Homework #2 Answers


**Answer:** By the symmetry of the game, the set of rationalizable pure actions is the same for both players. Call it $Z$. Consider $m \equiv \inf(Z)$ and $M \equiv \sup(Z)$. Any best response of player $i$ to a belief about player $j$ (whose support is a subset of $Z$) maximizes $E(a_i(1 - a_i - a_j))$, or equivalently, it maximizes $a_i(1 - a_i - E(a_j))$. Thus, player $i$’s best response to a belief about player $j$ depends only on $E(a_j)$, which can be written as $B_i(E(a_j)) = (1 - E(a_j))/2$. Because $m \leq E(a_j) \leq M$ must hold, $a_i \in B_i(E(a_j))$ implies $a_i \in [(1 - M)/2, (1 - m)/2]$. By the best response property of the rationalizable set, we have $m \in [(1 - m)/2, (1 - m)/2]$ and $M \in [(1 - M)/2, (1 - M)/2]$. Therefore, we have

\[
\begin{align*}
m & \geq \frac{1 - M}{2} \quad \text{and} \\
M & \leq \frac{1 - m}{2}.
\end{align*}
\]

It follows from (1) and (2) that $m \geq M$ holds, which can only occur if $m = M$. From (1) and (2), we have $m = M = 1/3$. Therefore, the only rationalizable strategy is the unique Nash equilibrium strategy, $a_i = 1/3$.

2. O-R, exercise 76.1.

**Answer:** The simplest example, in which it is common knowledge that two players have different posteriors about some event $A$, is the following. There are two states, with prior probability $1/2$ for each state. $\Omega = \{1, 2\}$ and $p(1) = p(2) = 1/2$. Player 1 cannot distinguish between the two states, $\varphi_1 = \{\{1, 2\}\}$, and player 2 can distinguish between the two states, $\varphi_2 = \{\{1\}, \{2\}\}$. Therefore, the meet of the two information structures is $\varphi_1 \land \varphi_2 = \{\{1, 2\}\}$. Let $A = \{1\}$. At $\omega = 1$, player 1’s posterior is 1, and player 2’s posterior is $1/2$. At $\omega = 2$, player 1’s posterior is 0, and player 2’s posterior is $1/2$. Because posteriors are different at all states, it is common knowledge that posteriors are different.

Let $E = \{\omega' : q_1(\omega') > q_2(\omega')\}$. Suppose $E$ is common knowledge at $\omega$. Let $M$ be the element of $\varphi_1 \land \varphi_2$ containing $\omega$. Then $M = \bigcup_j P_i^j$, where we
have the union of disjoint elements of \( \varphi_1 \), and \( M = \bigcup_j P_j \), where we have the union of disjoint elements of \( \varphi_2 \).

Because \( E \) is common knowledge at \( \omega \), we must have \( q_1(\omega') > q_2(\omega') \) for all \( \omega' \in M \).

Therefore, for all \( P_1 \subseteq M \), and all \( P_2 \subseteq M \), we have

\[
\frac{pr(A \cap P_1)}{pr(P_1)} > \frac{pr(A \cap P_2)}{pr(P_2)}
\]

Cross multiplying, \( pr(P_2)pr(A \cap P_1) > pr(P_1)pr(A \cap P_2) \).

Summing over (disjoint) \( P_1 \subseteq M \), we have \( pr(P_2)pr(A \cap M) > pr(M)pr(A \cap P_2) \).

Summing over (disjoint) \( P_2 \subseteq M \), we have \( pr(M)pr(A \cap M) > pr(M)pr(A \cap M) \), a contradiction.


**Answer:** The minmax payoffs are given by \( v_1 = v_2 = 1 \), which implies that player 1 must receive a payoff of at least 1 in any subgame perfect equilibrium of the repeated game. Player 2’s payoff exceeds player 1’s payoff by at least 1 at any action profile of the stage game, so player 2 must receive a payoff of at least 2 in any subgame perfect equilibrium of the repeated game. Suppose \( ((A, A), (A, A), ...) \) is the outcome path of a subgame perfect equilibrium. Player 2’s payoff is 3. By deviating to \( D \) in the first period, player 2 receives a payoff of 5 in period 1 and a continuation payoff of at least 2, because the continuation strategies after the deviation must form a subgame perfect equilibrium. Therefore, the deviation yields a payoff of at least

\[
(1 - \delta)(5 + \sum_{i=1}^{\infty} \delta^i 2) = \frac{7}{2},
\]

which is greater than 3.