1. Suppose that the demand for good $x$ is given by the equation

$$x = 100 - 2p_x.$$ 

(a) Derive an equation for the inverse demand function, $p_x(x)$.
(b) Derive formulas for the total revenue and marginal revenue functions (as functions of $x$).
(c) Find the price and quantity combination that maximizes total revenue.
(d) Calculate the price elasticity of demand for the price-quantity combination you found in part (c).

2. Suppose that the demand for wheat (measured in bushels) is given by

$$w = 100,000 - 5000p_w$$

where $w$ is the number of bushels demanded and $p_w$ is the price per bushel.

(a) This year’s harvest is 80,000 bushels. Calculate the price elasticity of demand at that point.
(b) If next year’s harvest were to fall below 80,000 bushels, would the revenues received by wheat farmers increase or decrease?

3. Suppose that you are a world-class sculptor, producing one sculpture per month. Let’s say that you currently sell half of your sculptures and keep half for yourself. If your popularity increases, so the amount that people are willing to pay for one of your sculptures increases, what factors would determine whether you increase or reduce the fraction that you sell? Explain in terms of income and substitution effects.

4. In an exchange economy between consumers 1 and 2, the initial endowments are

$$\bar{x}_1 = 150, \quad \bar{y}_1 = 150$$
$$\bar{x}_2 = 150, \quad \bar{y}_2 = 150.$$ 

Their utility functions are

$$u_1 = (x_1)^2y_1 \quad \text{and} \quad u_2 = x_2(y_2)^2.$$ 

Find the competitive equilibrium allocation and price ratio.