

Are Innovation-Based Endogenous Growth Models Useful?

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Abstract

A broad and popular class of innovation-based endogenous growth models implies that the steady-state growth rate of per capita output is increasing in the steady-state inducement to innovate. This paper presents evidence that in the sample periods 1890-1929 and 1947-1998, the US economy can be characterized as having fluctuated around two distinct steady states. It then quantifies the relationship between the changes in the steady-state inducement to innovate and the steady-state growth rate of per capita output. The relationship appears to be weak, suggesting that innovation-based endogenous growth models do not delineate an important dimension along which the trend growth rate is endogenous. Subsidies to innovation also appear to be an unpromising way to raise the US trend growth rate.

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1 Introduction

Recent years have witnessed the outpouring of a large empirical literature on growth.¹ Much of that literature has focused on testing whether countries have endogenous trend growth rates. In many ways, however, this focus has been misguided. First, in practical terms, there may not much difference between an endogenous growth model in which the trend growth rate depends on some variable and an exogenous growth model, the height of whose balanced growth path depends on the same variable. This is especially true if the exogenous growth model has slow transitional dynamics.

Second, all models are false by design because they abstract from the clutter of the real world. A critical consideration for any given model is therefore the range of issues for which it is a useful abstraction. Outside that range, other models should be used. For example, it may be appropriate for some purposes to use a model that takes the efficiency of labor to be exogenous and growing at a constant rate, while for other purposes, using it would be folly. For this reason, classical hypothesis testing, which posits that models are either true or false, can be a poor framework in which to evaluate models.

Third, history offers compelling evidence that for some purposes, the trend growth rate is indeed best modeled as endogenous. World per capita output grew on average by 1.21 percent a year between 1820 and 1992. Such rapid growth over such an extended period had never occurred previously. Indeed, the growth rate had been essentially zero on average for the previous 10,000 years.² Treating such a revolutionary change as a purely exogenous event is unattractive in the extreme.³

¹ A far from exhaustive list is Kormendi and Meguire (1985), Baumol (1986), De Long (1988), Barro (1991), De Long and Summers (1991), Barro and Sala-i-Martin (1992), Mankiw, Romer and Weil (1992), King and Levine (1993), Easterly, Kremer, Pritchett, and Summers (1993), Bernard and Durlauf (1995), Jones (1995a), Islam (1995), Bernard and Jones (1996a, 1996b, 1996c), Kocherlakota and Yi (1995, 1996), Quah (1996), Evans (1996, 1997, 1998), and Evans and Karras (1996a, 1996b). Barro and Sala-i-Martin (1995) and Barro (1997) provide useful surveys of this literature.

² See Maddison (1995, p.20). If the growth rate of per capita output had been even as high as 0.04 percent a year, Maddison's estimate for 1500-1820, per capita output would have had to have been about \$12 a year in 8000 BC in order to have arrived at Maddison's estimate of \$565 in 1500. Such a low per capita output is impossible.

³ Landes (1998) discusses of some of the explanations that have been offered for it.

In addition to these conceptual weaknesses, many papers in the empirical growth literature suffer from important econometric problems. For two particularly devastating econometric critiques of this literature, see Cho (1996) and Im, Pesaran and Shin (1995).

This paper pursues a different tack. It examines a popular class of endogenous growth models in order to identify a key feature of the US economic environment on which the US trend growth rate should strongly depend. It then investigates how strong this dependence can actually be, given the US experience. If the data are consistent with a strong dependence, these models can be reasonably regarded as useful. By contrast, their usefulness is suspect if the data are consistent only with a weak or nonexistent dependence.

The class of models considered here is broad as well as popular, including those of Romer (1990), Barro and Sala-i-Martin (1995), and Aghion and Howitt (1998). In these innovation-based models, the trend growth rate arises endogenously as innovators expend resources in order to invent specialized capital goods that enhance the efficiency of labor. Section 2 employs a variant of the Aghion and Howitt model in order to illustrate a key implication of these models; viz., in the steady state, the growth rate of per capital output should be increasing in the inducement to innovate.

The paper employs a novel empirical method. First, section 3 presents evidence that 1890-1929 and 1947-1998 are periods in which the US economy can be characterized as fluctuating around distinct steady states. Then, in section 4, it quantifies the relationship between the changes in a measure of the steady-state inducement to innovate and the steady-state growth rate of per capita output. This exercise indicates that the trend growth rate is unlikely to be very sensitive to the inducement to innovate. This finding suggests that innovation-based endogenous growth models may not be especially useful. Furthermore, subsidies to innovation appear to be an unpromising way of raising the US trend growth rate. Finally, section 5 summarizes the major findings of the paper.

2 Theory

In this section, I briefly sketch a variant of the Aghion and Howitt model of endogenous growth as expounded in Aghion and Howitt (1998, pp. 85-91, 94-96). In this model, growth occurs because innovators can earn profits by expending resources in order to enhance the efficiency of labor. My aim is to show that the steady-state growth rate in this model is increasing in the inducement to innovate and to devise a measure of it.

2.1 Households

Time is continuous and indexed by t . The population consists of L_t infinitely lived households at time t , and it grows at the constant positive rate l . Each household supplies one unit of labor inelastically, implying that the labor input is L_t at time t . The households face perfect capital markets and maximize identical time-separable objective functions characterized by a constant positive subjective discount rate ρ and a constant intertemporal elasticity of substitution for consumption α . As a result, in a steady state in which per capita net output grows at the rate g^* , the interest rate is $\rho+l+g^*/\alpha$.⁴

2.2 Final Output

A large number of profit-maximizing competitive firms produce final output according to the technology

$$Y_t = F(A_t L_t, K_t), \quad (2.1)$$

where Y_t is net output, L_t is the input of labor, A_t is the efficiency of labor, and K_t is the capital stock. The function $F(\bullet)$ is assumed to be strictly increasing and strictly concave in each argument and linear homogeneous. As a result, equation (2.1) can be rewritten as

$$y_t = f(k_t), \quad f' > 0, \quad f'' < 0, \quad (2.2)$$

where $y_t \equiv Y_t/A_t L_t$, $k_t \equiv K_t/A_t L_t$ and $f(k_t) \equiv F(1, k_t)$. The formulation (2.1) differs from that of Aghion and Howitt in two regards. First, I eschew the complications associated with

modeling cross-sectional differences in the efficiency of labor. Second, I do not model $F(\bullet)$ as Cobb-Douglas.

Every firm producing final output maximizes its profit and is a competitor in both its product and the factor markets. As a result, the net marginal product of capital is equated to its rental rate r_t :

$$f'(k_t) = r_t. \quad (2.3)$$

2.3 Technological Innovation

Innovators expend resources in order to develop technological innovations that enhance the efficiency of labor. I posit that the aggregate production function for innovation takes the form

$$\dot{A}_t = H(N_t / L_t, A_t), \quad (2.4)$$

where N_t is the amount of final output that innovators expend and $H(\bullet)$ is a function that is strictly increasing and strictly concave in each of its arguments and linear homogenous. As a result, equation (2.4) can be rewritten as

$$\dot{A}_t / A_t = h(n_t), \quad h' > 0, \quad h'' < 0, \quad (2.5)$$

where $h(n_t) \equiv H(1, n_t)$.

Several noteworthy assumptions are implicit in the production function (2.4). First, the amount that must be expended in order to enhance labor efficiency any given amount is proportional to the labor input. This assumption is a straightforward way of eliminating the scale effect that bedeviled the first vintages of endogenous growth models. As pointed out by Jones (1995b), these models imply that the steady-state growth rate g rises with the population level, an implication grossly at variance with observation. Besides simplicity, this assumption has the benefit of leaving the steady-state growth rate fully endogenous rather than semiendogenous. A common result in the theoretical literature is that eliminating scale effects makes g depend only on the population growth rate and exogenous parameters of technology and preference; e.g., Jones (1995b) and

⁴ The interest rate changes one-for-one with the population growth rate because households are assumed not to be dynastic even though they are immortal. In other words, current households do not provision new households with any assets.

Segerstrom (1998, 2000).⁵ Second, there are diminishing returns to the amount expended by innovators. This assumption is necessary for the existence of a steady state. These diminishing returns must be external to each individual innovator, however. Furthermore, each innovator must ignore his or her influence on the aggregate efficiency of labor. Each of them therefore faces a fixed marginal product of $L_t^{-1}h'(n_t)$. Third, returns to scale are constant, a restriction that Jones (1995b) has pointed out is not only special but also critical for growth to be endogenous rather than semiendogenous. Fourth, innovation uses factors in the same proportions as they are used in producing final output.

2.4 Innovator Choice

Innovations are embodied in specific capital goods that can be monopolized given the economy's legal environment. For this reason, the innovators responsible for the technologies currently in use can rent K_t units of capital from households at the interest rate i_t , costlessly transform them into K_t units of specialized capital goods that employ these technologies, and then rent the specialized capital goods to firms at the rental rate r_t . The profit from doing so is $K_t(r_t - i_t)$. According to equation (2.3), profit equals $K_t[f'(K_t / A_t L_t) - i_t]$, which is maximized when

$$f'(k_t) + k_t f''(k_t) = i_t. \quad (2.6)$$

Using equation (2.6) to eliminate i_t from the expression for profit yields the (maximized) profit accruing to the innovators responsible for the technologies currently in use. This quantity is

$$A_t L_t \pi_t \quad (2.7)$$

with

$$\pi_t \equiv -k_t^2 f''(k_t). \quad (2.8)$$

⁵ An endogenous way of eliminating the scale effect is to posit that innovators also expend resources on expanding the variety of final outputs available as in Howitt (1999). In such models, the R&D expenditure that enhances labor efficiency is proportional to variety, which is proportional in turn to population in the steady state. Variety in Howitt's model seems rather *ad hoc*, however, since it raises neither productivity nor utility. If variety were given an economic function, the scale effect would likely reemerge for an economically appropriate measure of output. Rather than trying to develop a viable method of eliminating scale effects without making growth semiendogenous, I adopt the admittedly *ad hoc* assumption of the text.

The quantity π_t is the income of innovators per efficiency unit of labor; it is positive and strictly increasing in the capital intensity k_t . For expositional convenience, I refer to it henceforth as the inducement to innovate.

These innovators expect to continue receiving the stream of profits $\{\pi_z, z \geq t\}$ until some future innovator displaces them with an innovation. The Poisson arrival rate for innovations can be calculated as follows. Let σ be the average size of the enhancements to labor efficiency. Then $\frac{1}{\sigma} \dot{A}_t dt$ is the total number of innovations made between times t and $t+dt$. A_t is a measure of the total number of innovators responsible for the technology in use at time t . The probability that any one of them is displaced between times t and $t+dt$ is thus $(\frac{1}{\sigma} \dot{A}_t dt) / A_t$, so the Poisson arrival rate is $\frac{1}{\sigma} (\dot{A}_t / A_t)$. The expected present value of the profit received by past innovators is therefore

$$\begin{aligned} V_t &\equiv \int_t^{\infty} A_z L_z \pi_z \exp \left\{ - \int_t^z [i_x + \frac{1}{\sigma} (\dot{A}_x / A_x)] dx \right\} dz \\ &= A_t L_t \int_t^{\infty} \pi_z \exp \left\{ - \int_t^z [i_x - l + (\frac{1}{\sigma} - 1) (\dot{A}_x / A_x)] dx \right\} dz. \end{aligned} \quad (2.9)$$

where i_t is the interest rate at time t and l is the population growth rate.

According to equation (2.5), expending dN_t in final output at time t results in $L_t^{-1} h'(n_t) dN_t$ innovations that afford the innovator a fraction $L_t^{-1} h'(n_t) dN_t / A_t$ of wealth V_t . Free entry of innovators equates these quantities, giving the arbitrage relationship

$$dN_t = [L_t^{-1} h'(n_t) dN_t / A_t] A_t L_t \int_t^{\infty} \pi_z \exp \left\{ - \int_t^z [i_x - l + (\frac{1}{\sigma} - 1) (\dot{A}_x / A_x)] dx \right\} dz$$

or

$$h'[h^{-1}(\dot{A}_t / A_t)] \int_t^{\infty} \pi_z \exp \left\{ - \int_t^z [i_x - l + (\frac{1}{\sigma} - 1) (\dot{A}_x / A_x)] dx \right\} dz = 1, \quad (2.10)$$

where I have substituted from equation (2.5).

2.5 Steady State

In the steady state, the interest rate i_t is $\rho + l + g^* / \alpha$. Equation (2.6) then implies that there is a unique steady-state capital intensity k^* satisfying the equation

$$f'(k^*) + k^* f''(k^*) = \rho + l + g^* / \alpha. \quad (2.11)$$

Equation (2.2) together with equation (2.11) then implies that per capita output and the efficiency of labor grow at the same rate g^* in the steady state. According to equation (2.11), there is a unique function relating the steady-state capital intensity of the economy to the steady-state growth rate and the population growth rate:

$$k^* = \kappa(g^*, l). \quad (2.12)$$

It is straightforward to show that all of the derivatives of $\kappa(\bullet)$ are negative.⁶ Equation (2.12) and (2.8) then imply that there is a unique steady-state value for the inducement to innovate, which I denote π^* . The rental rate r_t also has a unique steady-state value, given equations (2.12) and (2.3).

In the steady state, equation (2.10) reduces to

$$\left[\frac{\pi^*}{\rho + \left(\frac{1}{\alpha} + \frac{1}{\sigma} - 1\right) g^*} \right] h'[h^{-1}(g^*)] = 1 \quad (2.13)$$

after substituting $\rho + l + g^* / \alpha$ for the i s, g^* for the (\dot{A} / A) s, and π^* for the π s. Equation (2.13) determines the steady-state growth rate g^* implicitly. Suppose that the elasticity of intertemporal substitution and the average size of enhancements to labor efficiency are small enough that $\frac{1}{\alpha} + \frac{1}{\sigma} > 1$. It then follows from these equations that a unique function $\gamma(\bullet)$ exists relating the trend growth rate to π^* , the steady-state inducement to innovate:

$$g^* = \gamma(\pi^*). \quad (2.14)$$

It is straightforward to show that its derivative is positive.⁷

2.6 An Expression for π

Suppose that the production function takes the CES form

$$y_t = f(k_t) = \phi[(1 - \beta)(A_t L_t)^{(\varepsilon-1)/\varepsilon} + \beta K_t^{(\varepsilon-1)/\varepsilon}]^{\varepsilon/(\varepsilon-1)}, \quad \phi, \varepsilon > 0, \quad 0 < \beta < 1. \quad (2.15)$$

⁶ They are $\partial k^* / \partial g = 1/D\alpha$ and $\partial k^* / \partial l = 1/D$, where

$$D \equiv 2f'(k^*) \left\{ 1 + \frac{1}{2} \left[\frac{k^* f''(k^*)}{f'(k^*)} \right] \right\}.$$

The quantity in brackets above is an elasticity that lies strictly between -2 and -1 for well-behaved production functions. As a result, D and hence all of the derivatives are negative.

⁷ It is $1/\pi^* \left\{ -h''/h'^2 + \left(\frac{1}{\alpha} + \frac{1}{\sigma} - 1\right) / \left[\rho + \left(\frac{1}{\alpha} + \frac{1}{\sigma} - 1\right) g^* \right] \right\} > 0$.

The parameter ε is the elasticity of substitution between labor and capital. The parameter ϕ can be normalized to one since the units in which the efficiency of labor is measured are arbitrary. The parameter β controls the share of net output paid to capital.

Equations (2.15) and (2.3) imply that

$$r_t = f'(k_t) = \beta k_t^{-1/\varepsilon} [(1-\beta) + \beta k_t^{(\varepsilon-1)/\varepsilon}]^{1/(\varepsilon-1)} = \beta y_t^{1/\varepsilon} k_t^{-1/\varepsilon}. \quad (2.16)$$

It follows that from equations (2.16) and (2.8) that

$$\begin{aligned} \pi_t &= -k_t^2 f''(k_t) = -k_t^2 \beta \left[-\frac{1}{\varepsilon} y_t^{1/\varepsilon} k_t^{-1-1/\varepsilon} + \frac{1}{\varepsilon} y_t^{-1+1/\varepsilon} k_t^{-1/\varepsilon} (\partial y_t / \partial k_t) \right] \\ &= \frac{1}{\varepsilon} [r_t k_t - (r_t k_t / y_t) r_t k_t] = \frac{1}{\varepsilon} [1 - (r_t k_t / y_t)] (r_t k_t / y_t) y_t = \frac{1}{\varepsilon} (1 - b_t) b_t y_t, \end{aligned} \quad (2.17)$$

where b_t is the share of output paid to capital; i.e., $r_t K_t / Y_t$. Equation (2.15) can be rearranged to obtain

$$(1-\beta) y_t^{-(\varepsilon-1)/\varepsilon} + \beta (k_t / y_t)^{(\varepsilon-1)\varepsilon} = 1$$

or

$$y_t = \{(1-\beta) / [1-\beta (K_t / Y_t)^{(\varepsilon-1)/\varepsilon}]\}^{\varepsilon/(\varepsilon-1)}. \quad (2.18)$$

Substituting equation (2.18) into equation (2.17) then yields the following equation for the inducement to innovate:

$$\pi_t = \frac{1}{\varepsilon} b_t (1 - b_t) \{(1-\beta) / [1-\beta (K_t / Y_t)^{(\varepsilon-1)/\varepsilon}]\}^{\varepsilon/(\varepsilon-1)}. \quad (2.19)$$

Given values for the parameters ε and β , the expression in the right-hand member of equation (2.19) can be calculated from the share of output paid to capital and the capital-output ratio, which are observable data.

In practice, the quantity π_t is probably an upper bound to the inducement to innovate. First, some savings may become general-purpose capital, which cannot be monopolized. Second, the markup on specialized capital goods may be less than the monopoly level as defined by equation (2.6). This situation would arise if some innovations were insufficiently large for new specialized capital goods to dominate the old ones they replace, at the monopoly rental rate. In that case, the innovators would limit-price. I assume that the inducement to innovate is a fixed fraction of π_t , making the latter a perfect proxy for the former.⁸

⁸Some of income accruing to innovators may also stem from monopolizing specific human capital. In that case, π_t could conceivably underestimate inducement to innovate.

3 Empirical Analysis

This section provides evidence that the US economy can be characterized as having operated near two distinct steady states during the periods 1889-1929 and 1947-1998. I start with plots of the data and then move on to a formal econometric analysis of them. The data are described in the data appendix.

Figure 1 plots the growth rate of the real per capita net domestic product from 1890 to 1998. The growth rate does not appear to have trended in either the 1890-1929 or the 1947-1998 period. Furthermore, it appears to have approximately the same average level in the two periods. Figure 2 plots the ratio of the net capita stock to net output over the period 1926-1998, and Figures 3 and 4 plot the net rate of return paid to capital and the share of net output paid to capital over the period 1929-1998. Little, if any, trend is apparent in these three variables during the period 1947-1998. By contrast, between 1929 and 1947, the capital-output ratio plummeted, the rate of return on capital soared, and the share paid to capital fell substantially. These plots suggest that the US economy may have been near steady states during the periods 1890-1929 and 1947-1998 but that these steady states are distinct and indeed rather far apart.

I now turn to a formal statistical analysis of the growth rate, the logarithms of the capital-output ratio and the rate of return paid to capital, and the logistic of capital's share.⁹ I first examine whether these series have nonzero trends, conditional on their being stationary around whatever trends they may have. I then examine whether they are indeed stationary.

I estimated the trend in each of these series by applying ordinary least squares to the equation

$$z_t = \phi_0 + \phi_1 t + u_t, \quad t = 1, 2, \dots, T, \quad (2.20)$$

where t is henceforth a discrete index of time, z_t is the series under consideration, ϕ_0 and ϕ_1 are parameters, and u_t is an error term that is supposed to be stationary. One can then

⁹ I left the growth rate in its levels because it can lie anywhere on $(-\infty, \infty)$. I took logarithms of the capital-output ratio and the user cost of capital because they can lie anywhere on $(0, \infty)$. I took the logistic of the capital's share because it is constrained to lie on $(0, 1)$. As a result, the transformed variables can at least conceptually have trends with constant increments each period and covariance-stationary error terms. They can also be difference stationary.

test the null hypothesis $\phi_1 = 0$ against the alternative hypothesis $\phi_1 \neq 0$ by comparing τ , an appropriately calculated t -ratio for ϕ_1 , with critical values from τ 's distribution under the null hypothesis.

There is a difficulty, however. In order to calculate τ , one must account for the serial correlation in u_t , which is unknown *a priori*. I employ a variant of the method suggested by Andrews and Monahan (1992); see the econometric appendix for the details. This method has the desirable property that the resulting τ is asymptotically distributed as standard normal under the null hypothesis.

Table 1 reports estimates of the trends and their t -ratios for the series under consideration. According to the table, there is no evidence that any of the series trends.¹⁰ Table 2 reports Dickey-Fuller t -ratios for testing the null hypothesis that each of the series under consideration is difference stationary with no trend against the alternative that it is stationary around a constant mean. Considerable evidence exists that these series are mean stationary. For this reason, it appears to be reasonable to take the periods 1890-1929 and 1947-1998 as centered around steady states.

Given that the series are mean stationary, one may legitimately make unconditional inferences about their levels. The growth rate of per capita output averaged 0.01967 per year between 1890 and 1929 and 0.02016 per year between 1947 and 1998. The standard errors for these sample means are .00466 per year and .00303 per year, respectively.¹¹ These sample means do not differ statistically from each other at any conventional significance level.¹² Between 1947 and 1998, the logarithm of the capital-output ratio averaged 0.7603, the logarithm of the rate of return paid to capita averaged -2.3258 , and the logarithm of capital's share averaged -1.3304 . The standard errors of these sample means are 0.0132, 0.0290, and 0.0323, respectively.¹³ By contrast, the logarithm

¹⁰ These inferences, which are based on the standard normal asymptotic distribution, are conservative. Monte Carlo simulations reveal that using the asymptotic distribution results in too many rejections of the null hypothesis for samples of the size used here. See the econometric appendix for more detail.

¹¹ These standard errors were calculated using the Newey-West procedure with a window length of five years to account for any serial correlation and heteroskedasticity in the error term.

¹² The t -ratio for the difference is $(0.02016 - 0.01967) / (0.00303^2 + 0.00466^2)^{1/2} = 0.09$.

¹³ I calculated these standard errors using the formula

$$\sqrt{\frac{\hat{\gamma}_0 + 2 \sum_{j=1}^5 (1 - j/6) \hat{\gamma}_j}{T(1 - \hat{\phi})^2}}$$

of the capital-output ratio averaged 1.3668 between 1926 and 1929, the logarithm of the rate of return paid to capital was -2.6343 in 1929, and the logistic of capital's share was -0.9449 in 1929. These figures differ by about 46, 11, and 12 standard errors from the postwar sample means. Overwhelming evidence therefore exists that the steady state prevailing before the Great Depression differed greatly from the steady state prevailing over the postwar period.

4 Implications for How Endogenous Growth Is

The previous section provides compelling evidence against the common assumption that the aggregate production function takes the Cobb-Douglas form. The Cobb-Douglas functional form imposes a unitary elasticity of substitution between capital and labor, implying that the share of output paid to capital should be constant between distinct steady states. This restriction is grossly inconsistent with the fact that the capital's share declined substantially between 1929 and the postwar period while the rate of return on capital increased sharply.

The simplest production function consistent with the findings of the previous section takes the CES form (2.15). It follows immediately from equation (2.16) that if the economy operated near two distinct steady states during two the different sample periods,

$$\sigma = -\frac{\ln(K/Y)^{**} - \ln(K/Y)^*}{\ln r^{**} - \ln r^*}, \quad (3.1)$$

where $(K/Y)^*$ and r^* are the steady-state values around which the capital-output ratio and the rate of return to capital fluctuated during one of the sample periods and $(K/Y)^{**}$ and r^{**} are the steady-state values during the other sample period. In practice, $\ln(K/Y)^*$, $\ln r^*$, $\ln(K/Y)^{**}$ and $\ln r^{**}$ can be replaced with the sample means of $\ln(K_t/Y_t)$ and $\ln r_t$ during the two sample periods. The result of plugging the sample means reported in the previous section into equation (4.1) is $\varepsilon = 1.966$. Labor and capital are therefore substantially more substitutable than is customarily assumed.

where $\hat{\phi}$ is the ordinary-least-squares estimate obtained by regressing the variable under consideration on its value lagged one year and $\hat{\gamma}_j$ is the j th sample autocorrelation of the residuals from this regression. See the econometric appendix for discussion of this method.

According to equation (2.16),

$$\beta = r_t(K_t/Y_t)^{1/\varepsilon}. \quad (3.2)$$

A straightforward method for estimating β is to average $r_t(K_t/Y_t)^{1/1.966}$ over the postwar period. Doing so yields $\beta = 0.1442$.

I used equation (2.19) to calculate π_t , my measure of the inducement to innovate, for $\varepsilon = 1.966$ and $\beta = 0.1442$. The series π_t averaged 0.09889 between 1947 and 1998, and the standard error of this sample mean is 0.00064. By contrast, π_t was 0.14535 in 1929, which exceeds the postwar sample mean by almost 73 standard errors. The inducement to innovate should therefore have been substantially weaker in the postwar period than it was in the pre-Depression period.

The trend growth rate of the US economy is estimated to have increased slightly in response to this appreciable weakening of the inducement to innovate. On its face, this finding suggests not merely that the trend growth rate is unaffected by intentional innovation but also that additional inducement to innovate actually lowers the steady-state growth rate. Drawing this conclusion might be avoided in at least three ways.

First, the quantity π_t does not take into account the fact that research and development are subsidized and income is taxed. Direct government funding of research and development was no more important on average over the period 1947-1992, however, than it had been in 1929 (21.2 vs. 23.4 percent of the total).¹⁴ Income has been taxed much more heavily in the postwar period than it was in the pre-Depression period.¹⁵ Subsidies for privately funded R&D have no doubt been more pervasive in the postwar period than before the Depression, but they are probably dwarfed by the increased income taxation. On net, the inducement to innovate stemming from the government sector has probably been weaker in the postwar period than it was before the Depression, reinforcing the private-sector effect.

¹⁴ The numbers in the text are the ratio of real government research and development expenditures to total real research and development expenditures. I obtained the underlying data from a diskette provided by the Bureau of Economic Analysis on its R&D Satellite Accounts. These series are lines 15 and 1 of Tables 1.2h and 1.2. In interpreting these data, one should realize that much of the cost of innovative activity in the US economy may not be classified as R&D in the official accounts. Moreover, this downward bias in the official measure may be more pronounced in 1929 than in the postwar period.

¹⁵ The average marginal income tax rate was 0.018 between 1890 and 1929 and 0.180 between 1947 and 1994. These averages are based on the entries in column 3 of Table 1 of Stephenson (1998) and the fact that there was no income tax before 1913.

Second, some innovations may be embodied in specialized human capital rather than physical capital as assumed in this paper. Therefore, if the ratio of human capital to output increased sufficiently between the pre-Depression period and the postwar period, the inducement to innovate may actually have increased rather than fallen. This story has some surface plausibility since educational attainment has increased substantially.¹⁶ On closer inspection, however, the story fails. According to Kendrick (1976), the ratio of human capital to net output actually fell between 1929 and 1947.¹⁷

Third, the above estimates are only point estimates rather than revealed truth. In other words, the trend growth rate in the pre-Depression period may well have exceeded the trend growth rate in the postwar period, notwithstanding the fact that the point estimate of the latter exceeds the point estimate of the former. Indeed, the probability that the trend growth really decreased is approximately 0.465.¹⁸ Furthermore, the inducement to innovate may have declined less than the point estimates suggest.

Equation (2.14) relates g^* , the steady-state growth rate, to π^* , the steady-state inducement to innovate. To a first approximation, its derivative is

$$\gamma' = \frac{g^{**} - g^*}{\pi^{**} - \pi^*}, \quad (3.3)$$

where g^{**} and π^{**} are steady-state values of g_t and π_t between 1947 and 1998 and g^* and π^* are their steady-state values between 1890 and 1929. Taking a Bayesian perspective, I calculated the posterior distribution for δ as follows. First, I started with an uninformative prior distribution in which γ' is uniformly distributed on $(0, \infty)$ conditional on being positive; it has a probability p of being positive and $1-p$ of being zero; and p is uniformly distributed between zero and one. Then, I assumed that $g^{**}-g^*$ and $\pi^{**}-\pi^*$ are normally and independently distributed with means of 0.00049 and -0.04746 and

¹⁶ According to Table 8 of the *Digest of Education Statistics*, the fraction of high-school graduates in the population age 25 and over was 0.135 in 1910, 0.164 in 1920, 0.191 in 1930, 0.245 in 1940, 0.343 in 1950, 0.411 in 1960, 0.552 in 1970, 0.686 in 1980, and 0.776 in 1990. The fraction of college graduates was 0.027 in 1910, 0.033 in 1920, 0.039 in 1930, 0.046 in 1940, 0.062 in 1950, 0.077 in 1960, 0.110 in 1970, 0.170 in 1980, and 0.213 in 1990.

¹⁷ The real net stock of human capital increased by 53.0 percent between 1929 and 1947, while real net domestic product increased by 59.7 percent. The data on human capital come from Kendrick's Table B-28.

¹⁸ This is the fractile of the cumulative standard normal distribution corresponding to the t -ratio -0.09 .

standard deviations of 0.00555 and 0.00468, respectively.¹⁹ Next, I drew one million realizations from those distributions and calculated the realizations of γ implied by equation (4.3). Finally, I set the negative realizations to zero since the prior distribution assigns a probability of zero to negative values of γ . Table 3 reports some of the percentiles of the resulting frequency distribution for γ . Because this frequency distribution is based on a million realizations, it should closely approximate the posterior distribution of γ .

The table indicates that large policy measures are probably necessary in order to raise the US trend growth rate appreciably. As an example, consider a policy measure that induces one percent of the net domestic product to shift from labor to innovators. Such a policy measure is large in the sense that its enactment would no doubt require much political blood to be spilt. The third column reports the posterior distribution of the effects of this policy on the US trend growth rate.

The odds are less than even that the trend growth rate would even increase. There is only about one chance in five that it would increase by at least one-tenth of a percent point, only about one chance in twenty that it would increase by at least two-tenths of a percentage point, and only about one chance in a hundred that it would increase by at least three-tenths of a percent point.

5 Summary

In innovation-based endogenous growth models, innovators are induced to expend resources developing enhancements to the efficiency of labor because they anticipate profiting from doing so. Consequently, the steady-state growth rate of per capita income is an increasing function of the steady-state inducement to innovate.

¹⁹ $0.00049 = 0.02016 - 0.01967$ and $0.00555 = (0.00303^2 + 0.00466^2)^{1/2}$, where 0.02016, 0.01967, 0.00303, and 0.00466 are the sample means and standard errors of g_t in the periods 1947-1998 and 1890-1929. The latter calculation assumes that the two samples are drawn independently of each other, a reasonable presumption for samples 18 years apart. $-0.04746 = 0.09889 - 0.14635$ and $0.00468 = 0.00064 \cdot (1+52)^{1/2}$, where 0.09889 and 0.00064 are the sample mean and standard error of π_t in the 1947-1998 period and 0.14635 is its value in 1929. In this calculation, the standard deviation of the 1929 observation for π_t is being assumed to be the same as the standard deviation of each of the 52 postwar observations, and that observation is being assumed to be independent of the postwar sample.

The US economy is well characterized as having operated near steady states in the periods 1890-1929 and 1947-1998. Specifically, the growth rate of the real per capita net domestic product appears to be mean stationary in the two periods. Furthermore, the ratio of the domestic capital stock to the net domestic product, the net rate of return on capital, and the share of the net domestic product paid to capital appear to be mean stationary in the period 1947-1998.

Overwhelming evidence exists that these two steady states are distinct. Between 1929 and 1947, the capital-output ratio plummeted, the net real rate of return on capital soared, and capital's share in net output fell appreciably. These changes are highly significant. By contrast, the sample mean of the growth rate of per capital output hardly budged.

These observations are inconsistent with a Cobb-Douglas form for the aggregate technology and are consistent with a CES form in which the elasticity of substitution between labor and capital is about two.

Given that technology does take the CES form, a simple expression can be derived for the inducement to innovate. The derivation assumes that all innovations are embodied in specialized capital goods that innovators can monopolize and that all savings become specialized capital goods. This measure fell a great deal between 1929 and the postwar period. By contrast, the trend growth rate of per capita output appears not to have fallen much, if at all. The most likely explanation is that the trend growth rate is not very sensitive to the inducement to innovate. This finding suggests that innovation-based endogenous growth models may not be especially useful and that subsidizing innovation is an unpromising way to raise the US trend growth rate.

Data Appendix

My measure of the capital stock is the real net private stock of capital. The data come from Table 15KRE of "Net Stock Estimates of Fixed Reproducible Wealth, 1925-1997," which I downloaded from www.bea.gov/bea/dn2.htm as posted in July 1999. I shifted the BEA's data forward one year since I measure capital at the beginning of the year while they report it at the end of the year.

My measure of output (Y) is the real net domestic product for 1929-1998 and real net national product for 1889-1928. The former is line 12 from NIPA Table 1.10,²⁰ and the latter is series A6 of *Long Term Economic Growth*. I spliced the latter to the former at 1929.

I estimated the real income paid to capital using the formula:

$$NKI = \sigma NNP - NFI \quad (\text{A.1})$$

with

$$\sigma \equiv 1 - COMP / (NI - PI), \quad (\text{A.2})$$

where NKI is the real net domestic income paid to capital, NNP is the real net national product, NFI is real net foreign income, $COMP$ is nominal employee compensation, NI is nominal national income, and PI is nominal proprietors' income. Several assumptions are implicit in this calculation: the share of proprietors' income paid for the proprietors' labor services is the same as is paid for labor services elsewhere; all net foreign income is paid to capital; indirect taxes are shifted forward equally onto both labor and capital; and the share of real net national product paid to capital is the same as the share of nominal net national product paid to labor. The net rate of return paid to capital and the share of net output paid to capital are then NKI/K and NKI/Y , respectively. NNP is line 10 of NIPA Table 1.10, NFI is the difference between lines 10 (real net national product) and 12 (real net domestic product), and NI , $COMP$, and PI are lines 1, 2 and 9 of NIPA Table 1.14.

I downloaded the population data for 1900-1998 from the Census Bureau's web page www.census.gov/population/estimates/nation/popclockest.txt as posted in August 1999. The data for 1889-1899 are series A7 from *Historical Statistics of the United States*. Per capita net output is real NDP divided by this series.

Econometric Appendix

Following Andrews and Monahan (1992), I estimate the t -ratio for β as

²⁰ All NIPA data come from www.bea.doc.gov/bea/dn1.htm as posted in July 2000.

$$\tau \equiv \hat{\phi}_1 [1 - \min(\hat{\phi}, 1)] \sqrt{\frac{\sum_{t=1}^T t^2 - \left(\sum_{t=1}^T t\right)^2 / T}{\hat{\gamma}_0 + 2 \sum_{j=1}^J [1 - j/(J+1)] \hat{\gamma}_j}}, \quad (\text{A.3})$$

where $\hat{\phi}_1$ be the estimator of ϕ_1 obtained from applying ordinary least squares to equation (3.1), $\hat{\phi}$ is the ordinary least squares estimator of ϕ in the equation

$$\hat{u}_t = \phi \hat{u}_{t-1} + v_t, \quad t = 2, 3, \dots, T, \quad (\text{A.4})$$

$\hat{\gamma}_j$ is the j th sample autocovariance of $\hat{u}_t - \min(\hat{\phi}, 1)\hat{u}_{t-1}$, J is a positive integer, $\{\hat{u}_t\}$ are the residuals from applying ordinary least squares to equation (A.4), and v_t is an error term.

The basic idea underlying this estimator is straightforward. One first estimates ϕ in order to remove most of the serial correlation in u_t . This first step is a *sine qua non* for τ to have decent finite-sample properties when u_t exhibits appreciable serial correlation. The estimator for ϕ is, however, restricted to be no more than one since u_t is maintained to be stationary and *a fortiori* nonexplosive.²¹ One then applies the Newey-West (1987) nonparametric estimator in order to account for any remaining serial correlation.

Andrews and Monahan have shown that under the null hypothesis $\phi_1 = 0$, τ converges in distribution to standard normal as the sample size T and the window length J approach infinity while $J/T^{1/4}$ approaches zero and that τ diverges under the alternative hypothesis $\phi_1 \neq 0$. In practice, however, the resulting estimator performs fairly poorly for samples of the size considered in this paper.

Table 4 characterizes the distribution of τ on the assumption that the error term u_t is a normally distributed first-order autoregression, $J = 5$, and $T = 52$. The table compares various statistics of the standard normal distribution with those of the distributions associated with autoregressive parameters .00, .50, .75, .90, and .95. Clearly, these five distributions are appreciably wider than the asymptotic distribution. Furthermore, the larger the autoregressive parameter ϕ is, the wider they become. One therefore tends to reject the null hypothesis too often if one utilizes the standard normal distribution. For

²¹ I thank Masao Ogaki for pointing out that this restriction should be imposed on the estimator for ϕ .

that reason, one should definitely not reject the null hypothesis if a test employing critical values from the standard normal distribution fails to reject.

I estimated the statistics reported in Table 4 as follows. First, I carried out 20,000 simulations of the data-generating process

$$x_t = \varphi x_{t-1} + \varepsilon_t, \quad (\text{A.5})$$

with

$$x_1 = \text{NIID}\left(0, \frac{1}{1-\varphi^2}\right) \quad (\text{A.6})$$

and

$$\varepsilon_t \sim \text{NIID}(0,1), \quad t = 2, 3, \dots, 52, \quad (\text{A.7})$$

for the given values of φ . Then, with $J = 5$, I used equation (A.3) to calculate a realization of τ for each of these 20,000 samples. Finally, I calculated the mean, standard deviation, and the various fractiles of these 20,000 realizations of τ . On the maintained assumption that u_t is a normally and identically distributed first-order autoregressive process, these statistics are consistent estimates of the fractiles from the true distribution of the estimator τ for $T = 52$, $J = 5$, and the given value of φ .

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**Table 1. Estimates of ϕ_1 and Associated Statistics
from Trend Regressions**

<i>Series</i>	<i>Estimate of ϕ_1</i>	<i>T-Ratio</i>
<i>Growth Rate of Per Capita Output, 1890-1929</i>	-.0000489	-0.13
<i>Growth Rate of Per Capita Output, 1947-1998</i>	.0000203	0.01
<i>Logarithm of the Capital-Output Ratio, 1947-1998</i>	-.000780	-0.93
<i>Logarithm of the User Cost of Capital, 1947-1998</i>	.000554	0.85
<i>Logistic of Capital's Share, 1947-1998</i>	-.000284	-0.12

Note. A window length of 5 is used.

Table 2. Dickey-Fuller T-Ratios

<i>Series</i>	<i>T-Ratio</i>	<i>MSL</i>
<i>Growth Rate of Per Capita Output, 1890-1929</i>	-8.51 (0)	0.00000
<i>Growth Rate of Per Capita Output, 1947-1998</i>	-8.33 (0)	0.00000
<i>Logarithm of the Capital-Output Ratio, 1947-1998</i>	-2.96 (1)	0.04332
<i>Logarithm of the User Cost of Capital, 1947-1998</i>	-3.23 (0)	0.02245
<i>Logistic of Capital's Share, 1947-1998</i>	-3.18 (0)	0.02557

Note. The integer in parentheses is the number of lags of Δz_t that augment the Dickey-Fuller regression. The number of lags was selected in order to minimize the Akaike criterion subject to being no greater than four. The third column reports the marginal significance level of the t -ratio reported in the second column.

Table 3. Percentiles of the Distribution of δ

<i>Statistic</i>	<i>γ</i>	<i>Policy Effect</i>
50 th Percentile	0.000	0.000
55 th Percentile	0.004	0.005
60 th Percentile	0.019	0.022
65 th Percentile	0.034	0.041
70 th Percentile	0.051	0.060
75 th Percentile	0.069	0.081
80 th Percentile	0.088	0.103
85 th Percentile	0.111	0.131
90 th Percentile	0.140	0.165
95 th Percentile	0.184	0.216
99 th Percentile	0.268	0.315

Note. Based on 1,000,000 draws in a Monte-Carlo simulation. The third column is the second column multiplied by 1.1756, the average value of output per efficiency unit of labor between 1947 and 1998.

Table 4. Comparison of Estimated Statistics for τ with Those of the Standard Normal Distribution

<i>Statistics</i>	<i>N(0,1)</i>	<i>τ with</i>				
		<i>$\varphi = .00$</i>	<i>$\varphi = .50$</i>	<i>$\varphi = .75$</i>	<i>$\varphi = .90$</i>	<i>$\varphi = .95$</i>
Mean	0.00	0.00	0.01	0.01	0.02	0.00
Std. Dev.	1.00	1.24	1.28	1.47	1.97	2.46
.01 Fractile	-2.33	-3.06	-3.21	-3.94	-5.15	-6.86
.05 Fractile	-1.65	-2.02	-2.04	-2.35	-3.10	-3.84
.10 Fractile	-1.28	-1.54	-1.51	-1.67	-2.20	-2.68
.25 Fractile	-0.67	-0.78	-0.77	-0.80	-0.98	-1.19
.50 Fractile	0.00	0.01	-0.00	0.02	0.01	-0.00
.75 Fractile	0.67	0.79	0.79	0.82	1.00	1.19
.90 Fractile	1.28	1.54	1.58	1.72	2.21	2.70
.95 Fractile	1.65	2.01	2.10	2.37	3.18	3.90
.99 Fractile	2.33	3.04	3.25	3.82	5.38	6.93

Note. The statistics are calculated for $T = 52$ and $J = 5$.

Figure 1. The Growth Rate of Per Capita Net Output

1890-1998

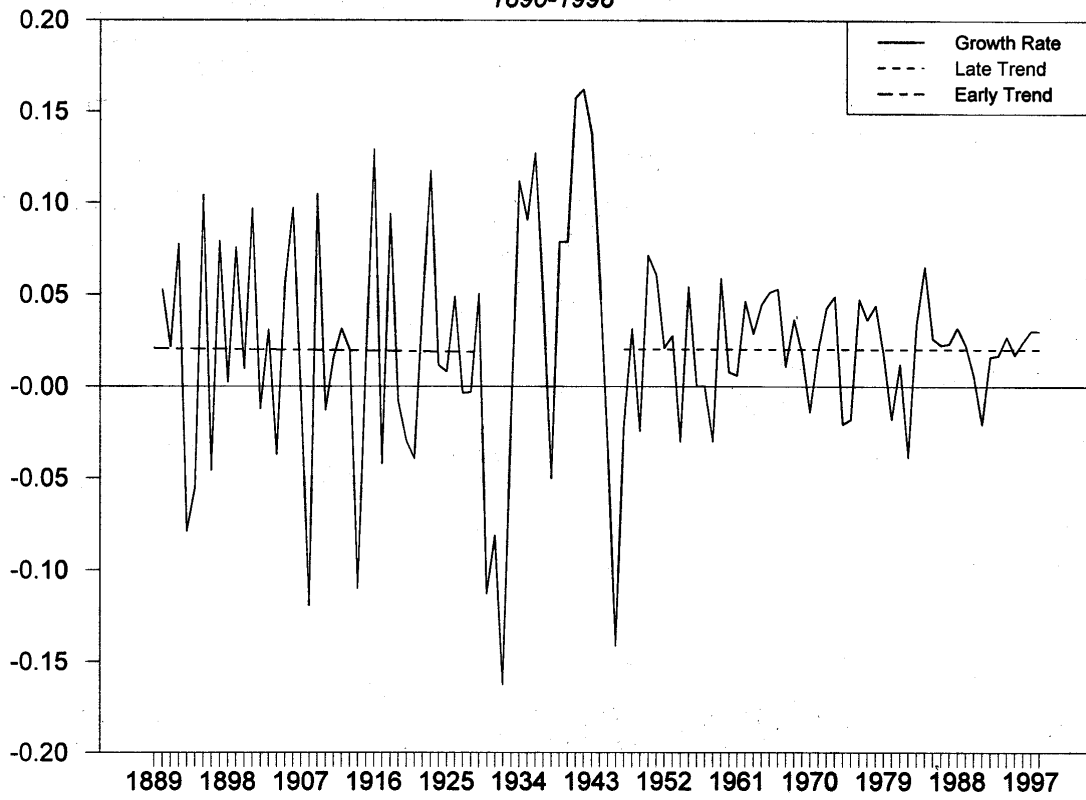


Figure 2. The Ratio of Net Capital to Net Output
1926-1998

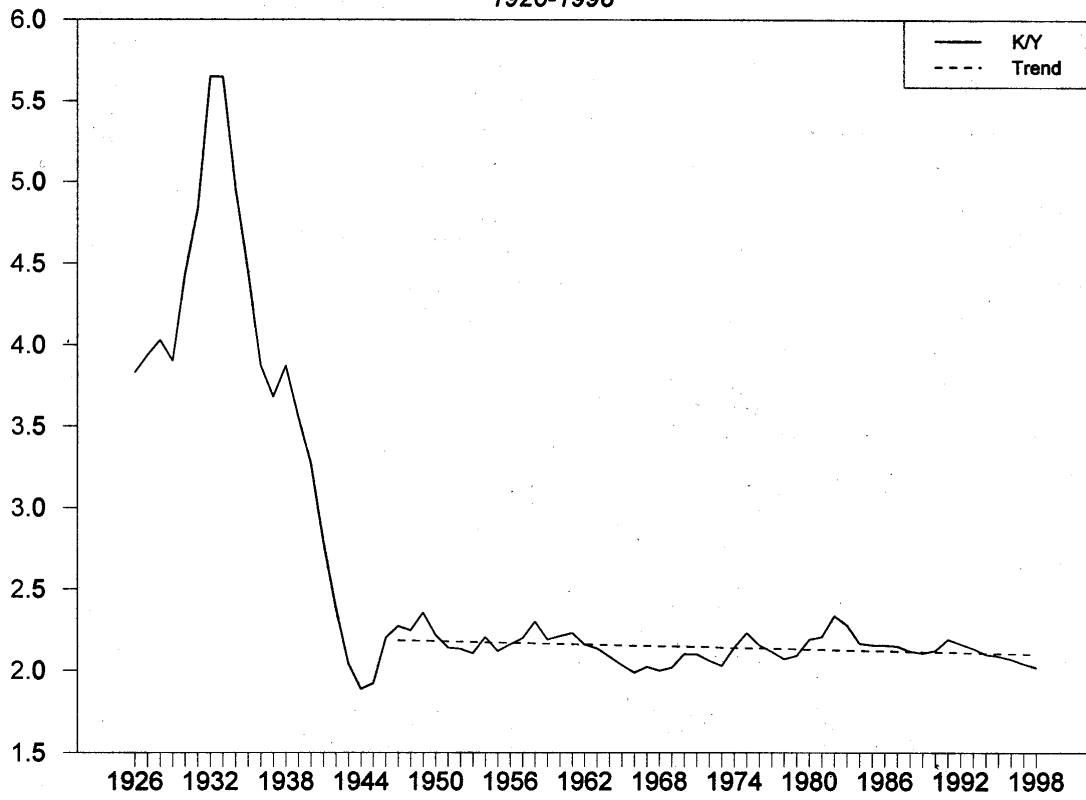


Figure 3. Net Rate of Return on Capital

1929-1998

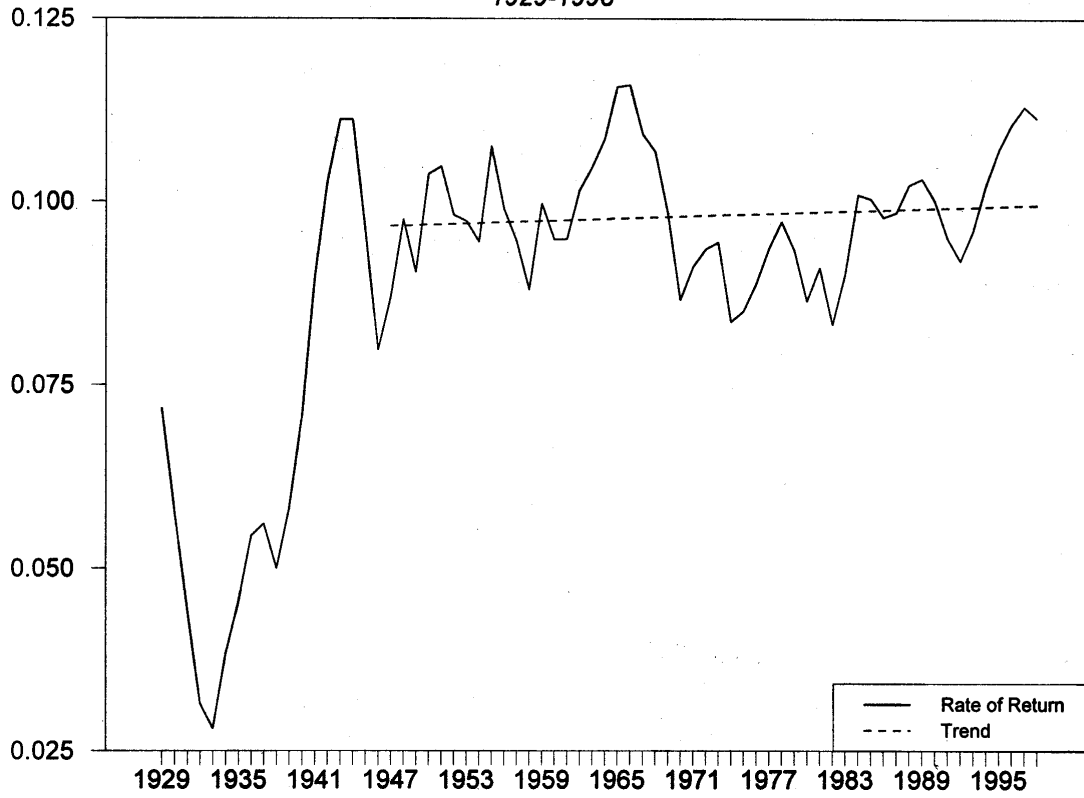


Figure 4. Net Share of Output Paid to Capital

1929-1998

