FOOD, NUTRITION, AND SUBSTITUTION IN THE LATE NINETEENTH CENTURY

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ABSTRACT

Economic historians hypothesize that households in the nineteenth century substituted away from carbohydrates and fiber and towards protein and fat as their incomes rose. Anthropometric historians assert that there was increased nutrient intake without any nutritional substitution. I test these hypotheses using the 1888 Cost of Living Survey. I fail to reject the hypothesis that the income elasticity of fiber is greater than or equal to the income elasticities of protein, fat, or sugar-- contrary to the nutritional substitution posited by economic historians. A food modified Engel curve reveals that the shares of carbohydrates, fat and sugar in the diet vary with household income, but the shares of protein and fiber do not. I do find, however, that the share of protein from animal sources increases with household income. I also find that the diets of late nineteenth century industrial workers were surprisingly balanced by modern standards.

I. Introduction

The quality of the industrial diet in the nineteenth century remains an open question in economic history. While there has been some consensus on the quality of pre-industrial diets, dietary change brought by the industrial revolution remains an object of speculation.[1] We know relatively little about how these early industrial diets differed from pre-industrial diets, which are known to contain primarily cereals, and we also know little about how balanced these diets were in relation to contemporary diets. More importantly, we do not know how responsive diets were to changes in income. While the income effect would lead to larger diets, the substitution effect may or may not lead to diets that are more balanced and nutritious.[2]

The extent of substitution in the diet is important, especially when one considers that food-based substitution is not the same as nutrient-based substitution. Indeed, it would be wrong to infer nutritional substitution from food substitution, as food groups do not fall easily into one nutrition group and vice versa. This distinction is important, especially if one would like to draw conclusions about health, energy capacity, work capacity, and changes in human physiology. Such conclusions, when drawn from nutritional balance sheets, may hide as much as they reveal about the diets of the working classes or, worse, give us false inferences based upon assumptions of the degree of food nutrition similarity. Some economic historians have stated that changes in diet among the working classes were the driving force of economic growth in the past, with dietary changes explaining as much as 50% of economic growth since 1750 (Fogel 1994a, 1994b). It is important, then, to investigate the precise type of dietary change that took place.

One problem, however, is that the relationship between income and nutrition has not been empirically validated for historical populations—there are competing hypotheses in the

[1] This is not to say that there is no work in the area. See, for example, Shergold (1982) and Oddy (1990).
[2] Berhman and Doelalikar (1987, 1989) found that the substitution effect dominates the income effect in rural South Asia.

literature. Economists have paid special attention to the role of changing diet quality, where households moved from cheap carbohydrates to more expensive protein and fat, in explaining gains in stature and other types of physiological capital over time, and this idea has been advanced explicitly and implicitly in the economics literature. Fogel (1994a) asserts that as income increased in the late nineteenth century the diets of workers shifted "from grains and food with high fiber content to sugar and meats. These dietary changes raised the proportion of ingested energy that can be metabolized" (p. 24). Anthropometric historians have asserted that households simply consumed more of the same nutrients, and in the same proportion, as their incomes grew. As Riley (2001) demonstrates, the anthropometric historian concept of nutritional stability asserts that the composition of the diet stayed the same as income increased, although the size of the diet did increase.

In this paper I use the 1888 Cost of Living Survey to obtain precise and robust estimates of the nutritional content of the diets of American and British industrial households in the late nineteenth century. One of the unique features of these data, that is particularly relevant to the study here, is that we can control for many socioeconomic, geographic, and industrial factors, allowing us to identify, as best we can with historical data, the relationship between income and nutrition. As such, this paper provides estimates of nutrient and food group demand among these industrial households. In addition, I test the validity of the two competing hypotheses about the relationship between income and dietary nutrition by looking at the nutrient content of the diet.

I find that nutrient demand was strong for all nutrient groups considered, although there was lower demand for carbohydrates. When looking at the implications of the two different hypotheses about the relationship between income and nutrition, I fail to reject the hypothesis that the income and expenditure elasticities of fiber are greater than or equal to the income and expenditure elasticities of protein, fat, or sugar. This runs counter to the nutritional substitution posited by economists. Using a food modified Engel curve, I further find the shares of carbohydrates, fat and sugar in the diet vary somewhat with total household income, but the shares of protein and fiber do not. In general, the results reveal that overall changes in the shares of the diet are not very large.

This does not decisively prove that economists are incorrect about nutritional substitution in the diet, although the evidence suggests they may have been incorrect about the type of substitution. One of the problems with the two hypotheses stated above is that they concentrate explicitly on the nutritional content of the diet in terms of nutrients. Consumers explicitly purchase food in its various forms (e.g. fish, rice, apples), and only implicitly purchase nutrients. To get a clearer picture of the issue I test the hypothesis that the income and expenditure elasticities of food group calories (e.g. meat calories, cereal calories, dairy calories, etc.) are equal. If Fogel's conception is correct, the elasticities of cereal calories will be significantly different from those of meat calories and dairy calories, provided that food groups are good proxies for nutritional groups. I reject the hypothesis of income and expenditure elasticity equality between cereal calories and meat calories, and also between cereal calories and dairy calories, which I take as strong evidence of marked food group substitution in the diets of these 19^{th} century familes.

In short, economists are correct about food group substitution and the anthropometric historians are correct about the lack of nutritional substitution. We can reconcile these two divergent results by noting that nutrition and food groups may not be highly correlated with one another. The elasticity estimates produced in this paper imply that while meat may be a good proxy for protein, cereals are not a good proxy for carbohydrates, and it does not appear that dairy is a good proxy for fat, or that fruits and vegetables are a good proxy for fiber. The problem may be that hypotheses about the relationship between income and nutrition tend to conflate food groups and nutrient groups. For example, economic historians have taken high demand for meat as high demand for protein without taking into account the fact that protein comes in many forms, and that increased demand for meat may be at the expense of other protein types. While high demand for meat will put more animal protein in the diet, it may be at the expense of non-animal protein, and the total share of protein in the diet may not increase as much as expected.

II. Data and Methodology

A. Data and Diets

The data analyzed in this paper comes from the "Cost of Living of Industrial Workers in the United States and Europe 1888-1890" survey (henceforth 1888CEX) published by the United States Department of Labor. The study was conducted under the direction of Carroll D. Wright, then the U.S. Commissioner of Labor, who at the time was given the task of empirically analyzing American industrial living standards relative to that of European industrial workers. The 1888CEX data set contains a sample of more than 8,500 families working in iron, steel, coal, textile, and glass industries in both Western Europe and the United States. There are 6,809 households in the survey from the United States, which come from more than 20 states. The European sub-sample of households comes from Germany, Switzerland, Belgium, Great Britain, and France, although the vast majority, 1,024, come from Great Britain.[3]

The timing of the survey is particularly important in terms of identifying the relationship between income and diet. It is important to have as many controls as possible so that the

[3] See Williamson (1967), Haines (1979) and Lees (1979) for more on the 1888CEX.

relationship between income and nutrition does not reflect the correlation of income with other variables.[4] In using a cross section at a point in time I am able to control for items such as inequality, technology, and food prices, which will be fixed at any point in time. Additionally, including the geographic location of the household acts as a way to control for food prices in the market, since the state is the smallest area of geographic detail in the 1888CEX. Including the industrial occupation of the household head allows us to use it as a proxy for labor organization, which will also be fixed at a point in time. The geographic location and the occupation of the household head can be used as a quasi-controls for public health and disease environment. This is advantageous since it is usually difficult to control for the disease environment in other studies of nutritional demand since it may change over time or interact with technology and public health, as the theory of technophysio evolution (Fogel and Costa 1997) suggests.

The 1888CEX survey contains detailed income and expenditure information for each household. In particular, annual expenditure on twenty-two food items is recorded.[5] These expenditures can be converted to nutrients using the report "Retail Prices and Wages: A Report by Mr. Aldrich" (1892). In the American sample, I took the annual average price of each food item as given in the Aldrich report for the twelve months from June, 1889 to May, 1890. I then divide the annual expenditure on each food item by the annual quantity price estimate for that food item in the household's state. For the British sample I divide the expenditure by the June, 1889 prices that are given in the Aldrich report. Lastly, I use a standard nutrition conversion table to convert the estimates of food quantity to their grams of fat, fiber, sugar, protein, and

[4] See Steckel (1995) for a schematic that outlines the relationships important to physiological capital.
[5] In particular, bread, coffee, milk, eggs, cheese, sugar, potatoes, poultry, rice, butter, pork, beef, meat, lard, molasses, tea, condiments, flour and meal, vegetables, fruit, fish, and "other foods" are the categories on which food expenditure is enumerated. In a detailed analysis of the enumerators comments, it was found that home production of nutrition was uniformally distributed over the range of income, and as such would not impact the elasticities reported here. Additionally, since the diet is measured at he household level, alcohol and tobacco are excluded from the nutritional values reported here as they are predominately consumed by adults-- the results are robust to their exclusion.

carbohydrates, which are the nutrient groups of interest in this analysis, and to their caloric values as well (Nutribase 2001).[6]

In this paper I concentrate on three major nutrient groups—protein, fat, and carbohydrates. Protein is the nutrient most responsible for human growth and the maintenance of muscle, hormones, and antibodies. Protein is the primary source of nitrogen for the human body, helps in the construction and maintenance of muscle and tissue, and also makes up the compounds that form the body's chemical reactions, including the flow of blood and the management of blood glucose levels. Fat aids in the absorption of key vitamins such as A, D, and E. Fat also acts to deposit itself around organs in the body with the aim of protecting them. Diets lacking in certain types of fat will lead to problems with kidneys and blood disorders. Carbohydrates provide the fuel for the human body. They are the major source of metabolic energy. In particular, carbohydrates serve as a component involved in the transportation of energy throughout the body.

Among the class of carbohydrates, this paper further considers two types of carbohydrates, sugars and fiber, which play very important roles in their own right. Sugars are the simpler carbohydrates (glucose, fructose, etc.). Sugars are comprised of monosaccharides (simple sugars), disaccharides (two simple sugars combined), and polyols (which are sugar based alcohols). Sugars are the main energy sources of carbohydrates. Some parts of the body,

[6] A word on the validity of the nutritional conversion is warranted. While it is true that foodstuffs in the distant past bear little resemblance to their current form, by the late nineteenth century food production was largely a regulated and standardized industry in the United States, and products were broadly similar to their twentieth century counterparts, particularly for dietary staples. As Law (2003) has shown, by the end of the nineteenth century nearly every state in the United States had passed legislation regulating food and dairy products in order to standardize food production and to ensure quality thresholds were met. These laws also ensured that foods were not adulterated and that there was truth in advertising for food products. What this tells us is that in the food products consumed in the late nineteenth century were standardized in such a way that the nutritional conversions can be applied since foods were largely what they were represented to be and adulteration was rare. Furthermore, the standardization of food production gives us confidence that the nutritional conversions here reasonably reflect the true nutritional content.

including the brain, only use glucose as an energy source since they do not use fats as an energy source. Fiber is also a carbohydrate, and is the portion of plant material that cannot be digested. One important function of fiber is that it maintains the intestinal track by increasing the mass in the intestine. Some fibers ferment in the large intestine, and this is how fiber promotes a feeling of fullness and helps with overall digestive health by regulating the intestinal track, and in this way diets lacking in fiber will usually result in intestinal and digestive problems that may make it difficult to absorb nutrients (Nutribase 2001). Given the prevalence of intestinal diseases in historical populations there may have been little fiber in historical diets.[7]

B. Empirical Strategy

I estimate the nutritional elasticities discussed herein with the log linear equation

$\ln\left(\frac{g_i}{n}\right) = \alpha_i + \beta_i \ln\left(\frac{x}{n}\right) + \Theta Z_i + \varepsilon_i$ where β is the elasticity, x is household resources (income or expenditure), and g is the nutrient (measured in grams), and n is the household size. I also include the log of family size, shares of the household in five year age categories, dummy variables for the industry that employs the head of the household, and dummy variables for the state of residence in the American sample, and these variables are represented by the vector Z. Income and expenditure are recorded separately in the 1888CEX, and therefore it is possible to estimate both income and expenditure elasticities of nutrients. To test the elasticity hypotheses, I regress the log of per capita grams of protein, fiber, fat, carbohydrates and sugar (respectively) on the log of per capita income and separately on the log of per capita expenditure. Using robust estimates of the standard errors, I test the hypothesis that the elasticity coefficients from

[7] In section III I find surprisingly low levels of fiber in the diet, at least in comparison to contemporary dietary recommendations.

the regressions are equal to one another for the protein, fiber, and sugar income and expenditure elasticities. The same procedure was used for food group calorie elasticities, where the dependent variable is the log of the per capita food group calories (e.g. meat calories, cereal calories, etc.).

In looking at the shares of the diet devoted to particular nutrient groups I use a food modified Engel curve that has not, to my knowledge, been presented previously in the literature. I modify the traditional Engel curve by regressing the fraction of the diet devoted to particular nutrients on household income, household size and household demographics. The modified Engel curve estimates changes in the nutrient distribution of the diet to household expenditure, household size, and the share of the household devoted to particular age categories. To test the shares of the diet hypotheses I regress the nutritional shares of the total diet (e.g. share of protein), as opposed to the total budget, on the log of per capita income and expenditure, respectively. For each nutrient share of the diet, w, I estimate

$$\frac{g_i}{d} \equiv w_i = \alpha_i + \beta_i \ln\left(\frac{x}{n}\right) + \varphi_i \left[\ln\left(\frac{x}{n}\right)\right]^2 + \sum_{k=1}^{K-1} \vartheta_{ik}\left(\frac{n_k}{n}\right) + \Theta Z_i + \varepsilon_i$$

Where g is either the grams of fat, protein, sugar, fiber, or carbohydrates, d is total nutrient grams available to the household, n is the size of the family, x is total expenditure, k is 5-year age categories (e.g. 5-9, 15-19, etc.), and Z is a vector that contains industry and state dummy variables. Diet shares and the log of household expenditure exhibit a type of joint normality.[8] Note that, in congruence with the Engel function, prices do not enter here as they are assumed constant and would collapse into the intercept. The hypothesis is that the coefficients of the income and expenditure terms are equal to zero.

[8] As Deaton (1997) explains, "the transformation of expenditures to budget shares and of total outlay to its logarithm induces an approximate normality in the joint density of the transformed variables, so that the regression function is approximately normal" (p. 231).

III. Empirical Results

A. Summary Results

Before turning to estimates of nutrient and food demand summary measures of the diet should be analyzed. Table 1 gives the food expenditure shares and nutrient shares by per capita expenditure decile. In terms of food expenditure, dairy and meat shares of the food budget increase with expenditure, while those for cereals do the opposite. This pattern is consistent with other dietary studies. Two distinctions between American and British households are the relatively large share of the food budget devoted to dairy in Great Britain and the relatively large share devoted to oils, fats, and sugars by American households. While these differences certainly reflect taste and dietary preferences to some extent, they may also reflect the agricultural (and therefore, to a certain extent, the nutritional) endowments of Great Britain and the United States in the late nineteenth century. Overall, however, the food expenditure patters of American and British households were quite similar.

The nutrient shares of the diet by per capita expenditure reveal a number of facts. First, the share of carbohydrates in the diet decrease with expenditure while the shares of fat and protein increase. Secondly, and perhaps surprisingly, the British have a greater share of their nutrients coming from protein. Indeed, the bottom ten percent (by income per head) of the British households in the survey have a protein share close to the mean of all American households. Lastly, the share of sugar in the diet is greatest at the mean, suggesting that sugar may be a concave function with respect to expenditure.

What is surprising, however, is that the predicted shares are very similar to what the USDA recommends for 2,000 and 2,500 calorie diets today. In the USDA's dietary recommendation approximately 15% of the nutrients should come from fat, 67% from non-fiber carbohydrates, 6% from fiber, and 12% from protein. As Table 1 shows, the diets of these nineteenth century industrial households are very close to USDA recommendations. On average, these late nineteenth century households had diets that were 69% non-fiber carbohydrates, 15% protein, 2% fiber, and 14% fat. Considering how monotonous historical diets are believed to have been, they conform quite well to what health officials have advised. Indeed, these historical diets are much closer to USDA recommendations than what the Centers for Disease Control found for American diets today (Wright, et. al. 2003). It is interesting to discover that the diets were, in percentage terms, nutritionally balanced, as this suggest that nutritionally rich diets would be achieved by eating "more of the same" as income increased, and in fact the large scale dietary substitution described by economists may be unwarranted.[9]

A. Elasticities of Nutrients

Demand for nutrients, as measured by their income and expenditure elasticities, was strong in both the United States and Great Britain. One feature of Table 2 that should be stressed is that these results help to establish plausible estimates of nutrient income and expenditure elasticities for industrial families in the late nineteenth century. As Table 2 shows, demand for all nutrients is strong, which we should expect from the narrative evidence from the late nineteenth century which tells us that industrial families were malnourished.[10] Another interesting feature of Table 2 is that fat, in particular, appears to be in great demand for both American and British households. Fogel (1994a) has stressed the role of fat in diets in historical populations, but most work continues to give the primary role to protein. High demand for fat

[9] This would, naturally, depend on whether one was willing to accept the USDA's recommendation as signifying an adequate diet.
[10] See Oddy (1990), Streightoff (1911), Byington (1910), and Chapin (1909) for more on the nutritional well-being of industrial families at the time.

may have implications for the energy requirements of industrial work in the late nineteenth century. The high income and expenditure elasticity estimates for fat suggest that fat should be discussed more often in the historical nutrition literature, and its role in physiological changes through the nineteenth century.

As discussed earlier, economic historians have asserted that households substituted away from fiber and towards sugar and protein as their incomes grew. If this is true, then the income and expenditure elasticities of fiber should be less than the income and expenditure elasticities of protein and sugar. I find, however, that I cannot reject the hypothesis that the income and expenditure elasticities of fiber are greater than or equal to the income and expenditure elasticities of protein and sugar. I also find that I cannot reject the hypothesis that the income and expenditure elasticities of fiber are equal to the income and expenditure elasticities of protein and sugar.

Table 2, which presents nutrient elasticity estimates, shows that the estimated income elasticity of fiber is greater than the estimated income elasticities of sugar and fat, and protein for the American households.[11] Indeed, I am unable to reject the hypothesis that the income elasticity of fiber is greater than or equal to the income elasticities of fat, protein, and sugar for American industrial households. Also, I cannot reject the hypothesis that the income elasticity of fiber is equal to the income elasticities of protein and sugar. That fiber has the largest income elasticity of all the nutrients is surprising considering the role that protein plays in the nutrition literature, both for economic historians and development economists.

For the British households, the income elasticity of fiber is greater than that of sugar and only slightly lower than the income elasticity of protein, a difference which is not statistically

[11] For convenience and to make the rejection of the null hypothesis least likely, the significance level used for all test is .001.

significant. For the British data, I am unable to reject the hypothesis that the income elasticity of fiber is greater than the income elasticity of protein and sugar. Furthermore, I fail to reject the hypothesis that the income elasticity of fiber is equal to the income elasticities of protein and sugar. The income elasticity results of Table 2 do not support the contention that households in the late nineteenth century substituted away from fiber and towards protein and sugar as income increased. The income elasticities show similar levels of demand responsiveness between protein, fiber and sugar. At best, these results support the argument that households had approximately uniform demand for all nutrients except non-sugar, non-fiber carbohydrates.

The nutrient expenditure elasticities reported in Table 2 mirror those of the income elasticities. As with the income elasticities, I fail to reject the hypothesis that the expenditure elasticity of fiber is greater than or equal to the expenditure elasticities of protein and sugar. I also fail to reject the hypothesis that the expenditure elasticity of fiber is equal to the expenditure elasticities of protein and sugar. The theory of nutritional substitution laid out by economic historians is not empirically supported by the elasticities.

Carbohydrates, however, appear to occupy a unique position here. It has long been established that demand for carbohydrates is less than demand for other nutrients since carbohydrates are so cheap and plentiful. Table 2 does show that carbohydrates have the lowest income elasticities of all the nutrients considered here. This result does conform to what has been found in other dietary studies in the developing world today. These results suggest that the demand for non-fiber, non-sugar carbohydrates must be particularly low, since carbohydrates as defined in these regressions include both fiber and sugar, and their separate elasticities are much higher than the income elasticity of total carbohydrates.

B. Diet Shares: The Modified Engel Curve

We should investigate how the nutritional shares of the diet vary with income. While the results of Table 1 give us some clues, we would like to see which nutrients vary, systematically, with expenditure. If the hypotheses advanced by economic historians are correct then the share of the diet devoted to fiber will decrease with income while the shares of protein and sugar in the diet will increase with income. By fitting food modified Engel curves I am able to see how responsive the shares of the diet are to changes in household income and expenditure. The food modified Engel curves also allow me to control for the fact that food becomes a smaller portion of the total budget as income and expenditure increase (Engel's Law) and that families with higher incomes tend to consume more food than lower income families.

I find that the shares of carbohydrates, fat and sugar in the diet vary with household income, but the shares of protein and fiber do not. Once again, the economic nutrition stature hypothesis fails to hold for the nutrient shares of the diet. Table 3 presents results of the modified nutrient-income Engel curve. Fat, carbohydrates, and sugar appear to behave in the expected manner. The shares of sugar and fat in the diet increase with income, and their income coefficients are statistically significant and economically large. The share of carbohydrates decreases with income. Also, the sign on the squared income coefficient is the opposite of that of the income term for carbohydrates, fat, and sugar, and for all three regressions they are significantly different from zero. The shares of protein and fiber, however, are not responsive to changes in income and neither their income nor the squared income coefficients are statistically different from zero.

What is most striking here is that the failure to reject the null hypothesis that the income coefficient is equal to zero is not due to less precise estimates of the standard errors in the fiber and protein modified Engel regressions. Protein and fiber have the most precise standard errors, and this further establishes the lack of protein or fiber variation with household income. The income coefficient estimates themselves are very close to zero, much closer than any of the other income coefficient estimates, and that is why I fail to reject the hypothesis of no income effect for the shares of fiber and protein in the diet. The shares of the diet devoted to protein and fiber are simply not responsive to changes in income.

Results for the British households are similar to those of the American sample, with sugar and fat increasing with income and carbohydrates decreasing with income. Once again, the carbohydrate, fat, and sugar regressions have statistically significant income and squared income coefficients. Also, the coefficients of income and squared income in the fiber and protein regressions are not statistically different from zero. The failure to reject the zero slope hypothesis is not a statistical artifact—both protein and fiber have some of the smallest standard errors, it is the coefficients themselves which are close to zero. The failure of the diet shares to move strongly with respect to income in either direction for fiber and protein is surprising as this implies that the share of protein in the diet did not vary with income.

In Table 3 the expenditure coefficient of the protein share regression is negative and the expenditure coefficient of the fiber regression is positive for the American sample, although neither coefficient estimate is statistically different from zero. Also, neither protein nor fiber's squared expenditure coefficients are statistically different from zero. Fat, sugar, and carbohydrates do behave in the same way as with the income relationship, with statistically significant expenditure and squared expenditure coefficients. The British expenditure-Engel estimates mirror those of the American expenditure estimates, with neither the expenditure nor squared expenditure-Engel coefficients in the protein and fiber regressions statistically different from zero. As with the income nutrient Engel curves, Table 3 shows that the fat, carbohydrate, and sugar regressions do have expenditure and squared expenditure coefficients that are statistically different from zero.[12]

C. Solving the Protein Puzzle

The most surprising finding thus far is the relative lack of dietary movement with respect to protein. Given the strong role that protein plays in the anthropometric literature one would expect large protein increases in the diet as income increased. The problem, however, is that the analysis so far has not distinguished between the type of protein in the diet. This becomes important because not all proteins are equal.

Complete proteins are proteins that allow for normal growth and development and reproduction. Most food protein contains a combination of proteins that would make them complete in combination, but this is not universally true. Furthermore, this does not mean that all combinations of protein rich foods will solve the problem of protein deficiency. For example, combinations of grains are more often than not inferior to milk as a source of protein because one would have to consume such a large amount of grains to get the same protein content present in milk. Also, red meats and poultry supply iron, vitamin B, and zinc, and these are usually found in very small amounts in plant based proteins. More importantly, animal protein also contains membrane fusion proteins (MFP) factors that aid in the absorption of iron and zinc from plant sources. By absorbing iron humans avoid anemia and the fatigue associated with it, and zinc helps the body to heal and aids the immune system in staving off infection.

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[12] Even though these coefficients are statistically different from zero, the estimates are small enough such that, over the range of per capita income and per capita expenditure, the changes in the diet shares are very small.

Given this special role in human physiology, it would seem that meat-based protein should play a significant role in physiological efficiency. Diets rich in animal protein not only provide protein, but they help the body absorb key micronutrients that allow humans to avoid fatigue and strengthen the immune system. Since this type of physiological feedback exists, then the share of protein coming from animal sources should increase with income.[13]

For American households moving from bottom to top decile in income and expenditure results in a 3% increase in the percentage of protein in the diet that is animal based. In Great Britain the share of protein that is animal-based increases by 6% as one moves from the bottom to top income deciles. These percent changes are larger than the changes of meat as a percentage of the diet calorically. Table 4 shows the results of a protein modified Engel curve, in which the dependent variable is the share of protein that is comes from meat sources. In the regression of Table 4 both the income and expenditure terms are large and significant. This tells us that although protein as a share of the diet does not appear to change with respect to income, the composition of that protein does vary with income. More importantly, this change in the composition of protein has important implications for human physiology. As humans consume more meat-based protein they are better able to absorb key vitamins and minerals that aid in their ability to work and to become more productive, and excess energy can then be used for improving the physiological structure (through taller stature, increased lung capacity, etc.). In this sense, physiological efficiency can be achieved by consuming more meat-based protein as that will increase both the amount of work (through iron, which will prevent anemia) and the ability to do more work (by staving off infection through greater zinc absorption). This implies

[13] I use a strict definition of animal protein here. Only protein coming from meat (e.g. beef, pork, poultry) and fish are included as animal based proteins. If I expand proteins to include animal by-products such as milk the qualitative implications remain the same.

that greater output can be achieved for the same input of animal protein than for the same amount of plant protein.

D. Caloric Food Group Elasticities

The preceding sections have shown that there was little nutritional substitution in these working class diets. But if the idea of nutritional substitution is extended to food groups then the income and expenditure elasticities of meat calories (which are high in protein) and dairy calories (which are high in fat) will be greater than those of cereal calories (which are high in fiber and carbohydrates in general). Table 5 shows the estimates of the income elasticity of food group calories. I reject the hypothesis that the income elasticity of meat calories is less than or equal to the income elasticity of cereal calories. I also reject the hypothesis that the income elasticity of dairy calories is less than or equal to that of cereal calories. That I reject the hypothesis of income elasticity equality between fruit and vegetable calories and cereal calories, while noting the nutrient elasticities reported above, may suggest that dietary fiber moves from being cereal based to being fruit and vegetable based as incomes increase. Much of this increase could be due to potatoes, which are grouped as vegetables here. The rejection of these hypotheses suggests that dietary substitution did take place among food groups.

The question that remains is whether we can safely take these food groups as substitutes for the nutrient elasticities. If so, then the results presented here pose a serious problem in that they have both confirmed and rejected the hypothesis of dietary substitution. By comparing the food group calorie elasticities to their nutrient proxies we can determine whether or not the assumption holds. All foods are made up of a combination of nutrients. Demand for a food, then, should be a weighted function of its contents. This can be represented by

$\xi_j = \sum_i \alpha_{ij} \beta_i$ where i is an index of nutrients, β is the nutrient demand elasticity, and α is the weight that food j has for nutrient i. If a food elasticity is a good proxy for a nutrient elasticity then α is close to 1 for some i and close to zero for all other i.

I find that, more often than not, foods are very bad proxies for nutrients. The income elasticity of dairy calories, which are usually high in fat, is greater than the income elasticity of fat and I reject the hypothesis that the two are equal to one another. Similarly, the income elasticity of fruit and vegetable calories, which will be high in fiber, is greater than the income elasticity of fiber, and I reject the hypothesis of equal income elasticities for the two. Similarly, carbohydrates and cereals are not good proxies for one another either. Indeed, Table 5 shows the income elasticity of cereals to be lower than the income elasticity of carbohydrates, and I reject the hypothesis that the two are equal to one another. Meat, however, appears to be a good proxy for protein. The income elasticity of meat calories is very close to the income elasticity of protein, and I am unable to reject the hypothesis that the two are equal to one another. The same is true for oil, fat, and sugar calories, which have an income elasticity close to that of sugar, and I am unable to reject the hypothesis of income elasticity equality between the two. The trends in the relation between food group and nutrient elasticities, however, do not appear to hold in general for the British households, but this is most likely due to smaller sample size and larger standard errors in the British sample.

Table 5's British food income elasticities largely agree with the American results. The income elasticities of meat and dairy calories are greater than that of cereal calories and these results also confirm the economic nutrition stature hypothesis for food groups. Table 5, which shows the expenditure elasticity results for the American and British samples, mirror those of the income elasticities, with the expenditure elasticities of meat calories and those of dairy calories being greater than those of cereal calories. Also, Table 5 shows that the expenditure elasticities of fruit and vegetable calories are greater than the expenditure elasticity of cereal calories. Indeed, Table 5 confirms that the expenditure elasticities of cereal calories is significantly lower than all of the other food group elasticities. Broadly, food group elasticities appear to behave in the way that the economic statement of the nutrition stature hypothesis has predicted.

IV. Conclusion

We found that the dietary substitution advocated by economists did not apply to nutrients, but did hold for food groups, and that the anthropometric nutrition hypothesis of diet stability did not apply to food groups, but did hold for nutrients. In particular, the demand elasticities for fiber were not statistically different from the demand elasticities for protein and sugar. Furthermore, the share of the diet devoted to each nutrient did not appear to vary much with household income. An important caveat, however, is that protein became more animal based as income increased, and this may play a large role in physiological changes over time. The story, then, may be more about the source of nutrients than the nutrients themselves.

Households in the nineteenth century did substitute among different food groups, preferring dairy and meat to oils and cereals as their incomes increased. This type of food substitution, however, did not result in diets that were markedly different nutritionally, but these diets would have different physiological effects as diets rich in animal products improve human development and growth by allowing people to better absorb nutrients. What this implies is that some nutrients, in particular fat and protein, went from being plant-based to animal-based as incomes increased. Similarly, there was relatively little fiber in these historical diets, but the demand elasticity of fiber was large. This hints, perhaps, at the fact that fiber demand may be related to better nutrition, the type seen with higher incomes. It also appears that non-sugar, non-fiber carbohydrates lost out in the substitution that took place, and that all of the other nutrient groups remained in fairly constant proportions in the diet.

A possible reason for the failure to find nutritional substitution could be due to the fact that nutrition groups are not closely associated with food groups. Indeed, we saw that most food groups were poor substitutes for their supposed nutrient proxy. As Riley (1994) has shown, even goods that would be classified as fiber or cereals can have a fairly large proportion of protein. For example, wheat flour is 17% protein, and wheat flour would certainly not be considered meat when classified as a food group. In short, these results point to the distinction between food groups and nutrient groups, and the problems inherent in posing hypotheses about one with information on the other. Overall, however, the diets are balanced enough that marked substitution in the diet would not be warranted. That this work finds the diets of stunted people to be nutritionally balanced is surprising, and further microeconomic work on historical diets should be performed to see if the results presented here hold for other historical populations.

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Table 1

Food Expenditure and Nutrient Shares by Type and Per Capita Expenditure (PCE) Decile, 1888CEX

United States Sample

	Food Expenditure Shares (%)				Nutrient Shares (%)		
	Bottom 10% PCE	Mean	Top 10% PCE		Bottom 10% PCE	Mean	Top 10% PCE
Dairy	13.7%	19.4%	20.6%	Fat	13.2%	15.2%	16.3%
Fruits and Vegetables	8.0%	8.8%	7.4%	Protein	13.6%	15.2%	17.2%
Oils, Fats, and Sugars	12.1%	10.7%	9.0%	Carbohydrates	73.1%	69.6%	66.5%
Meats	28.4%	29.7%	30.5%	Fiber*	1.5%	1.7%	1.8%
Cereals	21.3%	13.0%	8.0%	Sugar*	29.8%	32.8%	32.3%
Other Foods	16.5%	18.4%	24.5%	Carbs (non-sugar, non-fiber)*	41.8%	35.1%	32.4%

Great Britain Sample

	Food Expenditure Shares (%)				Nutrient Shares (%)		
	Bottom 10% PCE	Mean	Top 10% PCE		Bottom 10% PCE	Mean	Top 10% PCE
Dairy	19.8%	23.1%	24.2%	Fat	10.2%	12.2%	13.3%
Fruits and Vegetables	6.7%	6.3%	5.8%	Protein	15.0%	16.6%	18.5%
Oils, Fats, and Sugars	6.0%	5.0%	3.7%	Carbohydrates	74.8%	71.2%	68.2%
Meats	29.8%	31.7%	30.8%	Fiber*	1.5%	1.7%	2.2%
Cereals	19.3%	12.0%	7.8%	Sugar*	30.8%	34.4%	32.9%
Other Foods	18.4%	21.9%	27.7%	Carbs (non-sugar, non-fiber)*	42.5%	35.1%	33.1%

*Note: Suagr and Fiber are also carbohydrates

Table 2

Income and Expenditure Elasticity of Nutrients Estimates -- Constant Elasticity Functional Form, 1888 Cost Of Living Survey

	American Sample					British Sample				
	Protein	Carb	Fat	Fiber	Sugar	Protein	Carb	Fat	Fiber	Sugar
Intercept	6.209	8.669	6.285	3.619	7.183	3.951	7.079	3.003	1.542	5.899
	(.098)	(.106)	(.185)	(.211)	(.177)	(.291)	(.325)	(.404)	(.902)	(.483)
lnPCI	0.399	0.274	0.395	0.402	0.342	0.650	0.448	0.709	0.645	0.509
	(.008)	(.009)	(.016)	(.018)	(.015)	(.028)	(.032)	(.039)	(.088)	(.047)
R-Square	0.644	0.457	0.452	0.388	0.364	0.607	0.452	0.475	0.185	0.311
Intercept	3.652	6.811	3.346	0.923	4.795	2.329	6.033	0.983	-0.098	4.264
	(.106)	(.123)	(.217)	(.251)	(.210)	(.328)	(.376)	(.457)	(1.051)	(.556)
lnPCE	0.650	0.455	0.682	0.666	0.575	0.815	0.554	0.914	0.812	0.675
	(.009)	(.011)	(.019)	(.023)	(.019)	(.032)	(.037)	(.045)	(.104)	(.055)
R-Square	0.718	0.508	0.493	0.418	0.399	0.633	0.463	0.508	0.191	0.332
N	6809	6809	6809	6697	6698	1024	1024	1024	1014	1014

Coefficient estimates on lnPCI are income elasticity estimates of the respective nutrient group.
Coefficient estimates on lnPCE are expenditure elasticity estimates of the respective nutrient group.
Heteroskedasticty-consistent standard errors are listed under coefficient estimates in parentheses.
All Regressions include the log of household size and the share of the household in 5-year age categories.
Regressions also include dummy variables for geography and industry (there are no geographic variables for the British Sample).

Table 3

Nutrient Income and Expenditure Modified Engel Curve Estimates-- 1888 Cost of Living Survey

	American Sample					British Sample				
	ProtShare	CarbShare	FatShare	FiberShare	SuagrShare	ProtShare	CarbShare	FatShare	FiberShare	SugarShare
Intercept	0.016	1.628	-0.644	0.049	-0.615	-0.793	2.952	-1.159	0.091	-2.950
	(.082)	(.157)	(.123)	(.031)	(.253)	(.323)	(.541)	(.324)	(.190)	(.879)
lnPCI	0.021	-0.184	0.164	-0.009	0.191	0.190	-0.448	0.258	-0.018	0.684
	(.017)	(.033)	(.026)	(.007)	(.053)	(.069)	(.116)	(.069)	(.041)	(.188)
(lnPCI)^2	-0.0004	0.008	-0.008	0.001	-0.010	-0.009	0.022	-0.013	0.001	-0.036
	(.001)	(.002)	(.001)	(.0003)	(.003)	(.004)	(.006)	(.004)	(.002)	(.010)
R-Square	0.201	0.315	0.335	0.223	0.212	0.410	0.379	0.222	0.084	0.152
Intercept	0.108	1.907	-1.016	0.090	-1.121	-0.885	3.387	-1.503	0.152	-4.565
	(.106)	(.205)	(.160)	(.041)	(.331)	(.389)	(.649)	(.389)	(.230)	(1.058)
lnPCE	-0.007	-0.225	0.223	-0.018	0.306	0.205	-0.533	0.328	-0.033	1.031
	(.022)	(.043)	(.034)	(.009)	(.069)	(.084)	(.141)	(.085)	(.050)	(.229)
(lnPCE)^2	0.001	0.010	-0.011	0.001	-0.017	-0.010	0.026	-0.016	0.002	-0.054
	(.001)	(.002)	(.002)	(.0005)	(.004)	(.005)	(.008)	(.005)	(.003)	(.012)
R-Square	0.210	0.327	0.342	0.224	0.213	0.416	0.390	0.234	0.086	0.162
N	6809	6809	6809	6809	6809	1024	1024	1024	1024	1024

Heteroskedasticty-consistent standard errors are listed under coefficient estimates in parentheses.
All Regressions include the log of household size and the share of the household in 5-year age categories.
Regressions also include dummy variables for geography and industry (there are no geographic variables for the British Sample).

Table 4

Animal Based Protein Modified Engel Curve, 1888 Cost of Living Survey

	US	GB	US	GB
Intercept	-0.959	-5.713	-1.863	-8.025
	(.322)	(1.284)	(.422)	(1.544)
lnPCI	0.318	1.306		
	(.067)	(.275)		
(lnPCI)^2	-0.016	-0.068		
	(.004)	(.015)		
lnPCE			0.511	1.800
			(.089)	(.335)
(lnPCE)^2			-0.026	-0.095
			(.005)	(.018)
N	6809	1024	6809	1024
R-Square	0.180	0.146	0.181	0.157

Each column is a separate OLS regression where the animal (meat based) share of protein was the dependent variable.
All Regressions include the log of household size and the share of the household in 5-year age categories, and dummy variables for industry and geography (there are no geographic variables for the British Sample).
Heteroskedasticty-consistent standard errors are listed under coefficient estimates.

Table 5

Income and Expenditure Elasticity of Food Group Calorie Estimates -- 1888 Cost of Living Survey

	American Sample					British Sample				
	Dairy	FV	OFS	Meat	Cereals	Dairy	FV	OFS	Meat	Cereals
Intercept	5.609	3.629	8.939	7.414	9.989	4.761	3.936	8.261	4.529	9.546
	(.236)	(.264)	(.158)	(.157)	(.141)	(.356)	(.572)	(.381)	(.354)	(.479)
lnPCI	0.599	0.592	0.327	0.443	0.204	0.710	0.504	0.396	0.724	0.278
	(.021)	(.023)	(.014)	(.014)	(.012)	(.035)	(.056)	(.037)	(.035)	(.047)
R-Square	0.569	0.452	0.364	0.538	0.289	0.548	0.266	0.426	0.602	0.251
Intercept	2.432	1.325	6.578	4.203	8.256	2.869	2.945	7.049	2.488	8.912
	(.279)	(.321)	(.186)	(.179)	(.169)	(.404)	(.669)	(.439)	(.397)	(.561)
lnPCE	0.913	0.822	0.558	0.756	0.373	0.903	0.605	0.519	0.932	0.342
	(.025)	(.029)	(.017)	(.016)	(.015)	(.039)	(.066)	(.043)	(.039)	(.055)
R-Square	0.594	0.462	0.407	0.596	0.320	0.578	0.268	0.441	0.635	0.253
N	6506	6564	6660	6700	6691	1011	1006	1006	1016	1014

Coefficient estimates on lnPCI are income elasticity estimates of the respective food group.
Coefficient estimates on lnPCE are expenditure elasticity estimates of the respective nutrient group.
OFS is Oils, Fats, and Sugars; FV is Fruits and Vegetables
Heteroskedasticty-consistent standard errors are listed under coefficient estimates in parentheses.
All Regressions include the log of household size and the share of the household in 5-year age categories.
Regressions also include dummy variables for geography and industry (there are no geographic variables for the British Sample).

Appendix

The macronutrient elasticities and the modified Engel Curve are intimately related. To see this, consider the macronutrient elasticity β (without loss of generality, the fat elasticity) which is estimated in the regression

$$(1) \qquad \ln\left(\frac{FatGrams_i}{n_i}\right) = \alpha + \beta \ln\left(\frac{x_i}{n_i}\right) + \varepsilon_i$$

Where n is the household size and x is total income. It is easy to show that

$$(2) \qquad FatGrams_i = e^{\alpha} e^{\beta \ln(x_i)} e^{(1-\beta)\ln(n_i)} e^{\varepsilon_i}$$

Note that total grams in the diet has the same relationship as fat grams, with an elasticity of Ψ, such that

$$(3) \qquad TotalGrams_i = e^{\phi} e^{\Psi \ln(x_i)} e^{(1-\Psi)\ln(n_i)} e^{\upsilon_i}$$

Keeping in mind that the modified Engel curve, in the simplest case, takes the form

$$(4) \qquad \left(\frac{FatGrams_i}{TotalGrams_i}\right) = \Delta + \theta \ln\left(\frac{x_i}{n_i}\right) + \nu_i$$

where Δ is a constant and ν is the error term. If we substitute equations 2 and 3 into the modified Engel curve (4) we find

$$(5) \qquad e^{\alpha - \phi} x_i^{\beta - \Psi} n_i^{\Psi - \beta} e^{\varepsilon_i - \upsilon_i} = \Delta + \theta \ln\left(\frac{x_i}{n_i}\right) + \nu_i$$

The relationship between the modified Engel curve income coefficient, θ, and the dietary demand elasticities, β and Ψ, follows from equation 5. Note that Ψ is the same for each of the macronutrient modified Engel curves, such that the weighted average of β is equal to Ψ. The demand elasticity for the total diet must be a weighted average of the demand elasticities of the components of that diet. Taking the expectation of equation 5, and then differentiating with respect to income (x) gives the following relationship.

(6) $$e^{\alpha-\phi}(\beta-\Psi)x_i^{\beta-\Psi}n_i^{\Psi-\beta+1}=\theta$$

Keeping in mind that, empirically, x>1, n ≥ 1, 0<β<1, and 0<Ψ<1, the exponent on n will always be greater than zero and term $n_i^{\Psi-\beta+1}$ will therefore always be greater than 1. Also, since x>1 the sign of the left hand side of equation 6 is determined by β-Ψ. There are, then, three cases to consider. When β>Ψ (case 1) the growth of the nutrient in the diet is faster than the growth of the diet in general, and therefore its share of the diet grows as well, meaning that θ>0. When β=Ψ (case 2) the growth of the nutrient is equal to the growth of the diet, meaning that it does not increase or decrease in its share of the diet, so θ=0. When β<Ψ (case 3) the growth of the nutrient in the diet is slower than the growth of the diet in general, and therefore its share decreases, meaning that θ<0. What this means is that the sign of θ tells us how the nutrient's demand elasticity is related to the overall demand elasticity of the diet.

This interpretation of θ holds only in the linear case of the modified Engel curve, however. If, as in the paper, we include a squared log per capita income term in equation 4 the derivative in (6) would become

(7) $$e^{\alpha-\phi}(\beta-\Psi)x_i^{\beta-\Psi}n_i^{\Psi-\beta+1}=\theta+2\delta\ln\left(\frac{x_i}{n_i}\right)$$

Where δ is the coefficient of the squared income term in the modified Engel curve. In this case we can specify the relationship between θ and δ in the three cases. When β>Ψ (case 1), θ>2δln(x/n). When β=Ψ (case 2), θ=2δln(x/n). When β<Ψ (case 3), θ<2δln(x/n). These inequalities, however, do not constrain sign of the coefficients in the way the linear case had. The sign of the left hand side of (7) still tells us about the size of the nutrient elasticity in relation to the overall diet elasticity, however, and it is in this way that the relationship is conceptually the same as in the linear case.