Oligopoly Games and Voting Games

Cournot’s Model of Quantity Competition:

Suppose there are two firms, producing an identical good. (In his 1838 book, Cournot thought of firms filling bottles with mineral water taken without cost from the same spring.)

The strategic choice is quantity,

\[ S_i = \{q_i : q_i \geq 0\}. \]

The total quantity produced is sold on the market at the market-clearing price. The total quantity is \( Q = q_1 + q_2 \), and we assume that the "inverse" demand curve is given by

\[ p = 1000 - Q. \]

Thus, consumer behavior is determined by the demand curve, so consumers are not strategic players.
Also suppose that each firm incurs a cost of 100 per unit of output produced.

Then the payoff to firm 1 is

\[ u_1(q_1, q_2) = [1000 - q_1 - q_2]q_1 - 100q_1 \]

and the payoff to firm 2 is

\[ u_2(q_1, q_2) = [1000 - q_1 - q_2]q_2 - 100q_2. \]

A Nash equilibrium is a strategy profile, \((q_1^*, q_2^*)\), where (i) \(q_1^*\) is a best response to \(q_2^*\) and (ii) \(q_2^*\) is a best response to \(q_1^*\).
Player 1’s best response, to a belief that assigns probability one to player 2 choosing the pure strategy $q_2$, solves the following problem:

$$\max_{q_1} [1000 - q_1 - q_2]q_1 - 100q_1.$$ 

We can solve this maximization problem by setting the partial derivative with respect to $q_1$ equal to zero and then solving for $q_1$. We have

$$1000 - 2q_1 - q_2 - 100 = 0,$$

or

$$q_1 = 450 - \frac{q_2}{2}.$$ 

Put another way, player 1’s best response function (to pure strategy beliefs) is $BR_1(q_2) = 450 - \frac{q_2}{2}$.

The same analysis applied to player 2 yields player 2’s best response function, $BR_2(q_1) = 450 - \frac{q_1}{2}$. 
A Nash equilibrium solves

\[ q_1^* = 450 - \frac{q_2^*}{2} \quad \text{and} \]
\[ q_2^* = 450 - \frac{q_1^*}{2}. \]

If you do the algebra, the solution is \((q_1^*, q_2^*) = (300, 300)\).

Notice that the NE in the Cournot game is inefficient. Each firm receives profits of 90,000 in the NE. If instead the two firms agreed to reduce their output to 225, the price would go up from 400 to the monopoly price of 550, and profits would be \((450)(225) = 101,250\).

The efficient strategy profile yielding monopoly or cartel profits is not a NE. To see this, if firm 2 produced 225, then firm 1’s best response is to produce \(450 - 225/2 = 337.5\).
This is a good example of individual incentives vs. efficiency. Since (225,225), maximizes joint profits, if firm 1 were to increase output slightly, the revenue gain to both firms from increased output balances the revenue loss from a lower price. However, the revenue gain goes entirely to firm 1, and the revenue loss is shared by both firms.

Therefore, firm 1 (and firm 2 as well) has an individual incentive to produce more than the socially optimal quantity. The game is similar to the Prisoner’s Dilemma, if we think of $q_i = 225$ as the cooperative strategy and a higher output (say 300) as the defect strategy.
For a monopolist that knows the demand function, it does not matter whether the strategic choice is price or quantity. For the case of duopoly, it makes a big difference.

Bertrand’s Model of Price Competition:

Consider the same "inverse" demand function and cost structure as the Cournot example. Firms produce identical products, marginal cost is 100, and inverse demand is $p = 1000 - Q$. Then we can write demand as a function of price as

$$Q = 1000 - p.$$  

Under Bertrand competition, the strategic variable is price, which firms choose simultaneously

$$S_i = \{p_i : p_i \geq 0\}.$$
If the two firms set different prices, all of the market demand goes to the firm choosing the lower price, and the firm choosing the higher price receives no customers. If the two firms set the same price, each firm receives half of the market demand.

Then the payoff to firm 1 is

\[ u_1(p_1, p_2) = \begin{cases} 
(p_1 - 100)[1000 - p_1] & \text{if } p_1 < p_2 \\
(p_1 - 100)[1000 - p_1] \div 2 & \text{if } p_1 = p_2 \\
0 & \text{if } p_1 > p_2.
\end{cases} \]

Similarly, the payoff to firm 2 is

\[ u_2(p_1, p_2) = \begin{cases} 
(p_2 - 100)[1000 - p_2] & \text{if } p_1 > p_2 \\
(p_2 - 100)[1000 - p_2] \div 2 & \text{if } p_1 = p_2 \\
0 & \text{if } p_1 < p_2.
\end{cases} \]
What is the NE?

Recall that for the Cournot example, we set the derivative equal to zero. That is because player 1’s payoff function was a continuously differentiable and concave function of $q_1$. In the Bertrand game, payoffs are not continuous, so the calculus approach will not work.

A related "technical" concern is that player 1’s best response function, $BR_1(p_2)$, is not always well defined! For example, if player 2 sets a price of 200, player 1 wants to set a price slightly below 200 to capture the entire market, but for any such price, player 1 receives higher profits by increasing its price closer to (but still below) 200.

But we can still show that there is a unique NE in which each player sets a price equal to marginal cost, $p_1 = p_2 = 100$. 
First we show that \((p_1, p_2) = (100, 100)\) is a NE. Notice that each player receives a payoff of zero.

If player 1 increases its price, it will have no customers and continue to receive a payoff of zero.

If player 1 decreases its price, it captures the entire market but receives a negative payoff since its price is below its cost.

Therefore, \(p_1 = 100\) is a best response to player 2's strategy. By a symmetrical argument, \(p_2 = 100\) is a best response to player 1's strategy.
Next we show that no other profile \((p_1, p_2)\) can be a NE.

If \(p_1 > 100\) and \(p_2 > 100\), then whichever player has the highest price can slightly undercut the other’s price, thereby capturing the entire market and increasing profits. (Not a NE.)

If \(p_1 = 100\) and \(p_2 > 100\), then player 1 can increase its price to slightly below \(p_2\), thereby increasing its payoff. If \(p_1 > 100\) and \(p_2 = 100\), then player 2 can increase its price to slightly below \(p_1\), thereby increasing its payoff. (Not a NE.)

If one or both players sets a price below 100, the lowest price player (or both players if they tie) receives negative profits. (Not a NE.)

Notice that Bertrand competition gives the perfectly competitive outcome with zero profits. Competition is fiercer than in the Cournot game, because there is a strong incentive to undercut the rival’s price to capture the entire market.
Voting Games

Platform competition and the median voter theorem

Suppose the space of policy positions (from liberal to conservative) is represented by the unit interval $[0, 1]$.

Each voter has an ideal policy position in the unit interval, and assume that she will vote for whichever candidate’s position is closest to her own position. (Voters who are indifferent choose each of their preferred candidates with equal probability.)

Voters’ ideal positions are uniformly distributed over the unit interval.

The players are the $n$ candidates, who simultaneously choose a policy position, $s_i \in [0, 1]$. A candidate’s pay-off is the fraction of the votes he receives. (Note that candidates do not have policy preferences. They only care about their vote total.)
With 2 candidates, the only NE has each candidate choosing \( s_i = \frac{1}{2} \), which is the ideal position of the median voter.

To see that \((\frac{1}{2}, \frac{1}{2})\) is a NE, notice that each candidate receives half of the votes, and that any movement away from \( s_i = \frac{1}{2} \) causes candidate \( i \) to receive fewer than half the votes.

To see that \((\frac{1}{2}, \frac{1}{2})\) is the only NE, suppose there is a different NE profile, \((s_1, s_2)\). If \( s_1 \neq s_2 \), then candidate 1 can move his position closer to candidate 2’s position, and increase his votes. Thus, the only possibility for a NE is \( s_1 = s_2 \).

If \( s_1 = s_2 \neq \frac{1}{2} \), then candidate 1 can move his position closer to \( \frac{1}{2} \), and increase his votes. Thus, the only NE is \((\frac{1}{2}, \frac{1}{2})\).
This result, that both candidates will cater to the preferences of the median voter, is robust to non-uniform distributions of voter preferences.

Actual elections are often more polarized for several reasons that are not captured within this simple game.

1. Candidates have policy preferences themselves.

2. In order to become a candidate (for example, to win the primary), one must be somewhat extreme.

3. Our game assumes that everyone must vote and that voting is the only way to show support. If voter turnout or fundraising was an issue, maybe the NE would have more extreme candidates.

4. Our game assumes that voters know the preferences of the voters and that voters know the positions of the candidates.
Notice the connection of this voting game to the game with Ben and Jerry selling ice cream on the beach.

What would happen when there are three candidates, \( n = 3 \)?

As it turns out, there is no NE in pure strategies, \((s_1, s_2, s_3)\). Let us rule out all of the possibilities.

1. \( s_1 \neq s_2 \neq s_3 \). This cannot be a NE, because a candidate on the left or right can receive more votes by moving closer to the candidate in the middle.

2. Exactly two candidates choose the same position. This cannot be a NE, because the other candidate can receive more votes by moving closer to the other two.

3. \( s_1 = s_2 = s_3 \). This cannot be a NE, because one of the candidates can move slightly away from the other two, and increase his votes from \( \frac{1}{3} \) to nearly \( \frac{1}{2} \) (possibly even more than one half).
Strategic Voting

The platform competition game models voters as non-strategic. There are several reasons why voters may not always vote for the candidate whose platform they most prefer.

1. If a citizen must take an hour off from work in order to vote and drive to the polls, then she faces a cost of voting on the order of $10 to $20 or more. Suppose voting is costly, that the two candidates are already on the ballot, and that a citizen’s utility depends only on the cost incurred and the policy position of the winning candidate.

Then if a citizen decides to vote, she will vote for her preferred candidate. But for voting to be a best response, the dollar equivalent of having her preferred candidate in office multiplied by the probability of her vote being pivotal must exceed her cost. Especially in large elections, the probability of being pivotal is near zero, so why vote?
The NE of the game sketched on the previous slide involves mixed strategies, where the expected number of votes cast in a national election can be in the thousands, not millions.

Explaining voter turnout in games with strategic voters has been a puzzle.

We need to acknowledge that citizens receive utility from having voted (and maybe from others knowing that one voted), and not just from the outcome of the election.
2. With three or more candidates, voting for the candidate whose platform you most prefer might be "throwing away your vote."

Here is a game that illustrates the 2003 California Governor’s race, where Gray Davis had been recalled from office and the three most viable candidates were:

(conservative Republican) Tom McClintock

(moderate Republican) Arnold Schwarzenegger

(liberal Democrat) Cruz Bustamante

Suppose for simplicity that there are three players in the game corresponding to three voting blocks: player L, player M, and player C.
The voters’ preferences over candidate positions are as follows.

L’s ordering from most preferred to least preferred is: Bustamante, Schwarzenegger, McClintock

M’s ordering is: Schwarzenegger, Bustamante, McClintock

C’s ordering is: McClintock, Schwarzenegger, Bustamante

For $i = L, M, C$, the strategy set consists of the three candidates (who to vote for).

Player $i$’s utility is the sum of the payoff from her voting action (2 for most preferred, 1 for middle, and 0 for least preferred) and the payoff from who wins the election (4 for most preferred, 2 for middle, and 0 for least preferred).

Assume that the L voting block is slightly larger than the other two, so that if each candidate gets one vote the winner is the candidate that L votes for.
This game has one NE, where L votes for Bustamante, and M and C vote for Schwarzenegger.

Notice that C votes strategically against her preferred candidate. Voting for Schwarzenegger is a best response, yielding a utility of $1 + 2 = 3$, while "sincere" voting for McClintock allows Bustamante to win the election, yielding a utility of $2 + 0 = 2$.

This is why the California Republican Party endorsed Schwarzenegger, even though most Republicans favored McClintock.