Trade on Markets

A market economy entails ownership of resources. The *initial endowment* of consumer 1 is denoted by \((x_1, y_1)\), and the initial endowment of consumer 2 is denoted by \((x_2, y_2)\).

Both consumers’ initial endowments are represented by the same point in the Edgeworth Box, since

\[
x_1 + x_2 = x \quad \text{and} \quad y_1 + y_2 = y.
\]

Each consumer’s income now depends on prices:

\[
M_1 = p_x x_1 + p_y y_1 \quad \text{and} \quad M_2 = p_x x_2 + p_y y_2.
\]

What determines the prices?
The Auctioneer and Equilibrium

Market forces cause the price to fall when there is *excess supply*, and rise when there is *excess demand*. Eventually, things settle into *equilibrium*, where supply equals demand. We personify these forces into an imaginary “auctioneer.”

1. Think of the auctioneer as calling out prices.

2. Consumers solve their utility maximization problems, and announce their demands.

3. The auctioneer then adjusts prices, raising them for commodities where demand exceeds supply, and lowering them for commodities where supply exceeds demand.

4. We then repeat steps 2 and 3 until we find prices where supply equals demand. Consumers receive their utility maximizing bundle, based on the *equilibrium* prices.
In the Edgeworth Box, the budget line for both consumers is the line with slope $-p_x/p_y$ and running through the initial endowment point.

In the following diagrams, the initial endowment is in the northwest corner of the box, $(x, y) = (0, 1)$.

Excess demand for good $y$ ($p_x/p_y = 2$)
Excess demand for good $x \times (p_x/p_y = \frac{2}{3})$
Equilibrium \((p_x/p_y = 1)\)

Consumer 1 is a net buyer of \(x\) and a net seller of \(y\).
Consumer 2 is a net seller of \(x\) and a net buyer of \(y\).
Definition: An equilibrium is a price, \((p^*_x, p^*_y)\), and an allocation, \((x^*_1, y^*_1, x^*_2, y^*_2)\), such that

(i) For each consumer, \(i=1,2\), \((x^*_i, y^*_i)\) solves

\[
\max_{x_i, y_i} u_i(x_i, y_i)
\]

subject to:

\[
px x_i + py y_i = px x_i^* + py y_i^*
\]

(ii) \(x^*_1 + x^*_2 = \overline{x}\) and \(y^*_1 + y^*_2 = \overline{y}\).

Condition (i) reflects utility maximization, and condition (ii) reflects market clearing (demand = supply).
Equilibrium and Pareto Optimality

Utility maximization requires each consumer’s indifference curve to be tangent to their budget line:

\[
\frac{\partial u_1}{\partial x_1} = \frac{p_x}{p_y} = \frac{\partial u_2}{\partial x_2} = \frac{\partial u_2}{\partial y_2}
\]  (1)

From (3), we see that the consumers’ marginal rates of substitution are equal to each other.

*First Fundamental Theorem of Welfare Economics*: an equilibrium allocation is Pareto optimal.

This theorem extends to many consumers and many goods, and to the inclusion of production.

However, an efficient allocation is not necessarily fair. Is there a tradeoff between efficiency and fairness?
Suppose that we, as a society, could evaluate the “social welfare” of any allocation.

This social welfare function would prefer allocation $A$ over allocation $B$ if each individual preferred $A$ to $B$. Thus, the socially optimal allocation will be Pareto optimal.

Also implicit in the social welfare function would be our notions of fairness (for instance, equal weights or importance for each consumer’s utility).

In principle, we could calculate the socially optimal allocation, and assign each consumer his/her bundle. But do we trust Government to choose our consumption for us? How can the Government learn our utility functions and perform the calculation?
Second Fundamental Theorem of Welfare Economics: Any Pareto optimal allocation can be achieved as an equilibrium, by a suitable reassignment of the initial allocation.

We don’t have to abandon markets to implement a fair and efficient allocation, just redistribute income and let markets work.

Privatization in Russia: give everyone 1 share of stock in every firm. Minimal informational requirements.

Don’t take FFTWE and SFTWE too seriously:

1. We often make policy based on efficiency, and never do the redistributions (NAFTA).

2. Redistributions might not be feasible: tax someone $1,000,000 whether they become a doctor or an artist. An income tax is not a valid redistribution, because it is in response to choices on the labor market and capital market.

3. Imperfect competition or externalities make the equilibrium inefficient.
Solving for the Equilibrium: An Example

\[ u_1(x_1, y_1) = x_1 y_1 \] and \( (\bar{x}_1, \bar{y}_1) = (2, 1). \)

\[ u_2(x_2, y_2) = x_2 y_2 \] and \( (\bar{x}_2, \bar{y}_2) = (1, 2). \)

Step 1: normalize prices, so \( p_y = 1 \). We can do this because of homogeneity, only relative prices matter.

Step 2: solve for the demand functions.

For consumer 1, the Lagrangean expression is

\[ L = x_1 y_1 + \lambda [2p_x + 1 - px x_1 - y_1] \]

The first order conditions are:

\[ \frac{\partial L}{\partial x_1} = y_1 - \lambda p_x = 0 \] \hspace{1cm} (2)

\[ \frac{\partial L}{\partial y_1} = x_1 - \lambda = 0 \] \hspace{1cm} (3)

\[ \frac{\partial L}{\partial \lambda} = 2p_x + 1 - px x_1 - y_1 = 0. \] \hspace{1cm} (4)
Solving (4)-(6), we first eliminate $\lambda$ from (4) and (5), yielding the marginal rate of substitution equals the price ratio:

$$\frac{y_1}{x_1} = p_x. \quad (5)$$

Rearranging (7), we have

$$y_1 = p_xx_1 \quad (6)$$

Substituting (8) into (6), we have

$$2p_x + 1 - 2p_xx_1 = 0,$$

from which we solve for the demand for $x$:

$$x_1 = \frac{2p_x + 1}{2p_x}. \quad (7)$$

From (8) and (9), we have the demand for $y$:

$$y_1 = \frac{2p_x + 1}{2}. \quad (8)$$
For consumer 2, the Lagrangean expression is

\[ L = x_2 y_2 + \lambda [p_x + 2 - p_x x_2 - y_2]. \]

Solving the first order conditions, we have the marginal rate of substitution condition and the budget equation, from which we can solve for the demand functions:

\[ x_2 = \frac{p_x + 2}{2p_x} \quad \text{and} \quad y_2 = \frac{p_x + 2}{2} \quad (9) \]

Step 3: Use market clearing to solve for \( p_x \). Which market clearing condition do we use?

Market clearing for good \( x \) implies

\[ x_1 + x_2 = \frac{2p_x + 1}{2p_x} + \frac{p_x + 2}{2p_x} = 3. \quad (10) \]

Solving (12) for \( p_x \) yields \( p_x = 1. \)
Market clearing for good $y$ implies

$$y_1 + y_2 = \frac{2px + 1}{2} + \frac{px + 2}{2} = 3. \quad (11)$$

Solving (13) for $px$ yields $px = 1$.

You get the same answer both ways. Whenever supply equals demand for all markets except one, supply equals demand for the last market as well.

To get the equilibrium allocation, plug $px = 1$ into the demand functions:

$$x_1 = \frac{2px + 1}{2px} = \frac{3}{2}, \quad x_2 = \frac{px + 2}{2px} = \frac{3}{2},$$

$$y_1 = \frac{2px + 1}{2} = \frac{3}{2}, \quad y_2 = \frac{px + 2}{2} = \frac{3}{2}.$$

As a check, total consumption of good $x$ is 3, and total consumption of good $y$ is 3.