list prices are informative about a seller’s cost. In that case, list prices will influence with whom a buyer chooses to negotiate; a buyer will prefer those sellers expected to have low cost for then they are more likely to accept a low price. We show that, in equilibrium, list prices can be informative of a seller’s cost and, more specifically, a low list price signals a low cost (in expectation). The second step is establishing that firms can jointly raise profits by coordinating their
list prices. If, under competition, a high list price signals a high cost then a buyer
that negotiates with a seller with a high list price will end up with a relatively high
negotiated price. Thus, if sellers coordinate on announcing high list prices, they can
favorably influence buyers’ beliefs on their costs which will elevate transaction prices
in spite of no agreement on transaction prices.

Section 2 describes the model, while a review of some related research is provided
in Section 3. The existence and characterization of an equilibrium with informative
list prices is addressed in Section 4. The results of that section are then used to derive
conditions for a collusive practice of coordinating on list prices to be an equilibrium
in Section 5, with a class of examples investigated in Section 6. Section 7 illustrates
how the theoretical insight can be used in the context of cases. Unless otherwise
noted, proofs are in the appendix.

2 Model

Consider a market with two sellers offering identical products. A seller may be one
of two types, L or H, and type L occurs with probability q. Sellers’ types are
independent. A type t seller’s unit cost is assumed to be a random draw from the cdf
$F_t : [\underline{c}_t, \bar{c}_t] \to [0, 1], t \in \{L, H\}$. $F_t$ is continuously differentiable with positive density
everywhere on $(\underline{c}_t, \bar{c}_t)$. It is assumed that $F_H$ first-order stochastically dominates $F_L$
and, consequently, we will refer to a type L seller as a low-cost type and a type
H seller as a high-cost type. We also assume that the inverse hazard rate function,
$h_t(c) \equiv F_t(c)/F_t'(c)$, is non-decreasing, $h_t'(c) \geq 0$, which holds for most of the common
distributions such as uniform, normal, exponential, logistic, chi-squared, and Laplace.
In addition, distributions are assumed to be ranked in terms of their inverse hazard
rates: $h_L(c) > h_H(c)$ for all $c \in (\underline{c}_t, \bar{c}_t)$.

Each buyer is interested in buying 0 or 1 unit and $v$ denotes a buyer’s valuation.
There is a continuum of buyers and their valuations are represented by the cdf $G :$
$[v, \pi] \rightarrow [0,1]$. $G$ is continuously differentiable with positive density everywhere on $(v, \pi)$. A buyer may choose to solicit offers from either 1 or 2 sellers. What exactly it means to “solicit” an offer is described below. A fraction $b \in [0,1]$ of sellers are assumed to solicit an offer from a single seller and a fraction $1-b$ from two sellers. Whether a buyer approaches 1 or 2 sellers is assumed to be independent of a buyer’s valuation.\(^8\)

The modelling of the interaction between buyers and sellers is intended to capture many intermediate goods markets for which buyers are industrial customers. Sellers first choose list (or posted) prices. After observing those list prices, each buyer approaches either 1 or 2 sellers to negotiate. A buyer who approaches two sellers is presumed to engage in an iterative bargaining process whereby she uses an offer from one seller to obtain a better offer from the other seller or, alternatively, stated a discount off of the list price. Rather than explicitly model that process, we will use the second-price auction with a reserve price as a metaphor for it. More specifically, a buyer “invites” $w$ sellers to the auction, where $w \in \{1,2\}$. The buyer sets a reserve price and the $w$ sellers submit bids which, in equilibrium, will equal their cost. A transaction occurs if the lowest bid is below the buyer’s reserve price. In the case of having chosen just one seller, the mechanism is equivalent to the buyer making a take it or leave it offer. List prices are presumed to be chosen less frequently than negotiated prices and this has the implication that a seller knows its cost type when it chooses its list price but does not know its actual cost until the time of negotiation. In practice, this uncertainty about future cost may be due to volatility in input prices or not knowing the opportunity cost of supply because future inventories or capacity constraints are uncertain.

If a non-trivial number of transactions occur at list price then it is not difficult for its list price to be informative about a seller’s cost type and thus have information

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\(^8\)One would like to endogenize the number of sellers that are solicited by a buyer but the current model assumes it is exogenous for reasons of tractability.
pertinent to negotiations between a buyer and a seller. One would expect, however, that the informativeness of the list price when it comes to negotiations would be minimal if a small share of transactions actually occur at list. Those are the types of markets with which we are interested and for which it is a challenge to explain how collusion in list prices could actually influence final transaction prices. Given this research objective, it will make for a cleaner analysis if we go to the extreme and have that no transactions take place at list price by supposing a list price is a cheap talk message and not actually a price at which buyers can buy.

The extensive form is as follows:

- Stage 1: Sellers draw types from \( \{L, H\} \) (which is private information to each seller) and choose a list price (which is a cheap talk message) from \( \{l, h\} \).

- Stage 2: Buyers learn their valuations, observe sellers' list prices, and choose \( w \) sellers where \( w \in \{1, 2\} \).

- Stage 3: Each seller realizes its cost. If a seller is type \( t \) then its cost is a draw from \([c_t, \bar{c}_t]\) according to \( F_t \).

- Stage 4: For each buyer, the \( w \) selected sellers participate in a second-price auction with a hidden reserve price. Each seller submits a bid equal to its cost. Transactions and transaction prices are determined as follows:

  - If there are two sellers in the auction and

    * both bids are below the reserve price then the buyer buys from the seller with the lowest bid at a price equal to the second lowest bid.

    * one bid is below the reserve price and the other bid is above the reserve price then the buyer buys from the seller with the lowest bid at a price equal to the reserve price.

    * both bids are above the reserve price then there is no transaction.
– If there is one seller in the auction and
  * the bid is below the reserve price then the buyer buys from the seller at the reserve price.
  * the bid is above the reserve price then there is no transaction.

A strategy for a seller is a pair of functions: a list price function and a bid function. The list price function maps from \( \{L,H\} \) to \( \{l,h\} \) and thus has a seller select a list price based on its cost type. In the event a seller is selected by a buyer, a bid function assigns a bid depending on the seller’s cost type, the seller’s cost, the other seller’s list price, and whether the buyer selected one or two sellers. The weakly dominant bidding strategy for a seller is to bid its cost. From hereon, we will think of a strategy for a seller as a list price function and a bid function that has its bid equal to its cost. In that case, there are four strategies, associated with the four ways in which to map \( \{L,H\} \) to \( \{l,h\} \). For a buyer who is restricted to choosing one seller, a strategy selects a seller and a reserve price conditional on the observed list prices and the buyer’s valuation. If the buyer chooses two sellers, a strategy selects a reserve price conditional on the observed list prices and the buyer’s valuation. The solution concept is perfect Bayes-Nash equilibrium.

3 Literature Review

This paper pertains to two literatures. One body of work considers firms setting list prices and then offering discounts. A second line of research models directed search in a market setting. Directed search is present in our model in that list prices may direct buyers to negotiate with certain sellers.
3.1 Posted Prices with Discounts

Chen and Rosenthal (1996), García Díaz, Hernán González, and Kujal (2009), Raskovich (2007), and Lester, Visschers, and Wolthoff (2015) have sellers post a list price which is subsequently followed by either discounts or negotiation. Those papers do not consider collusion and the driving forces to their analyses are distinct from that which operates in our model. Gill and Thanassoulis (2016) is the only paper to consider collusion but firms coordinate on both list and discounted prices. Sellers choose list prices in stage 1 and discounted prices in stage 2. Some buyers always buy at list price and are referred to as “price takers.” The remaining buyers, referred to as “bargainers,” have an opportunity to receive the discounted price. Each buyer has some exogenous probability of receiving a seller’s discounted price and, if they do not, then they can still buy from any seller at list price. If a seller deviates by undercutting the collusive list price in stage 1, it can potentially sell to both price takers and bargainers but the other seller can respond by lowering its discounted price in stage 2; that makes deviation less profitable compared to when there are no discounts. Alternatively, a seller could deviate by undercutting the discounted price, while still setting the collusive list price. However, it will only be able to increase its share of bargainers because price takers do not negotiate. Compared to when discounts are not feasible, collusion is shown to be more difficult.

3.2 Directed Search in Auctions and Markets

To the extent that list prices are used as a basis for selecting seller(s) with which to negotiate, our model is related to models of indicative bidding. Indicative bidding is a two-stage auction process commonly used in the sales of business assets with very high values. In the first stage, potentially interested buyers submit non-binding bids. These bids are meant to be indicative of bidders’ interest in the item for sale. The

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9Boshoff and Paha (2016) also considers collusion with list and discounted prices but the model has some technical problems.
highest of these non-binding bids, in conjunction with decisions regarding bidders’ qualifications, are then used to establish a short list of final (second-stage) bidders who participate in a first-price sealed-bid auction.

Ye (2007) provides the first study of indicative bidding and allows for costly information acquisition. The analysis suggests that the current design of indicative bidding cannot reliably select the most qualified bidders for the final sale, as there does not exist a symmetric, strictly increasing equilibrium bid function in the indicative bidding stage; hence, bidders do not truthfully reveal their types. By restricting indicative bids to a finite domain, Quint and Hendricks (2015) explicitly model indicative bidding as cheap talk with commitment, and show that a symmetric equilibrium exists in weakly-monotone strategies. But again, the highest-value bidders are not always selected, as bidder types “pool” over a finite number of bids. Our current paper differs from Ye (2007) and Quint and Hendricks (2015) mainly in that no entry cost is assumed for bidders, and that the reserve price in the final selling mechanism depends on the list prices (indicative bids). As such, a separating equilibrium in the cheap-talk stage becomes possible in our setting.

Menzio (2007) considers cheap talk in a search model of the labor market. Employers have private information about the quality of their vacancies and can costlessly communicate with unemployed workers before they engage in an alternating offer bargaining game to determine the wage. It is shown that unless the labor market is either too tight or too slack, there exists an equilibrium in which noncontractual announcements (cheap talk) about compensation is correlated with actual wages and, therefore, it serves to direct the search of workers. As we explain later, our theory encompasses the key driving forces in Menzio (2007).

Finally, Kim and Kircher (2015) introduce cheap talk into an otherwise canonical competing auctions setting. In their model, auctioneers with private reservation values compete for bidders by announcing cheap-talk messages. They show that the choice of the trading mechanism is critical. When the first-price auction is used,
there always exists an equilibrium in which each auctioneer truthfully reveals her type. When the second-price auction is used, however, no informative equilibrium exists.

4 Competition

This section characterizes perfect Bayes-Nash equilibria when sellers compete. As this is a cheap talk game, there are always pooling equilibria which, in our setting, means uninformative list prices.\textsuperscript{10} We will focus on whether there exist equilibria in which a seller’s list price is informative of its cost type.

Consider sellers using the separating strategy that has a low-cost (high-cost) type post a low (high) list price:

\[
\phi(t) = \begin{cases} 
  l & \text{if } t = L \\
  h & \text{if } t = H
\end{cases}
\]  

(1)

The associated buyer beliefs for a perfect Bayes-Nash equilibrium assign probability one to the seller being type \( L \) if its list price is \( l \) and probability one to the seller being type \( H \) if its list price is \( h \). In determining when a separating equilibrium exists, the analysis will examine when \( b = 1 \) (all buyers negotiate with one seller), \( b = 0 \) (all buyers negotiate with both sellers), and finally the general case of \( b \in [0, 1] \).

4.1 All Buyers Negotiate with One Seller

Given a seller is believed to be type \( t \), a buyer’s expected payoff from reserve price \( R \) is \((v - R) F_t(R)\) given its valuation is \( v \). By assumption, \( F_L(R) \geq F_H(R) \) so a buyer (at least weakly) prefers a cost type \( L \) to a cost type \( H \) because any offer by the buyer

\textsuperscript{10}With those equilibria, a seller’s strategy is to choose \( h \) with some probability \( x \in [0, 1] \) if type \( L \) or \( H \), and then bid its realized cost if selected by a buyer. A buyer’s beliefs on a seller’s type are the prior beliefs: If \( l \) or \( h \) is observed then a seller is type \( L \) with probability \( q \). A buyer chooses an optimal reserve price based on those prior beliefs.
is more likely to be accepted. Thus, if the two sellers have different list prices, the buyer will choose the one with the lower list price. Summarizing, a buyer’s optimal strategy is:

- If both sellers chose $l$ then randomly choose a seller and choose the reserve price:

$$R_L (v) \equiv \arg \max (v - R) \cdot F_L (R).$$  \hfill (2)

- If one seller chose $l$ and the other seller chose $h$ then choose the seller who chose $l$ and choose the reserve price $R_L (v)$.

- If both sellers chose $h$ then randomly choose a seller and choose the reserve price:

$$R_H (v) \equiv \arg \max (v - R) \cdot F_H (R).$$  \hfill (3)

Lemma 1 establishes that optimal reserve prices are increasing in a buyer’s value and the reserve price is higher when the buyer is thought to be a high-cost type.

**Lemma 1** $R_t (v)$ is increasing in $v$ and $R_H (v) > R_L (v), \forall v$.

A low-cost type seller prefers to choose $l$ and signal it is a low-cost type if and only if

$$\left( \frac{q}{2} + 1 - q \right) \int_v^{R_L (v)} \int_{\xi_L}^{R_L (v)} (R_L (v) - c) \cdot dF_L (c) \cdot dG (v) \geq \left( \frac{1 - q}{2} \right) \int_v^{R_H (v)} \int_{\xi_L}^{R_H (v)} (R_H (v) - c) \cdot dF_L (c) \cdot dG (v).$$  \hfill (4)

On the LHS of the inequality is the payoff from choosing $l$. The seller is chosen for sure by the buyer when the other seller posted $h$, which occurs when the other seller is type $H$ (and that occurs with probability $1 - q$), and is chosen with probability $1/2$ when the other seller posted $l$, which occurs when the other seller is type $L$ (and that occurs with probability $q$). Thus, a seller who chooses a low list price is approached by a buyer with probability $\frac{q}{2} + 1 - q$. In that case, the buyer offers a price of $R_L (v)$.
and the seller accepts the offer if its realized cost is less than \( R_L(v) \). If the seller selects a high list price then it is approached by the buyer with probability 1/2 in the event that the other seller also posted a high list price, and is not approached when the other seller posted a low list price. Hence, a seller assigns probability \((1 - q)/2\) to being approached by a buyer and, in that situation, is offered \( R_H(v) \) because the buyer infers from the seller’s high list price that it is a high-cost type.

If instead a seller is a high-cost type then it prefers to choose \( h \) and signal it is a high-cost type if and only if

\[
\left( \frac{1 - q}{2} \right) \int_{\underline{c}}^{\overline{c}} \int_{R_L(v)}^{R_H(v)} (R_H(v) - c) \, dF_H(c) \, dG(v) \\
\geq \left( \frac{q}{2} + 1 - q \right) \int_{\underline{c}}^{\overline{c}} \int_{R_L(v)}^{R_H(v)} (R_L(v) - c) \, dF_H(c) \, dG(v) .
\]

In Section 6, a parametric class of distributions is provided such that (4)-(5) hold so that a separating equilibrium exist.

When a buyer selects one seller with which to negotiate, a seller’s list price plays two roles. First, it affects the likelihood that a seller is selected by a buyer. By setting a low list price \( l \), a seller is selected with probability \( 1 - (q/2) \), while the probability is only \((1 - q)/2\) if it chooses a high list price \( h \). This we refer to as the inclusion effect in that a lower list price makes it more likely a buyer will include a seller in the negotiation process. A low list price signals a seller has a low-cost distribution in which case it is more likely to accept the buyer’s offer (that is, it is more likely to draw a cost below the buyer’s offer so that the seller accepts the offer). The inclusion effect makes posting a low list price attractive because it induces more buyers to approach a seller and thereby results in more sales. However, there is a countervailing effect from a seller posting a low list price which is that the buyer negotiates more aggressively knowing it is more likely the seller’s cost is low given its list price revealed it is a low-cost type. This bargaining effect is captured by the buyer making a lower offer (in the form of a lower reserve price) in response to a low list price compared to a high list price: \( R_L(v) < R_H(v) \). Though not labelling them as such, the inclusion
and bargaining effects are present in Menzio (2007) in the context of a labor market with search.

In sum, posting a low list price makes it more likely that a buyer negotiates with a seller but then the buyer will demand a lower price in those negotiations. As we will later show, a separating equilibrium can exist - and therefore list prices are informative - because only the firm with the low-cost distribution finds it profitable to accept a weaker bargaining position in exchange for a higher chance of a buyer approaching it.\footnote{It is also worth noting that signalling here is related to that found in Farrell and Gibbons (1989). They consider a bargaining setting for which one might expect cheap talk to be uninformative because informative equilibria do not exist in games with pure conflict (Crawford and Sobel, 1982). However, two sides of a bargaining situation actually have a mutual interest in communicating that each believes bargaining can result in a Pareto-improving allocation and thus it is worthwhile to bargain. In our setting, a buyer and a seller have a mutual interest of conveying information to enhance the chances that a Pareto-improving sale takes place.}

### 4.2 All Buyers Negotiate with Both Sellers

When all buyers approach both sellers \((b = 0)\), separating equilibria do not exist. Define \(R_{t', t''}(v)\) as the optimal reserve price for a buyer with valuation \(v\) when one seller is believed to be type \(t'\) and the other to be type \(t''\):

\[
R_{t', t''}(v) = \arg \max_R \int_{\xi_{t'}}^R \int_{c_{t'}}^R (v - c_{t''}) dF_{t''}(c_{t''}) dF_{t'}(c_{t'}) + \int_{\xi_{t''}}^R \int_{c_{t''}}^R (v - c_{t'}) dF_{t'}(c_{t'}) dF_{t''}(c_{t''}) + (v - R) \left[ (1 - F_{t''}(R)) F_{t'}(R) + (1 - F_{t'}(R)) F_{t''}(R) \right].
\tag{6}
\]

Lemma 2 establishes that the optimal reserve price is higher when seller(s) are high-cost type(s).

**Lemma 2** \(R_{HH}(v) > R_{LH}(v) > R_{LL}(v), \forall v.\)
Consider buyers having beliefs based on sellers using the separating strategy in (1). The expected profit per buyer to a seller of type \( t_1 \) whose list price is \( m_1 \) (and thus inferred to be \( \phi^{-1}(m_1) \)) when the other seller’s type and list price are \( t_2 \) and \( m_2 \), respectively, is

\[
B(m_1, t_1; m_2, t_2) = \int_{\mathbb{R}} \int_{\Omega_2} \min\left\{ R_{\phi^{-1}(m_1)\phi^{-1}(m_2)}(v), c_2 \right\} \left( \min \left\{ R_{\phi^{-1}(m_1)\phi^{-1}(m_2)}(v), c_2 \right\} - c_1 \right) \times
\]
\[
dF_{t_1}(c_1) dF_{t_2}(c_2) dG(v).
\]

A buyer’s optimal reserve price is \( R_{\phi^{-1}(m_1)\phi^{-1}(m_2)}(v) \) given list prices \( m_1 \) and \( m_2 \). If seller 1’s bid (= cost) is less than \( \min \left\{ R_{\phi^{-1}(m_1)\phi^{-1}(m_2)}(v), c_2 \right\} \) then a buyer with valuation \( v \) buys from seller 1 and pays a price equal to \( \min \left\{ R_{\phi^{-1}(m_1)\phi^{-1}(m_2)}(v), c_2 \right\} - c_1 \). Hence, the probability that seller 1 makes a sale is weakly increasing in the reserve price \( R_{\phi^{-1}(m_1)\phi^{-1}(m_2)}(v) \), as is the profit conditional on making a sale which equals \( \min \left\{ R_{\phi^{-1}(m_1)\phi^{-1}(m_2)}(v), c_2 \right\} - c_1 \). For realizations of \( c_2 \) and \( v \) such that \( R_{\phi^{-1}(m_1)\phi^{-1}(m_2)}(v) \leq c_2 \), both are strictly increasing in the reserve price. \( B(m_1, t_1; m_2, t_2) \) is then increasing in the reserve price.

If seller 2 uses (1) then seller 1’s expected payoff from list price \( m_1 \) is

\[
qB(m_1, t_1; l, L) + (1 - q) B(m_1, t_1; h, H).
\]

Given \( B(m_1, t_1; m_2, t_2) \) is increasing in the reserve price, Lemma 2 implies

\[
qB(h, t_1; l, L) + (1 - q) B(h, t_1; h, H) > qB(l, t_1; l, L) + (1 - q) B(l, t_1; h, H), \ t_1 \in \{L, H\}.
\]

A seller then prefers to post a high list price regardless of its type. Hence, a separating equilibrium does not exist.

With buyers approaching both sellers, a seller’s list price does not affect the probability of being selected - so there is no inclusion effect - but it does affect how aggressively the buyer negotiates. A seller will always want to signal it has a high-cost distribution because it induces a buyer to set a higher reserve price. Compared
to posting a low list price, a high list price results in a reserve price of $R_{HH}(v)$ instead of $R_{LH}(v)$ (when the other seller is a high cost-type) or $R_{HL}(v)$ instead of $R_{LL}(v)$ (when the other seller is a low cost-type). Given that seller 2’s bid is not affected by seller 1’s list price, seller 1’s expected profit is then higher by posting a high list price. By a similar argument one can also show that semi-pooling equilibria do not exist.

4.3 General Case

Thus far, it has been shown that a separating equilibrium may exist when $b = 1$, and only pooling equilibria exist when $b = 0$. List prices can be informative when they influence a buyer’s decision as to which seller to approach to negotiate a deal. When buyers negotiate with a single seller and a low list price signals a low-cost distribution, a buyer will choose the seller with the lowest list price because it knows it can expect to be able to negotiate a lower price. Furthermore, only a low-cost firm is willing to post a low list price and thereby accept a weaker bargaining position in exchange for a higher chance of a buyer approaching it.

Now let us show that if a separating equilibrium exists when $b = 1$ then there exists $b^* \in (0, 1)$ such that a separating equilibrium exists if and only if $b \in [b^*, 1]$. Define the expected profit to a seller of type $t$ whose list price is $m$ from a buyer who approaches only that seller:

$$A(m, t) \equiv \int_{v} \int_{\mathbb{E}_t} R_{\phi^{-1}(m)}(v) \left( R_{\phi^{-1}(m)}(v) - c \right) dF_t(c) dG(v). \quad (8)$$

Given list price $m$, the inferred type of the seller is $\phi^{-1}(m)$ in which case the reserve price set by the buyer is $R_{\phi^{-1}(m)}(v)$.

When it sets its list price, a seller knows that a fraction $b$ of buyers will approach one seller (and will choose the one with the lowest list price) and a fraction $1 - b$ will approach both sellers. In that case, a type $L$ seller optimally chooses list price $l$ if
and only if
\[
W(l, L, b) \equiv b \left(1 - \frac{q}{2}\right) A(l, L) + (1 - b) [q B(l, L; l, L) + (1 - q) B(l, L; h, H)]
\]

\[
\geq b \left(1 - \frac{q}{2}\right) A(h, L) + (1 - b) [q B(h, L; l, L) + (1 - q) B(h, L; h, H)] \equiv W(h, L, b).
\]

A type $H$ seller optimally chooses list price $h$ if and only if
\[
W(h, H, b) \equiv b \left(1 - \frac{q}{2}\right) A(h, H) + (1 - b) [q B(h, H; l, L) + (1 - q) B(h, H; h, H)]
\]

\[
\geq b \left(1 - \frac{q}{2}\right) A(l, H) + (1 - b) [q B(l, H; l, L) + (1 - q) B(l, H; h, H)] \equiv W(l, H, b)
\]

From Section 4.2, we know that if $b = 0$ then (9) does not hold (as a type $L$ seller prefers to choose list price $h$) though (10) does hold. Suppose that (9)-(10) are satisfied when $b = 1$. Combining these conditions for $b = 0$ and $b = 1$ delivers:

\[
W(l, L, 1) - W(h, L, 1) > 0 > W(l, L, 0) - W(h, L, 0)
\]

\[
W(h, H, 1) - W(l, H, 1) > 0 > W(l, H, 0) - W(h, H, 0)
\]


By the linearity of the conditions in (11) with respect to $b$, it follows that there exists $b^* \in (0, 1)$ such that (9)-(10) hold if and only if $b \in [b^*, 1]$. We have then proven:

**Theorem 3** If a separating equilibrium exists for $b = 1$ then $\exists b^* \in (0, 1)$ such that a separating equilibrium exists if and only if $b \in [b^*, 1]$.

Setting $b = 1$ and using (8) in (9)-(10), those conditions can be re-arranged to conclude that a separating equilibrium exists if and only if

\[
\frac{q}{1} \leq q \leq \frac{q}{1}
\]

\[
\equiv \frac{q}{1} \leq q \leq \frac{q}{1}
\]

\[
\equiv \frac{q}{1} \leq q \leq \frac{q}{1}
\]

\[
\equiv \frac{q}{1} \leq q \leq \frac{q}{1}
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\]

\[
\equiv \frac{q}{1} \leq q \leq \frac{q}{1}
\]

\[
\equiv \frac{q}{1} \leq q \leq \frac{q}{1}
\]
If the probability that the other seller is a low-cost type is too low \((q < \bar{q})\) then a low-cost seller prefers to post a high list price in order to induce buyers to set a high reserve price. If the probability that the other seller is a low-cost type is too high \((q > \bar{q})\) then a high-cost seller prefers to post a low list price in order to attract more buyers.

Prior to learning its type, a seller’s expected profit at the separating equilibrium is

\[
E[\pi^{\text{comp}}] = b \left[ q \left( \frac{q}{2} + 1 - q \right) A(l, L) + (1 - q) \left( \frac{1 - q}{2} \right) A(h, H) \right] + (1 - b) \left[ q^2 B(l, L; l, L) + q(1 - q) B(l, L; h, H) \right. \\
+ \left. q(1 - q) B(h, H; l, L) + (1 - q)^2 B(h, H; h, H) \right].
\]

The first bracketed expression pertains to the fraction \(b\) of buyers that negotiate with only one seller. With probability \(q\), the seller is low cost and chooses list price \(l\) which signals to buyers it has a low cost distribution. From these buyers, it will attract half of them if the other seller also posts a low list price (which occurs with probability \(q\)) and all of them if the other seller’s list price is high (which occurs with probability \(1 - q\)). Thus, in expectation, a low list price will attract \(q^2 + 1 - q\) of the buyers who only approach one seller, and the expected profit earned on each of them is \(A(l, L)\). Now suppose this seller is a high-cost type, which occurs with probability \(1 - q\), and thereby chooses list price \(h\). For the buyers who approach only one seller, the seller will not attract any of them when the other seller posts a low price and will get half of them when the other seller posts a high list price. A high list price then attracts, in expectation, \((1 - q)/2\) of those buyers, and the seller earns expected profit of \(A(h, H)\) per buyer. The second bracketed expression in (13) is the expected profit coming from the fraction \(1 - b\) of buyers who negotiate with both sellers where \(B(m_1, t_1; m_2, t_2)\) is weighted by the probability that sellers are types \((t_1, t_2)\).\(^{12}\)

\(^{12}\)Let us point out that it is critical for the existence of a separating equilibrium that a seller has incomplete information on its cost when it chooses its list price. To see why, consider a buyer.
5 Collusion

After evaluating when coordination on list prices is profitable in Section 5.1, we then turn to examining the conditions under which collusion is stable in Section 5.2.

5.1 Profitability of Coordinating on List Prices

Suppose the market was initially characterized by competition so sellers have been making independent decisions with regards to their list prices (as described by the separating equilibrium). Sellers now decide to coordinate their list prices by agreeing to set a high list price, regardless of their types. A key assumption is what buyers believe about sellers’ strategies. Rather than suppose that buyers anticipate that sellers are now colluding, it is assumed that sellers continue to believe that sellers are competing with respect to their list prices (as well as bids). Thus, when buyers observe a seller post a high list price, they believe the seller is a high-cost type though actually a seller is choosing a high list price even when it is a low-cost type.

As it is not common in economic theory to assume an agent’s beliefs are wrong, let us explain why we think this assumption is appropriate for this setting. If buyers determine that firms are likely to be colluding, it is in their best interests to shut down the cartel as soon as possible which means reporting it to the competition authority and, in those jurisdictions for which it is an option, pursuing private litigation to claim customer damages. Thus, if buyers are able to accurately conjecture that sellers are colluding then cartel duration should be short. The facts contradict that claim.

who has selected one seller. Suppose that a seller’s type was its cost rather than the distribution from which it chooses cost, and that it chose a list price after learning its cost. Consider two cost types, $c' < c''$, and a seller separating strategy such that it chooses list price $l$ when its cost is $c'$ and list price $h$ when its cost is $c''$. A buyer will then optimally set a reserve price of $c'$ if the list price was $l$ and $c''$ if the list price was $h$. As then a seller’s profit is zero, a type $c'$ would do better to announce a list price $h$ so a buyer infers its cost is $c''$ which results in an expected profit of $((1 - q)/2)(c'' - c') > 0$. Thus, there is no separating equilibrium as a low-cost seller would want to mimic a high-cost seller.
Cartels typically operate for many years before they are discovered and are most often not reported by buyers. Average duration for discovered (illegal) cartels is around six years (Harrington and Wei, 2015), with some cartels operating for decades before being discovered.\footnote{For a survey of various studies on cartel duration, see Levenstein and Suslow (2006). Across studies, mean duration is around 5 to 8 years.} One of the few data sets reporting how a cartel was discovered found that only eight out of 47 cartels convicted by the Antitrust Division of the U.S. Department of Justice were reported by a customer or learned about through private litigation (Hay and Kelley, 1974). For the many years that a cartel operated prior to being prosecuted, it must then be the case that either buyers were unaware there was a cartel or they knew there was a cartel and chose not to report it (nor to pursue litigation). In light of that fact, one must either assume buyers have accurate beliefs and are irrational (that is, they are not reporting that there is a cartel even though it is optimal for them to do so) or buyers are rational and have inaccurate beliefs (that is, they are not reporting because they do not know there is a cartel). Given the choice between dispensing with rationality and with accurate beliefs, we prefer to maintain the assumption of rationality.\footnote{In the U.S., private litigation that results in a guilty verdict allows customers to receive payments equal to treble damages. In principle, it could then make sense for buyers to allow damages to accumulate before suing. There are, however, two flaws in this argument. First, most private litigation is settled out of court and single damages is typical (Connor and Lande, 2015). If single damages is anticipated (and given that interest on past harm is not assessed in the U.S.), it is not optimal for buyers to delay; it is best for them to have collusion terminated as soon as possible. Second, the long duration prior to prosecution is not limited to jurisdictions with a tradition of customer damages. For example, Abrantes-Metz, Connor, and Metz (2013) find comparable duration lengths for cartels in the United States and the European Union over 1990-2004. In sum, it is difficult to rationalize customers delaying the reporting of a cartel if they knew one existed.}

Suppose a seller sets a high list price regardless of its type and buyers believe a seller is a high-cost (low-cost) type when it posts a high (low) list price. Expected
profit to a seller under collusion is then

\[ E[\pi^{\text{coll}}] \equiv b \left[ \left( \frac{q}{2} \right) A(h, L) + \left( \frac{1-q}{2} \right) A(h, H) \right] \]

\[ + (1 - b) \left[ q^2 B(h, L; h, L) + q(1-q) B(h, L; h, H) \right] \]

\[ + q(1-q) B(h, H; h, L) + (1-q)^2 B(h, H; h, H) \].

For the fraction \( b \) of buyers who deal with one seller, each seller will end up negotiating with half of them and earn expected profit per buyer of \( A(h, L) \) when it is a low-cost type and \( A(h, H) \) when it is a high-cost type. For the fraction \( 1 - b \) of buyers who bargain with both sellers, a seller earns \( B(h, t_1; h, t_2) \) per buyer when its type is \( t_1 \) and the other seller’s type is \( t_2 \).

The incremental profit from colluding, \( E[\pi^{\text{coll}}] - E[\pi^{\text{comp}}] \), is re-arranged to:

\[ E[\pi^{\text{coll}}] - E[\pi^{\text{comp}}] = b \left[ \left( \frac{q}{2} \right) A(h, L) + \left( \frac{1-q}{2} \right) A(h, H) - q \left( \frac{q}{2} + 1 - q \right) A(l, L) - (1 - q) \left( \frac{1-q}{2} \right) A(h, H) \right] \]

\[ + (1 - b) \left[ q^2 [B(h, L; h, L) - B(l, L; l, L)] + q(1-q) [B(h, L; h, H) - B(l, L; H)] \right] \]

\[ + q(1-q) [B(h, H; h, L) - B(h, H; l, L)] + (1-q)^2 [B(h, H; h, H) - B(h, H; H)] \].

Consider the first bracketed term of \( E[\pi^{\text{coll}}] - E[\pi^{\text{comp}}] \) which is the profit differential (per buyer) between collusion and competition associated with the fraction \( b \) of buyers who only approach one seller. Re-arranging that term yields

\[ q^2 (1/2) [A(h, L) - A(l, L)] + q(1-q)(1/2) [A(h, L) - A(l, L)] \]

\[ + q (1-q) (1/2) [A(h, H) - A(l, L)] \).

First note that if both sellers are high-cost types then, whether colluding or not, they post high list prices and are inferred by buyers to be high-cost types. Given expected profit is the same under collusion and competition, there is no term in (16) corresponding to the event when both are high-cost types. The first term in (16) pertains to when both sellers are low-cost types which occurs with probability \( q^2 \). In
that case, a seller attracts half of the buyers under both collusion and competition, and makes additional expected profit per buyer under collusion equal to

\[
A(h, L) - A(l, L) = \int_{v_L}^{v_H} \int_{c_L}^{c_H} (R_H(v) - R_L(v)) \, dF_L(c) \, dG(v) + \int_{v_L}^{v_H} \int_{R_L(v)}^{R_H(v)} (R_H(v) - c) \, dF_L(c) \, dG(v).
\]

The first term in (17) is when the seller’s cost is less than \(R_L(v)\). As collusion has it charge a high rather than a low list price, it ends up selling at \(R_H(v)\) instead of \(R_L(v)\). Because buyers set a higher reserve price compared to when firms compete in their list prices, the seller earns higher profit of \(R_H(v) - R_L(v)\) conditional on selling, which we refer to as the price-enhancing effect. The second term in (17) is when the seller’s cost lies in \([R_L(v), R_H(v)]\). Setting a low list price under competition would result in not making a sale because the seller’s bid (which equals its cost) would exceed the buyer’s reserve price of \(R_L(v)\) (which is based on the buyer believing the seller is a low-cost type). In contrast, under collusion, the seller posts a high list price which induces a buyer to set the higher reserve price of \(R_H(v)\) and, given it exceeds the seller’s cost, results in a transaction at a price of \(R_H(v)\). Thus, collusion produces profit of \(R_H(v) - c\), while competition would have yielded zero profit. Interestingly, collusion allows a Pareto-improving transaction to take place that would not have occurred under competition because collusion causes buyers to bargain less aggressively. This we refer to as the transaction-enhancing effect.

Next consider when the seller is a low-cost type and the other seller is a high-cost type. Under competition, the seller attracts all buyers and earns \(A(l, L)\) per buyer, while under collusion it earns a higher profit per buyer of \(A(h, L)\) but only attracts half of the buyers. The second term in (16) captures the half of the market that the seller attracts under both collusion and competition. On those buyers, the profit per buyer is higher by \(A(h, L) - A(l, L)\), and the associated profit gain is \(b(1/2) [A(h, L) - A(l, L)]\). However, this gain is offset by an expected loss of \(b(1/2) A(l, L)\) corresponding to the half of buyers who no longer solicit a bid from the
seller under collusion. That profit loss appears in the third term in (16). But the seller gets those lost buyers back when the tables are turned and it is now a high-cost type and the other seller is a low-cost type. In that event, it would not have attracted any buyers under competition but gets half of the buyers under collusion and earns expected profit of $b(1/2)A(h, H)$. That profit gain is also in the third term in (16). Hence, the net profit impact is $b(1/2) [A(h, H) - A(l, L)]$, which gives us the third term in (16). Referred to as the business-shifting effect, it is the change in profit associated with half of the buyers no longer soliciting a bid from a firm when it is a low-cost type (under competition) and now soliciting a bid when it is a high-cost type (under collusion). This profit change could be positive or negative. While, ceteris paribus, it is better for a seller to attract a buyer when it is a low-cost type, the buyer’s reserve price is lower in that event. If the third term is non-negative then (16) is positive which means collusion increases expected profit earned on buyers who solicit one offer. If the third term is negative then the sign of (16) is ambiguous.\footnote{The welfare effect of the business-shifting effect is also unclear. That the cost of the seller is higher on average will tend to reduce the payoff in the event of a transaction and reduce the likelihood of a transaction. However, a seller being a high-cost type will result in a higher reserve price which means, holding cost constant, the chances of a transaction are higher.}

Returning to the incremental profit from collusion in (15), the second bracketed expression pertains to the $1 - b$ fraction of buyers who solicit bids from both sellers. $B(h, t_1; h, t_2) - B(\phi(t_1), t_1; \phi(t_2), t_2)$ is the difference in expected profit per buyer for a type $t_1$ seller under collusion and under competition. It can be shown that

$$
B(h, t_1; h, t_2) - B(\phi(t_1), t_1; \phi(t_2), t_2) = \int_{t_1}^{v} \int_{R_{HH}(v)}^{R_{Hh}(v)} \int_{c_1}^{c_2} \left( c_2 - R_{t_1t_2}(v) \right) dF_{t_1} (c_1) dF_{t_2} (c_2) dG (v)
$$

$$
+ \int_{t_2}^{V} \int_{R_{Hh}(v)}^{R_{HH}(v)} \int_{c_1}^{c_2} \left( c_2 - c_1 \right) dF_{t_1} (c_1) dF_{t_2} (c_2) dG (v)
$$
When \((t_1, t_2) = (H, H)\), all four terms are zero because, whether colluding or competing, they choose high list prices so the outcome is the same. For any other type pairs, each of these four terms is positive. The first and third terms are driven by the price-enhancing effect. Collusion raises the buyer’s reserve price which increases the price seller 1 receives from \(R_{t_1 t_2}(v)\) to \(c_2\) (in the first term) and to \(R_{HH}(v)\) (in the third term). The second and fourth terms capture the transaction-enhancing effect. By inducing the buyer to have a higher reserve price of \(R_{HH}(v)\), seller 1 sells for a price of \(c_2\) (in the second term) and \(R_{HH}(v)\) (in the fourth term). There is no business-shifting effects given that these buyers solicit bids from both sellers. In sum, coordination on list prices increases profits from buyers who solicit bids from both sellers.

5.2 Coordination on List Prices as an Equilibrium

Though list prices do not formally constrain transaction prices, coordination on high list prices influences transaction prices because it induces buyers to negotiate less aggressively as they now believe sellers have higher costs. The impact on buyers’ bargaining behavior is manifested in a higher reserve price which benefits a seller in two ways. First, for those transactions that would have occurred whether firms colluded or competed, a seller receives a higher price because a buyer’s reserve price is higher. Second, the higher reserve price means that a buyer is less likely to cause bargaining to break down which implies a seller earns positive profit under collusion (and the buyer earns positive surplus) because a transaction is consummated that would not have taken place under competition.

With regards to establishing the stability of collusion, the usual Folk Theorem
arguments apply. Suppose the situation between buyers and sellers repeats itself infinitely often and \( \delta \in (0, 1) \) is the common discount factor among sellers. Each period, a seller acquires some partial private information on its cost for the upcoming period. This information acquisition is represented by a seller learning its type. With that knowledge, it then chooses its list price. Each period it receives new information about its cost which is represented by independently drawing a new type.\(^{16}\)

For this infinitely repeated game, consider a strategy profile that has sellers choose list price \( h \) (regardless of type) as long as both sellers have chosen \( h \) in past periods, and otherwise sellers revert to the separating equilibrium in which a type \( L \) seller chooses list price \( l \) and a type \( H \) seller chooses list price \( h \). The equilibrium conditions are:

\[
\frac{b}{2} A(h, t) + (1 - b) [qB(h, t; h, L) + (1 - q)B(h, t; h, H)] + \left( \frac{\delta}{1 - \delta} \right) E [\pi^{\text{coll}}] \quad (19)
\]

\[
\geq bA(l, t) + (1 - b) [qB(l, t; h, L) + (1 - q)B(l, t; h, H)] + \left( \frac{\delta}{1 - \delta} \right) E [\pi^{\text{comp}}], \quad t \in \{L, H\}.
\]

Note that when a seller deviates by setting a low list price (RHS of (19)), it is ensured of attracting all buyers because the other seller will be posting a high list price. Rearrange (19) to:

\[
b \left[ \frac{1}{2} A(h, t) - A(l, t) \right]
+ (1 - b) [q (B(h, t; h, L) - B(l, t; h, L)) + (1 - q) (B(h, t; h, H) - B(l, t; h, H))]
+ \left( \frac{\delta}{1 - \delta} \right) \left\{ E [\pi^{\text{coll}}] - E [\pi^{\text{comp}}] \right\} \geq 0.
\]

We have already discussed the third term. Consider the first term, \( b \left[ \frac{1}{2} A(h, t) - A(l, t) \right] \).

Given that a separating equilibrium is presumed to exist then, for a type \( H \), if \( b = 1 \)

---

\(^{16}\)If a period is, say, six months then a firm knows its cost distribution for the next six months and, based on those beliefs, chooses a list price. Over the ensuing six months, a seller gets a cost draw when a buyer arrives at the seller and it is that cost which is relevant when bargaining with the buyer.
then (10) take the form:

\[
\left(1 - \frac{q}{2}\right) A(h, H) > \left(1 - \frac{q}{2}\right) A(l, H) \Leftrightarrow \\
\left(\frac{1}{2}\right) A(h, H) - A(l, H) > \left(\frac{q}{2}\right) [A(h, H) - A(l, H)].
\]  

(21)

Specifying (17) for a high-cost type, it can be shown that \(A(h, H) - A(l, H) > 0\). Hence, the RHS of (21) is positive which implies

\[
\left(\frac{1}{2}\right) A(h, H) - A(l, H) > 0.
\]  

(22)

Thus, the first term in (20) is positive for a high-cost type. However, for a low-cost type seller, the first term, \(\left(\frac{1}{2}\right) A(h, L) - A(l, L)\), need not be positive. For those buyers who negotiate with one seller, it is possible that a low-cost type seller may earn higher expected profit by posting a low list price and attracting all of them rather than setting a high list price as prescribed by the collusive strategy.

It is straightforward to show that the second term in (20) is positive,

\[
q [B(h, t; h, L) - B(l, t; h, L)] + (1 - q) [B(h, t; h, H) - B(l, t; h, H)] > 0.
\]  

(23)

For buyers who approach both sellers, this expression is the difference in expected profit per buyer between having a buyer think a seller is a high-cost type and is a low-cost type. That is always positive because a buyer’s beliefs do not affect the bid of the other seller (recall that sellers are not coordinating their bids) but do favorably affect the buyer’s reserve price. Hence, expected profit per buyer is higher. The volume of buyers is unchanged with collusion as all of these buyers approach both sellers.

In sum, if collusion is profitable, \(E[\pi^{\text{coll}}] > E[\pi^{\text{comp}}]\), then (19) always holds for \(t = H\), and holds for \(t = L\) when \(\delta\) is close enough to one. Under those conditions, the incentive compatibility constraints are satisfied and it is an equilibrium for firms to coordinate on high list prices. Recall that \(E[\pi^{\text{coll}}] - E[\pi^{\text{comp}}]\) in (15) is a weighted average of a positive term with weight \(1 - b\) and a term with an ambiguous sign with
weight \( b \). Thus, if collusion is profitable for \( b = 1 \) then it is profitable for all values of \( b \).

We will say collusion is “feasible” when list prices are informative under competition (that is, a separating equilibrium exists) and is “profitable” when \( E[\pi^{\text{coll}}] > E[\pi^{\text{comp}}] \). Summing up Sections 4 and 5, we have shown: 1) if collusion is feasible when \( b = 1 \) then it is feasible when \( b \) is sufficiently high; 2) if collusion is profitable when \( b = 1 \) then it is profitable for all \( b \); and 3) if collusion is feasible and profitable then collusion is sustainable as an equilibrium outcome when \( \delta \) is close enough to one. This finding is summarized as:

**Theorem 4** Assume a separating equilibrium exists ((12) is satisfied) and \( E[\pi^{\text{coll}}] > E[\pi^{\text{comp}}] \) when \( b = 1 \). Then there exists \( \tilde{b} \in (0, 1) \) and \( \tilde{\delta} \in (0, 1) \) such that if \( (b, \delta) \in [\tilde{b}, 1] \times [\tilde{\delta}, 1] \) then coordination on high list prices is an equilibrium outcome in the infinitely repeated game.

### 6 Collusion for a Class of Parametric Distributions

In this section, a parametric class of distributions is provided for which collusion is feasible and profitable when \( b = 1 \) which, by Theorem 4, implies coordination on list prices can occur when sufficiently many buyers negotiate with one seller.

Assume \( b = 1 \) and valuations and costs have support \([0, 1]\). Valuations are uniformly distributed: \( G(v) = v \). The cdf for a low-cost type is \( F_L(c) = c^\alpha \) and for a high-cost type is \( F_H(c) = c^\beta \), where \( 0 < \alpha < \beta \) so that the high-cost distribution first-order stochastically dominates the low-cost distribution. It is also easily verified that \( h_L(c) = c/\alpha > c/\beta = h_H(c) \) so the inverse hazard rate ranking is satisfied. Recall that a seller is a low-cost type with probability \( q \).

\(^{17}\)The proofs of the all results in this section are provided in an Online Appendix.
Theorem 5  Under the assumptions of Section 6, collusion is feasible if and only if

\[
q(\alpha, \beta) \equiv \frac{\beta^{\alpha+1}}{(\beta+1)^{\alpha+1}} - 2 \frac{\alpha^{\alpha+1}}{(\alpha+1)^{\alpha+1}} \leq q \leq \frac{\beta^{\beta+1}}{(\beta+1)^{\beta+1}} - 2 \frac{\alpha^{\beta+1}}{(\alpha+1)^{\beta+1}} \equiv q(\alpha, \beta) \tag{24}
\]

and is profitable if \( \alpha < 1 \).

Given the distributions, the necessary and sufficient conditions for a separating equilibrium described in (12) take the form in (24). It can be shown that \( \alpha < \beta \) implies the RHS of (24) exceeds the LHS. For example, if \( \alpha = 0.5 \) and \( \beta = 2 \) then collusion is feasible if \( q \in [0.453, 0.857] \). A sufficient condition for collusion to be profitable is that the low-cost distribution is concave, \( \alpha < 1 \).

For when \( (\alpha, \beta) \in [0, 1] \times [0, 2] \), Figure 1 reports the range of values for \( q, \bar{q}(\alpha, \beta) - q(\alpha, \beta) \), such that collusion is feasible and profitable (where the latter holds because \( \alpha < 1 \)).\(^{18}\) Depending on the values for \( (\alpha, \beta) \), there can be a wide range of values for \( q \) such that firms can effectively and profitably coordinate their list prices.

Figure 1: Range of Values for \( q \) for which

Collusion is Feasible and Profitable

\(^{18}\) \( \bar{q}(\alpha, \beta) \) and \( q(\alpha, \beta) \) are constrained to lie in \([0, 1] \). Hence, more exactly, Figure 1 reports \( \max \{ \min \{ \bar{q}(\alpha, \beta), 1 \}, 0 \} - \min \{ \max \{ q(\alpha, \beta), 0 \}, 1 \} \).
7 Tying Theory to Cases

Distilling the key elements of the theory, we find that effective coordination on list prices requires that the inclusion effect is sufficiently strong so list prices are informative which then allows sellers to induce buyers to bargain less aggressively through the posting of high list prices. In our simple set-up, the inclusion effect is sufficiently strong when enough buyers negotiate with only one of the two sellers. For the purpose of drawing some tentative insight into the market conditions conducive to collusion, let us extrapolate this result by conjecturing, more generally, that the inclusion effect is stronger when buyers negotiate with a smaller fraction of sellers. As each negotiation takes time and effort, a buyer is likely to negotiate with a smaller fraction of sellers when a market has more sellers. One also expects buyers to negotiate with fewer sellers when the magnitude of the expenditure over which negotiation is taking place is smaller (in which case the extent of possible savings from negotiation are less). Thus, a market for a low-expenditure item, such as some minor raw material in an industrial buyer’s production process, which is sold by many sellers should have a strong inclusion effect. Assuming the logic of our result extends, coordination on list prices would then be an effective form of collusion in that market. In contrast, a market for a high-expenditure item, such as some complex piece of equipment, sold by a small number of sellers is a market for which the inclusion effect is weak and, therefore, coordination on list prices is not likely to be effective.

To illustrate how this intuition can be applied to assess the credibility of a claim that coordination on lists price is an effective collusive strategy, let us consider two cases: large turbine generators around 1960 and urethane around 2000. A turbine generator is a device that converts mechanical energy into electrical energy.\(^\text{19}\) The particular market under consideration is large turbine generators purchased by electric utilities. It is a substantial custom-made piece of equipment which could cost in

\(^{19}\)For details of the turbine generators market and legal case, see Sultan (1974) and Harrington (2011).
excess of $10 million in the early 1960s (which is around $80 million in 2016 dollars). At the time, General Electric and Westinghouse were the only producers of turbine generators. In light of the item’s high expense to a buyer and the presence of only two sellers, it is quite likely that a buyer would negotiate with both sellers. In such a market, our theory suggests that list prices would be uninformative because most buyers would negotiate with both sellers. Therefore, coordinating on list prices would be an ineffective method of collusion because a seller’s list price would have little effect on a buyer’s beliefs about a seller’s cost and thus have minimal impact on bargaining and final transaction prices. While GE and Westinghouse did collude in this market, it is notable that they did so by first coordinating on a policy of not offering discounts in which case list prices became actual transaction prices. GE then acted as a price leader on list prices. Absent a move to a “no negotiation” policy, the theory of this paper suggests that coordination on list prices would have been ineffective.

Returning to a case discussed in the Introduction, the urethane market would appear to have features more conducive to coordination on list prices. Polyurethanes are used in various consumer and industrial products including mattress foams, insulation, sealants, and footwear. BASF, Bayer, Dow Chemical, Huntsman, and Lyondell either pled guilty or were convicted of coordinating their list prices over 2000-03. Their market shares in the sub-markets for polyether polyols, toluene diisocyanate (TDI), and methylene diphenyl diisocyanate (MDI) are shown in Figure 2. In contrast to turbine generators with two sellers, buyers of various categories of polyurethane had four or five suppliers from which to choose.21 While, at present, we do not have any data on the level of expenditure for a buyer, it is probably not a big ticket item like a large turbine generator. It would then seem unlikely that a buyer would negoti-

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20 The ensuing facts are from In re: Urethane Antitrust Litigation, 768. F.3d 1245 (10th Cir. 2014) and Class Plaintiffs’ Response Brief, In re: Urethane Antitrust Litigation, (10th Cir.), February 14, 2014.

21 Cartel members controlled the entire market for MDI and TDI and 79% of the market for polyether polyols.
ate with all or almost sellers. If that is so then the inclusion effect might be significant enough to support informative list prices which would be the basis for firms effectively colluding through coordination on list prices.

Figure 2: Market Shares in Urethane


8 Concluding Remarks

The primary contribution of this paper is to show that coordination on list prices can be an effective form of collusion even though firms are left unconstrained in the final prices they offer buyers. By coordinating on high list prices, sellers can cause buyers to believe that costs are likely to be high which will induce buyers to bargain less aggressively and that will serve to raise final negotiated prices. Notably, sellers continue to bargain in a competitive manner. We also offer some initial insight for what types of markets are suitable for this type of collusive practice.

A key assumption is that buyers do not suspect that firms have formed a cartel. As argued, that assumption is consistent with the evidence on cartel duration. Nevertheless, customers might eventually become suspicious after continually observing high list prices (and, more subtly, observing bids that are probabilistically unlikely if sellers were drawing from the high-cost distribution). Of course, if there is the prospect of buyers becoming suspicious then sellers might anticipate that response
which could lead them to not always choose high list prices. Modelling a game between customers seeking to detect a cartel and a cartel seeking to avoid detection is a technically challenging direction we leave for future research.²²

A natural question is why firms would choose to coordinate only on list prices rather than go that additional step and coordinate on final prices, especially given that express communication on either list or final prices is likely to be per se illegal. The answer may lie with a cartel’s concern about detection. For two reasons, detection by customers or the competition authority may be less likely when firms only coordinate on list prices. First, coordinating on final prices along with a market allocation (and the monitoring of sales) requires more extensive and frequent communication among cartel members which enhances the chances of the cartel’s discovery. Second, customers might be less inclined to think that firms are colluding when they offer different final prices, even though their list prices are similar. Competition in final prices could then delay buyers from suspecting that firms are colluding. A topic for future research is to understand when the incremental profit from coordination on final prices is sufficiently great compared to coordination on list prices that firms would prefer the former in spite of the higher chances of detection.

²²Besanko and Spulber (1989, 1990) consider a static game of incomplete information between a (possible) cartel and either customers or the competition authority who are trying to determine whether there is a cartel. Harrington (2004, 2005) and Harrington and Chen (2006) examine the impact of detection for the price path in a dynamic setting but the customers or competition authority are represented by a detection technology and thus not strategic.
9 Appendix

Proof of Lemmas 1 and 2: First to show Lemma 1, the first-order conditions of (2) and (3) are given by

\[ v - R_t(v) = h_t(R_t(v)), \quad t = L, H. \] (25)

Differentiating with respect to \( v \) on both sides and arranging, we have

\[ R'_t(v) = \frac{1}{1 + h'_t(R_t(v))} > 0. \]

To show that \( R_H(v) > R_L(v) \) \( \forall v \), suppose the negation so that \( R_H(v) \leq R_L(v) \) for some \( v \). It follows that

\[ 0 \leq - (R_H(v) - R_L(v)) = h_H(R_H(v)) - h_L(R_L(v)) \leq h_H(R_L(v)) - h_L(R_L(v)) < 0 \]

which is a contradiction.

Next to show Lemma 2, it can be verified that the first-order condition of (6) is given by

\[ 0 = (1 - F'(R_{t''}v)) f'_v(R_{t''}v) [(v - R_{t''}v) - h_v(R_{t''}v)] \]
\[ + (1 - F'(R_{t''}v)) f''_v(R_{t''}v) [(v - R_{t''}v) - h_v'(R_{t''}v)]. \]

If \( t' = t'' = L \) or \( t' = t'' = H \) then the above first-order condition reduces to (25), which implies \( R_{LL} = R_L \) and \( R_{HH} = R_H \). Now consider when \( t' = L \) and \( t'' = H \). Given the assumption that \( h_L(R_{LH}) > h_H(R_{LH}) \), we have

\[ (v - R_{LH}) - h_L(R_{LH}) < 0 < (v - R_{LH}) - h_H(R_{LH}), \]

which in turn implies

\[ R_{LH} + h_L(R_{LH}) > R_{LL} + h_L(R_{LL}); \quad R_{HH} + h_H(R_{HH}) > R_{LH} + h_H(R_{LH}) \]

Given that \( h'_t(z) \geq 0 \), we have the strict monotonicity of \( z + h_t(z) \). We thus have \( R_{HH} > R_{LH} > R_{LL} \).
References


Section 5: Collusion

To show that

\[
B(h, t_1; h, t_2) - B(\phi(t_1), t_1; \phi(t_2), t_2) = \int_{v_1}^{\pi} \int_{R_{t_1, t_2}(v)}^{R_{t_1, t_2}(v)} \left( c_2 - R_{t_1, t_2}(v) \right) dF_{t_1}(c_1) dF_{t_2}(c_2) dG(v)
\]
\[
+ \int_{v_1}^{\pi} \int_{R_{t_1, t_2}(v)}^{R_{t_1, t_2}(v)} \left( c_2 - c_1 \right) dF_{t_1}(c_1) dF_{t_2}(c_2) dG(v)
\]
\[
+ \int_{v_1}^{\pi} \int_{\tilde{R}_{t_2}(v)}^{\tilde{R}_{t_2}(v)} \left( R_{HH}(v) - R_{t_1, t_2}(v) \right) dF_{t_1}(c_1) dF_{t_2}(c_2) dG(v)
\]
\[
\int_{v_1}^{\pi} \int_{R_{t_1, t_2}(v)}^{R_{t_1, t_2}(v)} \left( R_{HH}(v) - c_1 \right) dF_{t_1}(c_1) dF_{t_2}(c_2) dG(v),
\]

consider

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\[ B(h, t_1; h, t_2) - B(t_1, t_1; t_2, t_2) = \int_{\mathbb{T}} \int_{\mathbb{T}} R_{HH}(v) \int_{\mathbb{T}} R_{LL}(v) \quad (c_2 - c_1) \, dF_t (c_1) \, dF_t (c_2) \, dG (v) \]

\[
+ \int_{\mathbb{T}} \int_{\mathbb{T}} R_{HH}(v) \int_{\mathbb{T}} R_{LL}(v) \quad (R_{HH} (v) - R_{t_1 t_2} (v)) \, dF_t (c_1) \, dF_t (c_2) \, dG (v) \\
+ \int_{\mathbb{T}} \int_{\mathbb{T}} R_{HH}(v) \int_{\mathbb{T}} R_{LL}(v) \quad (R_{HH} (v) - R_{t_1 t_2} (v)) \, dF_t (c_1) \, dF_t (c_2) \, dG (v) \\
+ \int_{\mathbb{T}} \int_{\mathbb{T}} R_{HH}(v) \int_{\mathbb{T}} R_{LL}(v) \quad (R_{HH} (v) - c_1) \, dF_t (c_1) \, dF_t (c_2) \, dG (v) \\
- \int_{\mathbb{T}} \int_{\mathbb{T}} R_{HH}(v) \int_{\mathbb{T}} R_{LL}(v) \quad (R_{t_1 t_2} (v) - c_1) \, dF_t (c_1) \, dF_t (c_2) \, dG (v) \\
- \int_{\mathbb{T}} \int_{\mathbb{T}} R_{HH}(v) \int_{\mathbb{T}} R_{LL}(v) \quad (R_{t_1 t_2} (v) - c_1) \, dF_t (c_1) \, dF_t (c_2) \, dG (v) \\
\]

and the following sequence of steps

\[ B(h, t_1; h, t_2) - B(t_1, t_1; t_2, t_2) = \int_{\mathbb{T}} \int_{\mathbb{T}} R_{HH}(v) \int_{\mathbb{T}} R_{LL}(v) \quad (c_2 - c_1) \, dF_t (c_1) \, dF_t (c_2) \, dG (v) \]

\[
+ \int_{\mathbb{T}} \int_{\mathbb{T}} R_{HH}(v) \int_{\mathbb{T}} R_{LL}(v) \quad (c_2 - c_1) \, dF_t (c_1) \, dF_t (c_2) \, dG (v) \\
+ \int_{\mathbb{T}} \int_{\mathbb{T}} R_{HH}(v) \int_{\mathbb{T}} R_{LL}(v) \quad (R_{HH} (v) - R_{t_1 t_2} (v)) \, dF_t (c_1) \, dF_t (c_2) \, dG (v) \\
+ \int_{\mathbb{T}} \int_{\mathbb{T}} R_{HH}(v) \int_{\mathbb{T}} R_{LL}(v) \quad (R_{HH} (v) - R_{t_1 t_2} (v)) \, dF_t (c_1) \, dF_t (c_2) \, dG (v) \\
+ \int_{\mathbb{T}} \int_{\mathbb{T}} R_{HH}(v) \int_{\mathbb{T}} R_{LL}(v) \quad (R_{HH} (v) - c_1) \, dF_t (c_1) \, dF_t (c_2) \, dG (v) \\
- \int_{\mathbb{T}} \int_{\mathbb{T}} R_{HH}(v) \int_{\mathbb{T}} R_{LL}(v) \quad (R_{t_1 t_2} (v) - c_1) \, dF_t (c_1) \, dF_t (c_2) \, dG (v) \\
- \int_{\mathbb{T}} \int_{\mathbb{T}} R_{HH}(v) \int_{\mathbb{T}} R_{LL}(v) \quad (R_{t_1 t_2} (v) - c_1) \, dF_t (c_1) \, dF_t (c_2) \, dG (v) \\
\]
\[ B(h, t_1; h, t_2) - B(t_1, t_1; t_2, t_2) = \]
\[ \int_{\Omega} \int_{\Omega_{t_1}}^{\Omega_{t_2}} \left( c_2 - R_{t_1 t_2} (v) \right) dF_{t_1} (c_1) dF_{t_2} (c_2) dG (v) \]
\[ + \int_{\Omega} \int_{\Omega_{t_1}}^{\Omega_{t_2}} (c_2 - c_1) dF_{t_1} (c_1) dF_{t_2} (c_2) dG (v) \]
\[ + \int_{\Omega} \int_{R_{t_1 t_2} (v)}^{\Omega_{t_1}} \left( R_{HH} (v) - R_{t_1 t_2} (v) \right) dF_{t_1} (c_1) dF_{t_2} (c_2) dG (v) \]
\[ + \int_{\Omega} \int_{\Omega_{t_1}}^{\Omega_{t_2}} (R_{HH} (v) - c_1) dF_{t_1} (c_1) dF_{t_2} (c_2) dG (v) \]
\[ - \int_{\Omega} \int_{R_{t_1 t_2} (v)}^{\Omega_{t_1}} \left( R_{t_1 t_2} (v) - c_1 \right) dF_{t_1} (c_1) dF_{t_2} (c_2) dG (v) \]

\[ B(h, t_1; h, t_2) - B(t_1, t_1; t_2, t_2) = \]
\[ \int_{\Omega} \int_{\Omega_{t_1}}^{\Omega_{t_2}} \left( c_2 - R_{t_1 t_2} (v) \right) dF_{t_1} (c_1) dF_{t_2} (c_2) dG (v) \]
\[ + \int_{\Omega} \int_{R_{t_1 t_2} (v)}^{\Omega_{t_1}} \left( c_2 - c_1 \right) dF_{t_1} (c_1) dF_{t_2} (c_2) dG (v) \]
\[ + \int_{\Omega} \int_{R_{t_1 t_2} (v)}^{\Omega_{t_1}} (c_2 - c_1) dF_{t_1} (c_1) dF_{t_2} (c_2) dG (v) \]
\[ + \int_{\Omega} \int_{\Omega_{t_1}}^{\Omega_{t_2}} (R_{HH} (v) - R_{t_1 t_2} (v)) dF_{t_1} (c_1) dF_{t_2} (c_2) dG (v) \]
\[ + \int_{\Omega} \int_{\Omega_{t_1}}^{\Omega_{t_2}} (R_{HH} (v) - c_1) dF_{t_1} (c_1) dF_{t_2} (c_2) dG (v) \]
\[ - \int_{\Omega} \int_{\Omega_{t_1}}^{\Omega_{t_2}} \left( R_{t_1 t_2} (v) - c_1 \right) dF_{t_1} (c_1) dF_{t_2} (c_2) dG (v) \]
\[ B(h, t_1; h, t_2) - B(t_1, t_1; t_2, t_2) = \int_{\mathbb{R}} \int_{\mathbb{R}^{t_1}} \int_{\mathbb{R}^{t_2}} \left( c_2 - R_{t_1 t_2}(v) \right) dF_{t_1}(c_1) dF_{t_2}(c_2) dG(v) \]

\[ + \int_{\mathbb{R}} \int_{\mathbb{R}^{t_1}} \int_{\mathbb{R}^{t_2}} \left( c_2 - R_{t_1 t_2}(v) \right) dF_{t_1}(c_1) dF_{t_2}(c_2) dG(v) \]

\[ + \int_{\mathbb{R}} \int_{\mathbb{R}^{t_1}} \int_{\mathbb{R}^{t_2}} \left( c_2 - c_1 \right) dF_{t_1}(c_1) dF_{t_2}(c_2) dG(v) \]

\[ + \int_{\mathbb{R}} \int_{\mathbb{R}^{t_1}} \int_{\mathbb{R}^{t_2}} \left( R_{HH}(v) - R_{t_1 t_2}(v) \right) dF_{t_1}(c_1) dF_{t_2}(c_2) dG(v) \]

\[ + \int_{\mathbb{R}} \int_{\mathbb{R}^{t_1}} \int_{\mathbb{R}^{t_2}} \left( R_{HH}(v) - R_{t_1 t_2}(v) \right) dF_{t_1}(c_1) dF_{t_2}(c_2) dG(v) \]

\[ + \int_{\mathbb{R}} \int_{\mathbb{R}^{t_1}} \int_{\mathbb{R}^{t_2}} \left( R_{HH}(v) - c_1 \right) dF_{t_1}(c_1) dF_{t_2}(c_2) dG(v) \]

\[ + \int_{\mathbb{R}} \int_{\mathbb{R}^{t_1}} \int_{\mathbb{R}^{t_2}} \left( R_{HH}(v) - c_1 \right) dF_{t_1}(c_1) dF_{t_2}(c_2) dG(v) \]

Section 6: Collusion is Feasible and Profitable for a Class of Parametric Distributions

As all buyers approach only one seller, optimal reserve prices (depending on the seller’s revealed type) are

\[ R_L(v) = \arg \max_R(v - R) F_L(R) = \arg \max_R(v - R) R^\alpha \Rightarrow R_L(v) = \left( \frac{\alpha}{\alpha + 1} \right) v. \]

\[ R_H(v) = \arg \max_R(v - R) F_H(R) = \arg \max_R(v - R) R^\beta \Rightarrow R_H(v) = \left( \frac{\beta}{\beta + 1} \right) v. \]
It can be shown that a buyer’s expected utility is $v + 1 + v + 1 + 1 + 1$ from a low-cost and high-cost seller, respectively. Given $\alpha < \beta$, expected utility is higher from a low-cost seller so a buyer prefers to solicit a bid from that seller type.

It is straightforward but tedious algebra to show that $(\text{??})$ takes the form in $(\text{??})$. Let us show that if $\alpha < \beta$ then the RHS of $(\text{??})$ exceeds the LHS. First note that the denominators are positive because $\frac{\beta}{\alpha + 1} > \frac{\beta}{\alpha + 1}$. Letting $C \equiv \frac{\alpha + 1}{(\alpha + 1)^{\beta + 1}}$, $E \equiv \frac{\alpha + 1}{(\alpha + 1)^{\beta + 1}}$, and $D \equiv \frac{\beta + 1}{(\beta + 1)^{\beta + 1}}$, $(\text{??})$ becomes

\[
\frac{C - 2E}{C - E} \leq \frac{D - 2F}{D - F} \iff (C - 2E)(D - F) \leq (C - E)(D - 2F) \\
\iff CD - 2ED - FC + 2EF \leq CD - ED - 2FC + 2EF \\
\iff FC \leq ED \iff \frac{\alpha^{\beta + 1}}{(\alpha + 1)^{\beta + 1}} \leq \frac{\beta^{\alpha + 1}}{(\alpha + 1)^{\beta + 1}} \iff \frac{\alpha^{\beta + 1}}{(\alpha + 1)^{\beta + 1}} \leq \frac{\alpha^{\beta + 1}}{(\alpha + 1)^{\beta + 1}} \iff \frac{\alpha^{\beta + 1}}{(\alpha + 1)^{\beta + 1}} \leq \frac{\alpha^{\beta + 1}}{(\alpha + 1)^{\beta + 1}} \iff \frac{\alpha^{\beta + 1}}{(\alpha + 1)^{\beta + 1}} \leq \frac{\alpha^{\beta + 1}}{(\alpha + 1)^{\beta + 1}}
\]

which holds because $\beta > \alpha$ and $\frac{\beta}{\alpha + 1} \geq \frac{\alpha}{\alpha + 1}$. Thus, collusion is feasible if the probability of a seller being a low-cost type lies between 0.453 and 0.857.

Next we prove that if $\alpha < 1$ then $E[\pi^{\text{coll}}] > E[\pi^{\text{comp}}]$. Under competition, the ex-ante expected profit for a seller is

\[
E[\pi^{\text{comp}}] = q \left( 1 - \frac{q}{2} \right) \int_0^1 \int_0^{\frac{\alpha + 1}{\alpha + 1}} \left( \frac{\alpha}{\alpha + 1} - v - c \right) \alpha c^{\alpha - 1} dc dv \\
+ \left( 1 - q \right) \left( 1 - \frac{q}{2} \right) \int_0^1 \int_0^{\beta + 1} \left( \frac{\beta}{\beta + 1} - v - c \right) \beta c^{\beta - 1} dc dv
\]

\[
E[\pi^{\text{comp}}] = q \left( 1 - \frac{q}{2} \right) \left( \frac{\alpha^{\alpha + 1}}{(\alpha + 2)(\alpha + 1)^{\alpha + 2}} \right) + \left( 1 - q \right) \left( 1 - \frac{q}{2} \right) \left( \frac{\beta^{\beta + 1}}{(\beta + 2)(\beta + 1)^{\beta + 2}} \right).
\]

\[
E[\pi^{\text{comp}}] = \frac{1}{2} \int_0^1 \left[ q \int_0^{\frac{\beta}{\beta + 1}} \left( \frac{\beta}{\beta + 1} - v - c \right) \alpha c^{\alpha - 1} dc + \left( 1 - q \right) \int_0^{\frac{\beta}{\beta + 1}} \left( \frac{\beta}{\beta + 1} - v - c \right) \beta c^{\beta - 1} dc \right] dv
\]

\[
E[\pi^{\text{coll}}] = \frac{1}{2} q \left( \frac{\beta^{\beta + 1}}{(\beta + 2)(\beta + 1)^{\beta + 2}} - \frac{\alpha^{\alpha + 1}}{(\alpha + 2)(\alpha + 1)^{\alpha + 2}} \right) + \frac{1}{2} \left( 1 - q \right) \left( \frac{1}{\beta + 1} \right) \left( \frac{\beta}{\beta + 1} \right)^{\beta + 1}.
\]

We require $(2) > (1)$ which can be shown to be equivalent to

\[
q \left( \frac{\beta^{\beta + 1}}{(\beta + 2)(\beta + 1)^{\beta + 2}} - \frac{\alpha^{\alpha + 1}}{(\alpha + 2)(\alpha + 1)^{\alpha + 2}} \right) + \frac{1}{2} \left( 1 - q \right) \left( \frac{1}{\beta + 1} \right) \left( \frac{\beta}{\beta + 1} \right)^{\beta + 1}.
\]
If \( \frac{\beta^{\alpha+1}}{(\beta+1)^{\alpha+2}} - \frac{\alpha^{\alpha+1}}{(\alpha+2)(\alpha+1)^{\alpha+2}} > 0 \) then (3) becomes

\[
q < 1 + \frac{1}{(\alpha+1)(\alpha+2)} \left( \frac{\beta}{\beta+1} \right)^{\alpha+1} - \frac{\alpha^{\alpha+1}}{(\alpha+2)(\alpha+1)^{\alpha+2}}.
\]

As the RHS is always greater than 1, this condition holds. If instead \( \frac{\beta^{\alpha+1}}{(\beta+1)^{\alpha+2}} - \frac{\alpha^{\alpha+1}}{(\alpha+2)(\alpha+1)^{\alpha+2}} < 0 \) then (3) becomes

\[
q > \frac{\beta^{\alpha+1}}{(\beta+1)^{\alpha+2}} - \frac{2 \alpha^{\alpha+1}}{(\alpha+2)(\alpha+1)^{\alpha+2}} + \frac{1}{(\alpha+1)(\alpha+2)} \left( \frac{\beta}{\beta+1} \right)^{\alpha+1}.
\]  

(4)

By assumption the denominator is negative. Letting \( \kappa(\beta) \) denote the numerator, note that \( \kappa(\alpha) = 0 \) and it can be shown that \( \kappa'(\beta) > 0 \) when \( \alpha < 1 \). Hence, \( \kappa(\beta) \geq \kappa(\alpha) = 0 \) which implies the RHS of (4) is negative in which case it is always true. Thus, a sufficient condition for collusion to be profitable is that the low-cost distribution is concave: \( \alpha < 1 \).