Reserve Price Competition with Demand Uncertainty

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Abstract

We consider duopoly competition under aggregate demand uncertainty, where firms compete by choosing reserve prices and holding uniform-price auctions. Consumers observe their valuation, but not the demand state, commit to a firm and participate in its auction. Our model captures the features of several important markets, for example, local delivery or power markets, where demand is high during peak periods and low during off-peak periods. Equilibrium reserve prices depend on the overall market demand elasticities in the high and low demand states. If market demand is sufficiently elastic, then equilibrium reserve prices do not bind and the allocation is efficient. If market demand in the low state is sufficiently inelastic, then every equilibrium in which firms employ pure strategies involves at least one firm choosing a binding reserve price causing inefficiency. We also show that more demand uncertainty softens competition.

1 Introduction

We consider duopoly competition under aggregate demand uncertainty, where firms compete by choosing reserve prices and holding uniform-price auctions. Active consumers, after observing their valuation but not the demand state, commit to a firm and participate in its auction. We model a consumer as a point on a demand curve, so the only interesting decision faced by consumers is which firm to choose; once at a firm, consumers have a weakly dominant strategy to bid their valuation. For each reserve price pair, (R^1, R^2), we characterize an equilibrium to the resulting "consumer" subgame. There are five types of consumer equilibria, depending on how large R^1 is relative to R^2, in which consumers endogenously sort themselves across the two firms. Whether or not competition drives the equilibrium reserve prices to zero depends on the overall market demand elasticities in the high and low demand states.

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If market demand is sufficiently elastic, then equilibrium reserve prices do not bind and the allocation is efficient, coinciding with the competitive equilibrium that would arise in a hypothetical centralized economy with price-taking consumers and firms. However, if market demand in the low state is sufficiently inelastic (made precise in Proposition 4), then every equilibrium in which firms employ pure strategies involves at least one firm choosing a binding reserve price and not allocating all its capacity in the low demand state. Withholding capacity due to a binding reserve price is the only form of inefficiency. The equilibrium allocation is quasi-efficient, in the sense that consumption is received by the consumers with the highest valuations.[1]

This paper belongs to the literatures on competing auctions and competing mechanisms. These literatures, discussed in the next section, primarily have not focused on markets in which sellers have multiple units of output to sell. Also, these literatures primarily have not focused on markets in which consumers have correlated valuations. Both of these features are crucial to our setting. We are interested in duopoly markets with many consumers and aggregate demand uncertainty. If there were two demand states, high and low, then whether one "potential" consumer is an active participant must be correlated with whether another potential consumer is an active participant; if the activity of consumers (and their valuations) were independent, then there would be only one aggregate demand state.

Our model captures some of the features of several important markets. For example, consider local food delivery markets, where demand is high during peak periods and low during off-peak periods. Uber Eats and Doordash offer surge pricing, so during periods of peak demand, delivery prices rise above their normal level. There is no explicit auction, but surge prices could reflect the market clearing price in the high demand state. Consumers choose a firm by downloading one of the apps and "bid" by either accepting or rejecting a price offer. Normal prices during off-peak periods could be market clearing (corresponding to our Regime 5) or they could reflect one of the firms setting a binding reserve price with excess capacity, while markets clear at the other firm (corresponding to our Regime 4). The point is that consumers come to anticipate the prices prevailing at each firm in each state and choose firms accordingly. If the firms employed fixed-price mechanisms without surge pricing, the outcome would be highly inefficient as some high-valuation consumers would be rationed during peak-demand periods.

Consider electricity markets, especially with renewable energy sources where capacity is fixed in the short run and marginal cost is nearly zero. Although power must flow on a single network grid, in Ohio and other locations, consumers can contract with one of several competing providers. The technology exists to measure usage at hourly intervals,

[1] Quasi-efficiency is obvious when considering the consumers at a particular firm, but less obvious when considering consumers at different firms.

so in principle a competitor can hold an auction with its customers to clear their market during peak periods. The main reason that this form of competition does not yet exist is that consumers have no way to monitor the price.[2] However, with advances in smart-home technology, consumers will be able to install meters that can monitor the price offered by their supplier and program which of their appliances to turn off as the price rises. With deregulation and technology advances, reserve price competition could well emerge in this market.[3]

In Section 2, we provide a literature review. The model is presented in Section 3. Section 4 characterizes the consumer equilibrium when one of the firms sets a reserve price of zero, and Section 5 characterizes the consumer equilibrium for general (R^1, R^2). Section 6 considers the full game and provides some results about when competition drives reserve prices to zero. Under some conditions when the low demand state is sufficiently price inelastic to preclude equilibrium with zero reserve prices, we show that there is an equilibrium in which one firm sets a reserve price that binds only in the low demand state and the other firm sets a reserve price of zero. We also show that, in a precise sense, more demand uncertainty softens competition. Section 7 works out an example and Section 8 contains some concluding remarks. The Appendix addresses a technical issue that arises in the consumer subgame (Regime 3), and it contains all proofs except for the proof of Proposition 1.

2 Literature Review

Reserve price competition plays a prominent role in the competing mechansims literature. Much of this literature, including the papers discussed in this paragraph, assume that sellers have a single indivisible unit to sell and buyer valuations are independently distributed. McAfee (1993) considers the steady state of a dynamic process with many buyers and sellers. In each period, sellers announce a mechanism from a broad class and buyers choose a seller and commit to its mechanism. Sellers are assumed to ignore their effect on overall market utility and, in equilibrium, all sellers choose an efficient auction with a reserve price equal to the seller's valuation (zero in our context). Peters and Severinov (1997) consider competing sellers who offer a second-price auction with a reserve price. They offer a limit equilibrium concept for the infinite economy and show that the symmetric

[2] In Texas during February 2021, many customers had electricity bills that were tied to the spot market price of a kilo-watt hour, when a major storm hit. The spot price jumped from \$0.12 per kilo-watt hour to \$9.00 per kilo-watt hour, and some residents faced bills of over \$7000 for one week's worth of electricity. See Najmabadi (2021).

[3] An interesting alternative is for a local government to organize a single auction, where consumers can program which appliances to turn off at different prices and suppliers can decide how much power to supply. Since this market would have an element of quantity competition and reserve price competition is closer to price competition, it is unclear which structure would be more efficient.

equilibrium reserve price is zero, thereby justifying McAfee's assumption that sellers ignore their effect on the market. Burguet and Sakovics (1999) show that, when there are only two sellers engaging in reserve price competition and the number of buyers is finite, sellers choose mixed strategies and equilibrium reserve prices are bounded above zero. Virag (2010) considers large finite markets and shows that the limiting reserve price converges to zero in distribution as the market becomes large. Pai (2014) studies competition between two sellers who select an "extended auction," a class of mechanisms that includes auctions and posted prices. Without imposing strong conditions on the distribution of valuations, sellers employ mechanisms that are not quasi-efficient, ruling out auctions with reserve prices.

Peck (2018) considers a model with a finite number of firms, each of whom has a large capacity of output. Firms choose mechanisms from a broad class that includes fixed prices, entry fees, and auctions with reserve prices. There is a continuum of consumers who demand multiple units and are drawn independently from a distribution with a finite number of types. In general, firms do not choose reserve price mechanisms in equilibrium. However, the online appendix considers the model in which consumers demand a single unit and there is a continuum of valuation types. In that case, when demand is sufficiently elastic, the competing mechanisms game has an equilibrium in which all firms choose auctions with a zero reserve price. The present paper introduces aggregate demand uncertainty and restricts attention to reserve price competition. With a continuum of consumers, aggregate demand uncertainty requires correlation in the valuations of consumers. Fixed-price mechanisms perform poorly in this situation, because firms may sell too little output in some states and require inefficient rationing in other states. The only other paper in this literature we can find with competing sellers and correlated valuations is by Peters (2014). Buyers have unit demands and sellers, each with one indivisible unit, choose mechanisms from a broad class. Under a regularity assumption on demand and a market payoff taking assumption (reasonable if there are many sellers), Peters (2014) shows that there is a unique equilibrium outcome, equivalent to each seller choosing a second price auction with a zero reserve price. The main message from Peters (2014) is that competition produces simple mechanisms in equilibrium.

The present paper is unique in this literature, in that we model competition by a small number of sellers who sell to a large number of buyers, in the presence of aggregate demand uncertainty. This structure allows us to shed light on oligopoly markets with surge pricing. We find that equilibrium could be perfectly competitive, like in McAfee (1993), Peters and Severinov (1997), Virag (2010), and Peters (2013). This is the case, even though our model has only two firms. Equilibrium in our model can be perfectly competitive, unlike the duopoly model of Burguet and Sakovics (1999), but it could involve a binding reserve price and inefficient withholding of output, which is always the case in Burguet and Sakovics

(1999). In the later situation, there can be an equilibrium where one firm sets a zero reserve price and the other sets a binding reserve price; in Burguet and Sakovics (1999), both firms must be choosing mixed strategies.

Our model is related to parts of the directed search literature. Directed search is a huge topic, so see Wright et al. (2021) for a thorough survey. Coles and Eeckhout (2003) consider a model with two sellers each with one indivisible unit, and two buyers. Sellers post prices that can be contingent on whether one or two buyers arrive at the firm, which allows for auction mechanisms. It is shown that there are many equilibria, but the sellers prefer the equilibria in auctions. Eeckhout and Kircher (2010) consider competition in mechanisms and several types of "search frictions," including a purely non-rival technology that corresponds to our setting. In that case, second-price auctions emerge as equilibrium mechanisms. Again, our model differs in that buyer valuations are correlated and sellers have a large capacity, not a single indivisible good.

We have not found theoretical models of competition between, say, Uber Eats and Doordash. However, casual observation suggests that currently consumers are likely to prefer one service over another and are unlikely to check prices from multiple platforms. These platforms use surge pricing, arguably to clear the market in high demand states, which is what we are studying.[4] Especially in markets where many restaurants are located near each other in a town center, drivers do not have to cruise in search of orders, so it is reasonable to ignore spatial issues as we do in our model. Anecdotal evidence suggests that Doordash fires those drivers who do not accept enough orders, so it is reasonable to assume that Uber Eats and Doordash have their own pool of drivers.[5] Thus, delivery competition fits our model fairly well. However, it should be noted that we ignore the driver side of this market and instead treat each supplier as a single entity. We ignore fluctuations in capacity (the number of drivers), although that could certainly fit into our framework.[6]

Wolak (2014) provides a non-technical survey of the electricity industry and lays out the technological features of the industry. The only sensisble structure is for all power to flow on the same grid. Fabra and Llobet (2022) study centralized electricity auction formats with capacity uncertainty. Firms submit bids specifying the quantity they are willing to supply and the minimum price at which they are willing to supply it. Firms' minimum prices are

[4]In India, food delivery duopolists Swiggy and Zomato offer loyalty programs that effectively lock in their customers. These programs offer discounts but, interestingly, explicit pricing policies and reports from subscribers indicate that discounts are modified or excluded during peak days and times.

[5]See the report on Last Week Tonight (host John Oliver) on March 31, 2024.

[6]Our model resembles some of the features of the rideshare market, especially when spatial issues are minimized. The best fit would be the market for rides from an airport, since drivers are located in a waiting area rather than cruising for rides. However, several aspects of Uber/Lyft competition are problematic for our model. The crucial assumption that consumers choose a single platform is violated to the extent that consumers download both apps and compare prices. Even some drivers work for both firms. Waiting times are an important part of the market. See Rosaia (2020) and the references therein.

somewhat similar to reserve prices, except they are set by bidders and not the auctioneer. The auction price is the minimum of the market clearing price and an exogenously given "market reserve price." Our analysis differs substantially from Fabra and Llobet (2022). First, their market reserve price is actually a price ceiling. Second, we consider demand uncertainty rather than supply uncertainty, although both are present in electricity markets. Third, rather than a single centralized auction, we consider competition by firms, each of whom offers its own auction. Competition by suppliers already exists in deregulated markets such as Ohio and elsewhere. However, in Ohio, competing firms currently offer fixed-price contracts.[7] The technology exists to measure a consumer's usage on an hourly basis, so the potential is there for a provider to charge the maximum of their reserve price and the market clearing price each hour. However, to avoid the problems of skyrocketing prices faced in Texas in 2021, a technology must also be developed for each consumer to monitor the price they are facing and program the circumstances at which they turn off their access.

3 The Model

The environment is one with 2 firms, each with marginal costs normalized to zero and with the same capacity of a homogeneous good, normalized to 1. There are two aggregate demand states, H and L, with prior probabilities π_H and π_L. Consumers demand either zero or one unit of the good. The support of consumer valuations is $[a, b]$, and the measure of active consumers in state s, with valuation greater than or equal to $v \in [a, b]$ is given by $\alpha_s D(v)$. State H is the high-demand state and state L is the low-demand state, so $\alpha_H > \alpha_L$ holds. We assume that $D(v)$ is continuously differentiable, strictly decreasing, and that the absolute value of the price elasticity of market demand is increasing in price (demand becomes more elastic as we increase price). Assuming that the process that determines the activity and valuation of consumers is symmetric across "potential" consumers, and using Bayes' rule, the probability of state s, conditional on being an active consumer with valuation v, is[8]

$$\pi_s(v) = \frac{\pi_s \alpha_s}{\pi_H \alpha_H + \pi_L \alpha_L}. \tag{1}$$

Intuitively, active consumers update their priors because they are more likely to be active in states with more active consumers, and all valuation types share the same posterior beliefs because demand uncertainty enters demand in a multiplicative fashion.

The timing of the game is as follows. First, firms simultaneously announce a reserve price, R^f. Then nature selects which consumers are active and selects their valuations. Active consumers observe their valuation and the fact that they are active. They also observe

[7] See https://www.energychoice.ohio.gov/ for more information.

[8] To see how expressions like this can be derived from Bayes' rule, see Deneckere and Peck (1995).

the reserve prices, and choose which firm to visit. Consumers visiting a firm participate in that firm's auction. The price is the maximum of the highest rejected bid and the reserve price. At the auction stage, it is a weakly dominant strategy for consumers to bid their valuation. Thus, a firm's auction price is the reserve price or the market clearing price based on supply and demand at that firm, whichever is higher.

We denote the Reserve Price Game by Γ, and we denote the consumer subgame following reserve price R^1 for firm 1 and R^2 for firm 2 as $CS(R^1, R^2)$. Omitting the dependence on the reserve prices, we denote the probability that a type v consumer chooses firm f by $\beta^f(v)$, and we denote the price prevailing at firm f in state s by p_s^f. Also we denote the market clearing price for the whole economy in state s by p_s^c, satisfying

$$\alpha_H D(p_H^c) = 2 \text{ and} \tag{2}$$

$$\alpha_L D(p_L^c) = 2. \tag{3}$$

Our solution concept is perfect Bayesian equilibrium (PBE), but the structure of the game allows us to simplify the notation and exposition. The only relevant beliefs are about the aggregate state, H or L. Firms receive no information about the state when selecting reserve prices, so their beliefs coincide with the priors, π_H and π_L. Because reserve prices signal nothing about the state, both off-path and on-path consumer beliefs are given by (1). We assume that, at the auction stage, consumers adopt their weakly dominant strategy of bidding their valuation. Therefore, with a slight abuse of the terminology we refer to a PBE of Γ as a subgame perfect equilibrium (SPE).

We first characterize a consumer equilibrium for each subgame $CS(R^1, R^2)$, and we denote this consumer equilibriun by $CE(R^1, R^2)$. Then we work backwards to find equilibrium reserve prices. There are five "regimes," where there is a consumer equilibrium in one of the five regimes for each $CS(R^1, R^2)$.

In *Regime 1*, all consumers choose firm 2 and, clearly, firm 1's reserve price binds in both states.

In *Regime 2*, there is an interior cutoff, $\overline{v}$, below the highest valuation type, which depends on (R^1, R^2), such that all consumers with $v > \overline{v}$ choose firm 1 and all consumers with $v < \overline{v}$ choose firm 2. Consumers who choose firm 1 are indifferent between the two firms, and firm 1's reserve price binds in both states.

In *Regime 3*, there is a cutoff, $\overline{v}^*$, such that all consumers with $v > \overline{v}^*$ choose firm 1 and all consumers with $v < \overline{v}^*$ choose firm 2. Consumers who choose firm 1 are indifferent between the two firms. With the cutoff, $\overline{v}^*$, the measure of consumers with $v \geq R^1$ at firm 1 in state H is exactly equal to the supply, 1. As long as (R^1, R^2) is within Regime 3, the cutoff $\overline{v}^*$ is such that the measure of consumers at firm 1 is exactly one, so this cutoff does not depend on (R^1, R^2).

In *Regime 4*, consumers with $v > p^c_H$ choose each firm with probability one half, $\beta^1(v) = \beta^2(v) = \frac{1}{2}$, and consumers with lower valuation choose firm 2 with some probability, $\beta < 1$, not necessarily equal to one half. We have $p^1_L = p^2_L = R^1$ and $p^1_H = p^2_H = p^c_H$. At firm 1, R^1 binds in state L. The mixing probability β is determined by the condition that the market clearing price at firm 2 in state L is exactly R^1.

In *Regime 5*, all consumers choose each firm with probability one half, $\beta^1(v) = \beta^2(v) = \frac{1}{2}$. Firm 1's reserve price does not bind, and prices are given by $p^1_L = p^2_L = p^c_L$ and $p^1_H = p^2_H = p^c_H$.

4 Consumer Equilibrium with $R^2 = 0$

Assume that $R^2 = 0$ holds. Here we characterize the equilibrium of the consumer subgame $CS(R^1, 0)$, for all values of R^1.

We show below that, as R^1 falls, the consumer equilibrium crosses a threshold from one regime to the next, starting in Regime 1 and ending in Regime 5. Furthermore, there are no gaps or overlaps, so each consumer subgame $CS(R^1, 0)$ has an equilibrium in exactly one of the five regimes. Next, we describe these regimes, and then summarize this analysis in Proposition 1 .

4.1 Regime 1

If R^1 is high enough, consumers prefer to pay the price at firm 2 rather than the price R^1 at firm 1. Prices at firm 2 are given by

$$p^2_H = D^{-1}(\frac{1}{\alpha_H}) \text{ and } p^2_L = D^{-1}(\frac{1}{\alpha_L}).$$

All consumers choosing firm 2 constitutes a consumer equilibrium if and only if the expected price faced by consumers at firm 2 is weakly less than R^1, or

$$\pi_H \alpha_H D^{-1}(\frac{1}{\alpha_H}) + \pi_L \alpha_L D^{-1}(\frac{1}{\alpha_L}) \leq \pi_H \alpha_H R^1 + \pi_L \alpha_L R^1. \tag{4}$$

The reason is that (4) is necessary for a consumer that buys in both states to prefer firm 2. A consumer with a valuation less than p^2_H finds choosing firm 2 to be even more beneficial, due to the option value of not purchasing in state H. The lowest R^1 consistent with Regime 1 occurs when (4) holds with equality. Thus, we have a consumer equilibrium in Regime 1 for

$$R^1 \geq \frac{\pi_H \alpha_H D^{-1}(\frac{1}{\alpha_H}) + \pi_L \alpha_L D^{-1}(\frac{1}{\alpha_L})}{\pi_H \alpha_H + \pi_L \alpha_L} \equiv \widehat{R}^1.$$

4.2 Regime 2

When R^1 is below the threshold determined by (4), some consumers will visit firm 1. In Regime 2, there is an interior cutoff valuation, $\overline{v}$, such that all consumers with $v > \overline{v}$ go to firm 1 and all consumers with $v < \overline{v}$ go to firm 2. Furthermore, there is excess supply at firm 1 in both states, so we have $p_H^1 = p_L^1 = R^1$. For this to be consistent with consumer equilibrium, we have $p_H^2 > R^1 > p_L^2$ and the indifference condition,

$$\pi_H \alpha_H p_H^2 + \pi_L \alpha_L p_L^2 = \pi_H \alpha_H R^1 + \pi_L \alpha_L R^1. \tag{5}$$

To see that (5) is required for a consumer equilibrium in Regime 2, if the right side of (5) exceeded the left side, then all consumers would prefer firm 2 and we would be in Regime 1. If the left side of (5) exceeded the right side, then all consumers with valuation greater than R^1 would prefer firm 1; however, market clearing at firm 2 would imply $p_H^2 < R^1$, contradicting the supposition that the left side of (5) exceeded the right side.

With condition (5), consumers who would purchase in both states at firm 2 are indifferent, and consumers who would would only purchase in state L at firm 2 strictly prefer firm 2, thereby justifying the consumer choices as sequentially rational. Given the cutoff valuation, market clearing prices at firm 2 are given by

$$\begin{aligned} \alpha_H D(p_H^2) - \alpha_H D(\overline{v}) &= 1 \quad (6) \\ \alpha_L D(p_L^2) - \alpha_L D(\overline{v}) &= 1 \quad (7) \end{aligned}$$

Given R^1, (5), (6), and (7) can be solved for p_H^2, p_L^2 and $\overline{v}$. For these equations to characterize a consumer equilibrium within Regime 2, there must be excess supply at firm 1 in state H:

$$\alpha_H D(\overline{v}) < 1.$$

The lower limit of $\overline{v}$ consistent with Regime 2, which we denote by $\overline{v}^*$ therefore satisfies the condition that the measure of consumers at firm 1 in state H is exactly equal to firm 1's supply,[9]

$$\alpha_H D(\overline{v}^*) = 1. \tag{8}$$

As R^1 falls within Regime 2, $\overline{v}$, p_H^2, and p_L^2 all fall. At the threshold satisfying (8), from (6), we have

$$\alpha_H D(p_H^2) = 2,$$

[9] It will be convenient to use the terminology "lowest" $\overline{v}$ consistent with Regime 2 as $\overline{v}^*$, even though, strictly speaking, it is a lower limit since the reserve price does not bind in both states when $\overline{v} = \overline{v}^*$. This should not cause confusion since Proposition 1 is precisely stated.

so the lowest p_H^2 in Regime 2 is p_H^c. From (7) and (8), we see that the lowest p_L^2 in Regime 2, which we denote by p_L^{2*}, satisfies

$$D(p_L^{2*}) = \frac{1}{\alpha_L} + \frac{1}{\alpha_H}. \tag{9}$$

The lowest R^1 within Regime 2, which we denote by R^{1*}, satisfies the indifference condition,

$$\pi_H \alpha_H p_H^c + \pi_L \alpha_L p_L^{2*} = (\pi_H \alpha_H + \pi_L \alpha_L) R^{1*}. \tag{10}$$

It follows from (10), and the fact that market clearing prices are higher in state H than in state L, that $R^{1*} < p_H^c$ holds. Also, from (2) and (8), it follows that $\overline{v}^* > p_H^c$ holds.

4.3 Regime 3

At the cutoff, $\overline{v}^*$, satisfying (8), the measure of consumers choosing firm 1 in state H is exactly equal to firm 1's supply. Thus, any p_H^1 between R^1 and $\overline{v}^*$ clears the market at firm 1 in state H. What will be the highest rejected bid when the cutoff is $\overline{v}^*$? The highest rejected bid would be $\overline{v}^*$ if a single consumer out of the continuum is not awarded a unit, and there would be no rejected bids if all consumers are awarded a unit. An important technical issue is that our application of the law of large numbers cannot resolve whether p_H^1 should be R^1 or $\overline{v}^*$. In the Appendix, we consider sequences of consumer equilibria of auctions with a large finite number of consumers, when $CE(R^1, 0)$ is in Regime 3 . We show that the limiting equilibrium cutoff approaches $\overline{v}^*$ and the p_H^1 solving (11) below is the limiting expected price at firm 1 in state H, as the number of consumers approaches infinity. In all these sequences, the excess demand or excess supply at firm 1 in state H, as a fraction of total supply, approaches zero, but uncertainty remains about whether demand (slightly) exceeds supply or supply (slightly) exceeds demand. This justifies our characterization of $CE(R^1, R^2)$ in which p_H^1 is in between R^1 and $\overline{v}^*$, and satisfies the condition that a consumer with valuation $\overline{v}^*$ is indifferent between which firm to choose. We should think of p_H^1 as the expected price at firm 1, conditional on state H.

As R^1 falls below R^{1*}, $\overline{v}$ remains constant at $\overline{v}^*$, and p_H^1 rises above R^{1*}.[10] Since the cutoff remains at $\overline{v}^*$ for all R^1 in Regime 3, the prices at firm 2 are given by $p_H^2 = p_H^c$ and $p_L^2 = p_L^{2*}$. The prices at firm 1 are given by $p_L^1 = R^1$ and, for p_H^1, the solution to the indifference condition,

$$\pi_H \alpha_H p_H^c + \pi_L \alpha_L p_L^{2*} = \pi_H \alpha_H p_H^1 + \pi_L \alpha_L R^1. \tag{11}$$

[10] As R^1 crosses below R^{1*}, one might think that there would be a consumer equilibrium in which $\overline{v}$ would adjust to fall below $\overline{v}^*$, but this is not the case. The reason is that the measure of consumers choosing firm 1 in state H would rise above the supply, so p_H^1 would rise discontinuously from R^{1*} to $\overline{v}^*$.

The equation (11) guarantees that consumers who buy in both states are indifferent between firms, since the expected price at each firm is equated. At the upper boundary of Regime 3 (highest R^1), we have $p_H^1 = R^{1*}$.

What is the lower boundary of Regime 3 (lowest R^1)? Sequential rationality of all consumers with $v < \overline{v}^*$ choosing firm 2 requires $p_L^{2*} \leq R^1$. Therefore, the lower boundary of Regime 3 occurs at $R^1 = p_L^{2*}$ and the corresponding highest p_H^1 consistent with $CE(R^1, R^2)$ in Regime 3 is p_H^c.

4.4 Regime 4

For $R^1 < p_L^{2*}$, we no longer have a cutoff equilibrium to the consumer subgame characterized by $\overline{v}$, above which consumers choose firm 1 and below which consumers choose firm 2. In Regime 4, there is a consumer equilibrium in which we have $p_L^1 = p_L^2 = R^1$ and $p_H^1 = p_H^2 = p_H^c$. Consumers with valuations greater than p_H^c choose each firm with probability one half, so we have the competitive, market clearing outcome in state H. Consumers with valuations between R^1 and p_H^c choose firm 2 with some probability, $\beta < 1$, such that the market clearing price at firm 2 is exactly R^1 in state L. Thus, β is determined by

$$\frac{1}{2}\alpha_L D(p_H^c) + \alpha_L \beta[D(R^1) - D(p_H^c)] = 1. \tag{12}$$

Firm 1 has excess supply in state L, so R^1 binds. Obviously, since the prices in each state are equated across firms, consumers' firm choices are sequentially rational.

For higher values of the reserve price, R^1, more consumers must be choosing firm 2. Therefore, the supremum of reserve prices consistent with Regime 4 occurs when all consumers with valuations between R^1 and p_H^c choose firm 2, $\beta = 1$. When this occurs, the market clearing condition at firm 2 in state L is

$$\frac{1}{2}\alpha_L D(p_H^c) + \alpha_L D(R^1) - \alpha_L D(p_H^c) = 1. \tag{13}$$

The first term on the left side of (13) reflects the fact that half of the consumers with valuations greater than p_H^c choose firm 2; the second and third terms reflect that fact that all consumers with valuations between R^1 and p_H^c choose firm 2. Equation (13) can be simplified to

$$D(R^1) = \frac{1}{\alpha_L} + \frac{1}{2}D(p_H^c). \tag{14}$$

Since, by definition, p_H^c satisfies $\alpha_H D(p_H^c) = 2$, we have

$$\begin{aligned} D(R^1) &= \frac{1}{\alpha_L} + \frac{1}{2} \cdot \frac{2}{\alpha_H}, \text{ or} \\ D(R^1) &= \frac{1}{\alpha_L} + \frac{1}{\alpha_H}. \end{aligned} \tag{15}$$

From (9), the R^1 solving (15) is exactly p_L^{2*}, so p_L^{2*} is the upper limit of R^1 consistent with Regime 4.

The lower limit of R^1 consistent with Regime 4 is p_L^c, which occurs when consumers with valuations between R^1 and p_H^c choose firm 2 with probability, $\beta = \frac{1}{2}$. To see this, when $\beta = \frac{1}{2}$ holds, it follows from (12) that $R^1 = p_L^c$ holds.[11] If $R^1 < p_L^c$ were to hold, (12) would require $\beta < \frac{1}{2}$. With more than half the consumers choosing firm 1, we would have $p_L^1 > p_L^c > R^1$, so R^1 does not bind in state L, but this is a requirement for Regime 4. More to the point, we would have $p_L^2 < p_L^c$, so prices differ across firms, clearly inconsistent with Regime 4.

4.5 Regime 5

For $R^1 \leq p_L^c$, there is a consumer equilibrium in which all consumers choose each firm with probability one half, essentially ignoring the reserve prices since they do not bind in either state. We refer to this regime, which occurs for the lowest reserve prices, as Regime 5.

The analysis above has established the following proposition.

Proposition 1: *For each $R^1 \geq 0$, $CS(R^1, 0)$ has a consumer equilibrium in exactly one of the five regimes, characterized as follows:*

Regime 1: $\widehat{R}^1 \leq R^1$

Regime 2: $R^{1*} < R^1 < \widehat{R}^1$.

Regime 3: $p_L^{2*} \leq R^1 \leq R^{1*}$.

Regime 4: $p_L^c < R^1 < p_L^{2*}$.

Regime 5: $R^1 \leq p_L^c$.

[11] In this boundary case, all consumers are mixing with probability one half, and markets clear at all firms in all states. Thus, this boundary case is in Regime 5 and not Regime 4, since R^1 does not bind.

Remark 1: The previous analysis makes clear that, as R^1 falls, the consumer equilibrium smoothly crosses a threshold from one regime to the next, starting in Regime 1 and ending in Regime 5. There are no gaps or overlaps in the regimes. Since many of the regimes require indifference on the part of consumers, some of whom choose firm 1 and some choose firm 2, there will be multiple consumer equilibria based on which indifferent consumer types choose which firms. However, we know of no other consumer equilibria in which the consumer subgame prices differ from those in the above analysis.

Since p_L^{2*} plays a prominent role in the analysis, it is useful to build intuition for the role of this price. The price, p_L^{2*}, is the market clearing price at firm 2 in state L when half the consumers with $v > p_H^c$ and all the consumers with $v < p_H^c$ choose firm 2. Armed with this intuition, it is clear why the consumer equilibrium transitions smoothly from regime to regime as R^1 is lowered. When R^1 drops below the point at which (4) holds, we can no longer sustain a consumer equilibrium in which all consumers choose firm 2, so we move from Regime 1 to Regime 2, with an interior cutoff, $\overline{v}$. At the lowest R^1 in Regime 2, R^{1*},the cutoff is $\overline{v}^*$ and the measure of consumers at firm 1 in state H is exactly one, so $p_H^2 = p_H^c$ holds. Since consumers with $v < p_H^c$ strictly prefer firm 2, the market clearing price at firm 2 in state L is exactly p_L^{2*}. As R^1 falls below R^{1*}, we transition continuously from Regime 2 into Regime 3. Since the cutoff remains at $\overline{v}^*$, prices at firm 2 do not vary. As R^1 falls within Regime 3, $p_L^1 = R^1$ falls accordingly. To maintain the indifference condition, (11), the market clearing p_H^1 must rise. The lowest that p_L^1 can fall, while maintaining optimal consumer behavior satisfying the cutoff property, is to the price at firm 2, p_L^{2*}. Therefore, the lowest R^1 in Regime 3 is p_L^{2*}. When R^1 falls below p_L^{2*}, we transition to Regime 4. Consumers with $v < p_H^c$ choose firm 2 with probability $\beta \in [\frac{1}{2}, 1]$. At the highest R^1 in Regime 4, we have $\beta = 1$, so all consumers with valuation below p_H^c choose firm 2 and half the consumers with valuation above p_H^c choose firm 2. Thus, the market clearing price at firm 2 in state L is exactly p_L^{2*}. At the lowest R^1 in Regime 4, we have $\beta = \frac{1}{2}$, so half of all consumers choose each firm, and we have the competitive prices at each firm. For even lower R^1, we continue to have competitive prices and half of all consumers choosing each firm, as we continuously transition into Regime 5.

5 Consumer Equilibrium for Arbitrary (R^1, R^2)

In this section, we characterize the consumer equilibrium for each $CS(R^1, R^2)$, where both reserve prices are positive. Assume without loss of generality that $R^1 > R^2$ holds.[12]

First, we establish Lemma 1: If $R^2 < R^1$ and $R^2 \leq p_L^{2*}$ hold, then there is a consumer

[12] If the two reserve prices are equal, there is a consumer equilibrium in which all consumers choose each firm with probability $\frac{1}{2}$.

equilibrium which is identical to $CE(R^1, 0)$. That is, we will show that with $R^2 < R^1$ and $R^2 \leq p_L^{2*}$, consumers essentially ignore R^2 and they treat it exactly as they treat $R^2 = 0$. Thereafter, we need only consider the cases where both $R^1 > R^2$ and $R^2 > p_L^{2*}$ hold.

Lemma 1: *If $R^2 < R^1$, and $R^2 \leq p_L^{2*}$ hold, then there is a consumer equilibrium $CE(R^1, R^2)$ in which consumer behavior and prices are exactly as in $CE(R^1, 0)$.*

Now we consider consumer equilibrium for $CS(R^1, R^2)$ such that $R^1 > R^2 \geq p_L^{2*}$ holds. Note that there are no consumer equilibria satisfying $R^1 > R^2 \geq p_L^{2*}$ in Regime 4 or Regime 5, so we can restrict attention to Regimes 1-3. The roadmap of the analysis is to consider each value of $R^1 > p_L^{2*}$, and for each such value of R^1, consider each $R^2 \in [p_L^{2*}, R^1)$. Proposition 2 characterizes $CE(R^1, R^2)$ for all (R^1, R^2). The proof consists of characterizing the consumer equilibria in Regimes 1-3 for the cases in which Lemma 1 does not apply. For each value of R^1, we analyze the consumer equilibrium as we change R^2 in $[p_L^{2*}, R^1)$. From Lemma 1 and Proposition 2, it is clear that the regimes in which $CE(R^1, R^2)$ exists covers (R^1, R^2), and that there is no overlap. Each (R^1, R^2) fits into exactly one of the regimes.

Proposition 2: *For each $R^1, R^2 \geq 0$ with $R^1 > R^2$, $CS(R^1, R^2)$ has a consumer equilibrium in exactly one of the five regimes, characterized as follows:*

Regime 1: Either $R^1 \geq D^{-1}(\frac{1}{\alpha_H})$, or $R^1 \in [\widehat{R}^1, D^{-1}(\frac{1}{\alpha_H}))$ and $R^2 \leq R^1 - \frac{\pi_H \alpha_H}{\pi_L \alpha_L}(D^{-1}(\frac{1}{\alpha_H}) - R^1)$.

Regime 2: Either (i) $R^1 \in [\widehat{R}^1, D^{-1}(\frac{1}{\alpha_H}))$ and $R^2 > R^1 - \frac{\pi_H \alpha_H}{\pi_L \alpha_L}(D^{-1}(\frac{1}{\alpha_H}) - R^1)$ and $R^2 \leq R^1 - \frac{\pi_H \alpha_H}{\pi_L \alpha_L}(p_H^c - R^1)$ or (ii) $R^1 \in [R^{1*}, \widehat{R}^1)$ and $R^1 \geq p_H^c$, or (iii) $R^1 \in [R^{1*}, \widehat{R}^1)$ and $R^1 < p_H^c$ and $R^2 \leq R^1 - \frac{\pi_H \alpha_H}{\pi_L \alpha_L}(p_H^c - R^1)$.

Regime 3: Either (i) $R^1 \in [R^{1*}, D^{-1}(\frac{1}{\alpha_H}))$ and $R^1 < p_H^c$ and $R^2 > R^1 - \frac{\pi_H \alpha_H}{\pi_L \alpha_L}(p_H^c - R^1)$ or (ii) $R^1 \in [p_L^{2*}, R^{1*}]$.

Regime 4: $p_L^c < R^1 < p_L^{2*}$ (identical to $CS(R^1, 0)$ for these values).

Regime 5: $R^1 \leq p_L^c$ (identical to $CS(R^1, 0)$ for these values).

From Proposition 2, we see that every consumer equilibrium is quasi-efficient, in the sense that whenever a consumer with valuation v' consumes in a given state, all consumers with valuation $v'' > v'$ consume in that state. In Regime 1, all consumers choose firm 2, and p_H^2 and p_L^2 determine the valuation above which a consumer consumes. In Regimes 2 and 3, all consumers with $v > \overline{v}$ choose firm 1 and consume, so again p_H^2 and p_L^2 determine the valuation above which a consumer consumes. In Regimes 4 and 5, the prices are the same at each firm, and all consumers consume if and only if their valuation is above the relevant price. This leads to the following corollary.

Corollary to Proposition 2: *Each $CE(R^1, R^2)$ is quasi-efficient, in the sense that consumption is allocated to the consumers with the highest valuations. The only source of inefficiency is that some capacity is not allocated when a reserve price binds.*

6 The Reserve Price Stage

The first question we investigate in this section is when zero reserve prices is consistent with subgame perfect equilibrium. Throughout, we consider $CE(R^1, R^2)$ as characterized in Proposition 2.

Proposition 3: *Fix the consumer equilibria as specified in Proposition 2. Under conditions (1) and (2) below, $R^1 = R^2 = 0$ are SPE strategies of the reserve price game, and any SPE where firms follow pure strategies is outcome-equivalent to the equilibrium of Γ with $R^1 = R^2 = 0$.*

Conditions:
(1) The elasticity of demand at p_L^c is greater than $\frac{1}{2}$, so we have

$$-\frac{D'(p_L^c)p_L^c}{D(p_L^c)} \geq \frac{1}{2}.$$

(2) Demand is elastic at p_L^{2*}, so we have

$$-\frac{D'(p_L^{2*})p_L^{2*}}{D(p_L^{2*})} > 1.$$

The proof of Proposition 3 (in the Appendix) relies on two lemmas. Lemma 2 shows that with sufficient elasticity at the two prices p_L^c and p_L^{2*} (Conditions 1 and 2, respectively, of Proposition 3), the best response to $R^2 = 0$ is $R^1 = 0$, and vice versa. Lemma 3 then shows that under Condition 2 of Proposition 3, i.e. if demand is elastic at p_L^{2*}, and when the consumer equilibrium is as characterized in Proposition 2, then there cannot be a SPE with R^1 and R^2 both strictly positive pure strategies and one or both of the reserve prices binding in either state.

The sketch of the proof for Lemma 2 is as follows. Consider one of the reserve prices, say R^2, fixed at 0. Clearly, setting an R^1 in Regime 1 is stricly worse than $R^1 = 0$, while an R^1 in Regime 5 is equivalent to $R^1 = 0$. The proof of Lemma 2 first shows that with sufficient elasticity at p_L^c (Condition 1) the profit from increasing R^1 above p_L^c into the interior of Regime 4 drives the profit of firm 1 lower than from setting $R^1 = 0$. Since elasticity is always increasing in price (under our maintained assumption) the profit disadvantage of an R^1 in Regime 4, relative to $R^1 = 0$, keeps increasing as R^1 is increased and brought closer to the lower bound of Regime 3. Thus, for R^1 equal to the lower bound of R^1 in Regime 3, the profit is strictly lower than from $R^1 = 0$.

Then, we show that firm 1's profit in Regime 3 remains constant for all R^1 in Regime 3 (and therefore strictly lower than the profit from $R^1 = 0$), which in-turn is equal to the profit from setting $R^1 = R^{1*}$. Condition 2, i.e., elastic demand at p_L^{2*}, is sufficient to show that the highest profit for all R^1 in Regime 2 with $R^1 \geq R^{1*}$ occurs at $R^1 = R^{1*}$. This is because with elastic demand, raising R^1 costs more in quantity decline than it pays in price increase. Thus, we show that under Conditions 1 and 2, $R^1 = 0$ is a best response to $R^2 = 0$.

Lemma 3 shows the "outcome uniqueness" of the $R^1 = R^2 = 0$ SPE among equilibria where firms follow pure strategies and the consumer subgame is as characterized in Proposition 2. Here, by the outcome of an SPE we mean the consumers' choices, and the sale prices at each firm in each state. In Lemma 3, we rule out the possibility of both R^1 and R^2 strictly greater than p_L^{2*}. The proof shows that for any (R^1, R^2) with both R^1 and R^2 strictly greater than p_L^{2*}, some unilateral profitable deviation is available to one of the firms. Outcome uniqueness then holds, since by Lemma 2, if $R^2 < p_L^{2*}$ holds, then there cannot be an equilibrium with $R^1 > p_L^{2*}$ as firm 1 would not be best-responding. Finally, we show that if both R^1 and R^2 are less than p_L^{2*}, they must both be less than p_L^c, or else the firm with the higher reserve price is better off undercutting (both firms are better off undercutting if they set the same reserve price between p_L^c and p_L^{2*}).

Both conditions of Proposition 3 are elasticity conditions. Condition 2 allows us to focus on $R^1 = R^{1*}$ in Regime 2. A careful reading of the proof indicates that Condition 2 is stronger than what is needed.[13]

We show in Proposition 4 below that, when Condition 1 (in Proposition 3) does not hold and consumer equilibrium is as characterized in Proposition 2, then $R^1 = R^2 = 0$ is

[13] Rather then requiring demand to be elastic at p_L^{2*} (and therefore elastic at all prices greater than p_L^{2*}), it is sufficient to ensure that the left side of (48) is positive (see the Appendix). Demand does not necessarily have to be elastic at p_L^2 and p_H^2, since the condition resembles a weighted average of demand elasticities across the two prices.

inconsistent with SPE. If reserve prices do not bind, there is always a profitable deviation to a binding reserve price.

Proposition 4: *Fix the consumer equilibria as specified in Proposition 2. If the elasticity of demand at* p_L^c *is less than* $\frac{1}{2}$*, so we have*

$$-\frac{D'(p_L^c)p_L^c}{D(p_L^c)} < \frac{1}{2}, \tag{16}$$

then in every subgame perfect equilibrium in which firms follow pure strategies, at least one firm chooses a reserve price that binds in state L.

When the condition of Proposition 4 is met, so $R^1 = R^2 = 0$ is inconsistent with equilibrium, the game resembles a Hawk-Dove game. When, say, firm 1 sets a binding reserve price and firm 2's reserve price is not binding, most of the benefit goes to firm 2. The intuition is clearest when we have an equilibrium of the form $(R^1, 0)$ in Regime 4. Here, both firms set the same price, R^1, in state L, and both firms set the same price, p_H^c, in state H. Firm 2 sells all its output while firm 1 does not sell all its output in state L. The reason that firm 1 is best responding is that demand in state L is sufficiently inelastic that firm 1 is better off with the higher price in state L, even though it does not sell all its output.

Due to the Hawk-Dove nature of the game, it would seem that, when a binding R^1 is a best response to $R^2 = 0$, then $R^2 = 0$ is a best response to R^1. We show in Proposition 5 that, whenever R^1 is a best response to $R^2 = 0$ and $(R^1, 0)$ in Regime 4, then $R^2 = 0$ is a best response to R^1.

Proposition 5: *Suppose* $R^1 \in (\ p_L^c, p_L^{2*})$ *is a best response to* $R^2 = 0$*, so* $(R^1, 0)$ *is in Regime 4. Then* $(R^1, 0)$ *are equilibrium reserve prices.*

Proposition 6 (below) provides two conditions, related to the elasticity of demand, which together are sufficient for an R^1 in Regime 4 to be a best response to $R^2 = 0$. Then, we utilise Proposition 5 to conclude that under these conditions $(R^1, 0)$ are firm strategies in a subgame perfect equilibrium.

Proposition 6. *If (1) the elasticity of demand at* p_L^c *is strictly lower than* $\frac{1}{2}$*, so we have*

$$-\frac{D'(p_L^c)p_L^c}{D(p_L^c)} < \frac{1}{2},$$

and if (2) demand is elastic at p_L^{2*}*, so we have*

$$-\frac{D'(p_L^{2*})p_L^{2*}}{D(p_L^{2*})} > 1,$$

then $(R^1, 0)$ *are equilibrium reserve prices for some* R^1 *such that* $R^1 \in (\ p_L^c, p_L^{2*})$ *holds.*

Proposition 7 provides a sense in which more demand uncertainty leads to a softening of competition. Define p_α^c to be the solution to $\alpha D(p) = 2$. An equivalent way of writing our demand functions in the two states is in terms of α and ε, where we have

$$\alpha_L = \alpha - \varepsilon \text{ and } \alpha_H = \alpha + \varepsilon. \tag{17}$$

Then the case of no demand uncertainty corresponds to $\varepsilon = 0$, and as we increase ε, we increase the amount of demand uncertainty. If the price elasticity of demand is greater than one half at p_α^c and less than one half at some price below p_α^c, then prices are competitive when there is no uncertainty but when there is sufficient uncertainty, some prices exceed their competitive levels.

Proposition 7: *Suppose demand is represented by (17), that the price elasticity of demand is greater than one half at* p_α^c*, and that the price elasticity of demand is less than one half at some price less than* p_α^c*. Then in any SPE in which firms follow pure strategies and the consumer equilibrium is as specified in Proposition 2, prices are competitive when there is no demand uncertainty* ($\varepsilon = 0$) *and some prices exceed their competitive levels when there is enough demand uncertainty* (ε *is high enough).*

The intuition behind Proposition 7 is that, with demand uncertainty, the condition ruling out competitive prices is based on the elasticity of demand at the market clearing price in state L. The more uncertainty there is, the lower p_L^c is therefore the more inelastic demand is at p_L^c.

7 An Example

In this section, we solve a family of examples with linear demand, $D(v) = 1 - v$. Uncertainty is captured by the parameter, ε, where $\alpha_L = 3.1 - \varepsilon$ and $\alpha_H = 3.1 + \varepsilon$. We assume that each state occurs with probability one half. We only consider values of ε less than or equal to 1.1, because the non-negativity constraint on the competitive market clearing price in state L binds for higher ε. The market clearing prices are given by

$$p_H^c = 1 - \frac{2}{3.1+\varepsilon} \text{ and } p_L^c = 1 - \frac{2}{3.1-\varepsilon}$$

and we have

$$p_L^{2*} = 1 - \frac{1}{3.1+\varepsilon} - \frac{1}{3.1-\varepsilon}.$$

When there is no uncertainty, $\varepsilon = 0$, the price elasticity of demand is slightly greater than one half at the market clearing price. By Proposition 7, prices are competitive when there is no demand uncertainty. Our approach is to fix $R^2 = 0$ and find firm 1's best response by considering choices in Regime 2, Regime 4, and Regime 5. Since firm 1's profit in Regime 3 is the same as at $R^1 = R^{1*}$ at the "bottom" of Regime 2, this case does not need to be considered. Once we have firm 1's best response for a given ε, we verify that firm 2 is best responding to firm 1 by choosing $R^2 = 0$.

Consider $(R^1, 0)$ in Regime 2. Firm 1's profit, as a function of $\overline{v}$ and R^1, is given by $3.1(1-\overline{v})R^1$. Using (5), (6), and (7), firm 1's profit can be written as a function of R^1 only, given by $2.1 - 6.2R^1$. Since this expression is decreasing in R^1, the optimal R^1 within Regime 2 occurs at $R^1 = R^{1*}$, at the boundary between Regime 2 and Regime 3. Firm 1's profits at R^{1*} are the same as in Regime 3, which in turn are the same as firm 1's profits at the "top" of Regime 4 with $R^1 = p_L^{2*}$. Therefore, firm 1 has a best response to $R^2 = 0$ either in Regime 4 (binding R^1) or Regime 5 (non-binding R^1).

Now consider $(R^1, 0)$ in Regime 4. Firm 1's profit is derived from (35), given by

$$\frac{1}{2} - \frac{1}{3.1+\varepsilon} + \frac{(3.1-\varepsilon)R^1(1-R^1)}{2} - \frac{R^1}{2}. \tag{18}$$

Differentiating the profit with respect to R^1, setting the expression equal to zero, and solving yields our candidate for an interior solution:

$$R^1 = \frac{2.1-\varepsilon}{2(3.1-\varepsilon)}. \tag{19}$$

Because the profit expression is quadratic in R^1, (19) determines the optimal R^1 in Regime 4 whenever it is between p_L^c and p_L^{2*}. It is straightforward to verify that R^1 lies in this range for any $\varepsilon \in [0.1, 1.1]$.

For $\varepsilon < 0.1$, (19) yields a value of R^1 than is less than p_L^c, which implies that firm 1 receives higher profits in Regime 5 where R^1 is not binding. For this range, we have an equilibrium in which both firms set a reserve price of zero, as each firm is best responding to the other.

For $\varepsilon > 0.1$, we can substitute (19) into (18), yielding a complicated expression for firm

1's highest profit in Regime 4, as a function of ε. This profit can be compared to firm 1's profit in Regime 5, with a non-binding reserve price. The profit in Regime 5 is given by

$$1 - \frac{1}{3.1 + \varepsilon} - \frac{1}{3.1 - \varepsilon}.$$

It turns out that, for all $\varepsilon \in [0.1, 1.1]$, firm 1's profit from choosing the reserve price (19) is greater than the profit with a non-binding reserve price. Therefore, (19) is a best response to $R^2 = 0$. At this reserve price, $CE(R^1, 0)$ is in Regime 4, so Proposition 6 implies that firm 2 is best-responding to R^1 and $(R^1, 0)$ are equilibrium reserve prices.

To pin the example parameters completely, if $\varepsilon = 0.5$ holds, in equilibrium we have

$$\begin{aligned} R^1 &= 0.30769 \text{ and } R^2 = 0 \\ p_L^1 &= p_L^2 = 0.30769 \text{ and } p_H^1 = p_H^2 = 0.44444. \end{aligned}$$

Profits for firm 1 are 0.34530 and profits for firm 2 are 0.37607. The equilibrium can be compared to the outcome with zero reserve prices, where the prices in state L would be 0.23077 and profits for each firm would be 0.33761.

We have thus characterized the equilibrium for this family of examples. When $\varepsilon < 0.1$ holds, there is not enough uncertainty to support an equilibrium with a binding reserve price. When $\varepsilon > 0.1$ holds, demand is sufficiently inelastic in state L to induce one of the firms to set a reserve price that binds in state L.

8 Concluding Remarks

We study duopoly competition by firms who set reserve prices in the presence of demand uncertainty, followed by active consumers choosing one of the firms and participating in its auction. We characterize the consumer equilibrium following every reserve price pair, which is surprisingly complicated given the simplicity of the strategy spaces. There are five regimes or types of consumer equilibria, in which consumers sort themselves based on their valuations. The equilibrium reserve prices depend on the price elasticity of demand at the hypothetical competitive equilibrium prices in each state. If demand is sufficiently elastic at price p_L^c, then equilibrium reserve prices are zero and the allocation is what would prevail at the competitive equilibrium of a centralized market. If demand is sufficiently inelastic at price p_L^c and elastic at price p_H^c, then the equilibrium is in Regime 4, with one firm choosing a zero reserve price and the other firm choosing a reserve price that binds only in the low state. We do not have results for the case in which demand is inelastic at price p_H^c, but we conjecture that firms must be choosing mixed strategies in equilibrium. Proposition 7 provides a sense in which more demand uncertainty can serve to soften competition. More uncertainty serves to increase p_H^c and decrease p_L^c, which, in turn, makes demand at price p_L^c more inelastic. If demand is inelastic enough in state L, equilibrium allocations are no longer competitive.

One of our motivations for studying this model is that it might relate to increasingly common competition by firms who use "surge pricing." The surge price could be the auction price in the high demand state, while the normal price is the auction price in the low demand state. The price in the low demand state could be market clearing (Regime 5), or it could reflect a binding reserve price set by one of the firms (Regime 4). In both regimes, we have $p_L^1 = p_L^2$ and $p_H^1 = p_H^2$, so consumers receive the same price whichever firm they choose. The only way to identify which regime prevails would be to observe whether one of the firms has excess capacity in the low demand state. Our analysis relates to Uber Eats/Doordash competition, especially when the restaurants are located in a town center or restaurant district. In such circumstances, an auction framework could be a good approximation.

When deregulation and smart home technology advance, our model could be relevant for electricity markets. Transmission must be delivered on a central grid but providers could contract directly with consumers and potentially hold auctions with reserve prices. What is interesting about this (future) market is that governments could decide whether to allow providers to sign up consumers for their auctions in advance or, instead, have a centralized auction, where as a function of a single market clearing price, suppliers choose how much power to offer and consumers decide the price at which to shut off power. Our simulations for the centralized model (outside the scope of this paper) indicate that reserve price competition often yields higher economic welfare than a centralized auction in which firms can withhold capacity.

9 Appendix

Understanding Regime 3

In this subsection, we show that a consumer equilibrium in Regime 3, with p_H^1 (between R^1 and p_H^c) satisfying the required indifference condition, is the limit of consumer equilibria of the finite economy as the number of consumers approaches infinity. Along the sequence, the expected price at firm 1, conditional on state H, converges to p_H^1. We should interpret p_H^1 the same way in the continuum economy, because, although p_H^1 is one of the continuum of market clearing prices, it cannot be the highest rejected bid.

Consider a finite economy of "size" n, defined as follows. As in the continuum economy, there are two aggregate demand states, H and L, with prior probabilities π_H and π_L. Consumers demand either zero or one unit of the good. In state H, there are $\alpha_H n$ active consumers, with valuations drawn independently, such that the probability of receiving a valuation greater than or equal to v is $D(v)$. In state L, there are $\alpha_L n$ active consumers, with valuations drawn independently, such that the probability of receiving a valuation greater than or equal to v is $D(v)$. Again, we assume that the process that determines the activity and valuation of consumers is symmetric across "potential" consumers, so using Bayes' rule, the probability of state s, conditional on being an active consumer with valuation v, is given by (1). Each firm has a supply or capacity of n units. By the law of large numbers, the market clearing prices converge in probability to (2) and (3) as n approaches infinity.

Suppose we have reserve prices $(R^1, 0)$ in the large finite economy such that $CE(R^1, 0)$ is in Regime 3 in the continuum economy. That is, suppose we have $p_L^{2*} < R^1 < R^{1*}$.

Claim: Suppose we have $p_L^{2*} < R^1 < R^{1*}$. For all sufficiently small $\varepsilon > 0$, there is an N, such that $n > N$ implies there is a consumer equilibrium characterized by a cutoff, $\overline{v}^n$, where consumers with higher valuations choose firm 1 and consumers with lower valuations choose firm 2.

Proof of Claim. Fix a small ε and and consider cutoff strategies characterized by $\overline{v}$. Because $CE(R^1, 0)$ is in Regime 3 in the continuum economy, there is $p_H^1 \in (R^1, p_H^c)$ such that

$$\pi_H \alpha_H p_H^c + \pi_L \alpha_L p_L^{2*} = \pi_H \alpha_H p_H^1 + \pi_L \alpha_L R^1 \tag{20}$$

holds. For the large finite economy, if we have $\overline{v} = \overline{v}^* - \varepsilon$, then for sufficiently large n, (i) the price at firm 2 in state H is almost surely less than p_H^c and the price at firm 2 in state L is almost surely less than p_L^{2*}, and (ii) there will almost surely be excess demand at firm 1 in state H and excess supply in state L, so the price at firm 1 in state H is almost surely equal to $(\overline{v}^* - \varepsilon)$ and the price at firm 1 in state L is almost surely equal to R^1. Denote prices in the large finite economy with tildas. From (20) and $p_H^1 < \overline{v}^* - \varepsilon$, we have

$$E[\pi_H \alpha_H \widetilde{p}_H^2 + \pi_L \alpha_L \widetilde{p}_L^2] < E[\pi_H \alpha_H \widetilde{p}_H^1 + \pi_L \alpha_L \widetilde{p}_L^1]. \tag{21}$$

If we have $\overline{v} = \overline{v}^* + \varepsilon$, then for sufficiently large n, (i) the price at firm 2 in state H is almost surely greater than p_H^c and the price at firm 2 in state L is almost surely greater than p_L^{2*}, and (ii) there will almost surely be excess supply at firm 1 in state H and in state L, so the price at firm 1 in state H and in state L is almost surely equal to R^1. From (20) and $p_H^1 > R^1$, we have

$$E[\pi_H \alpha_H \widetilde{p}_H^2 + \pi_L \alpha_L \widetilde{p}_L^2] > E[\pi_H \alpha_H \widetilde{p}_H^1 + \pi_L \alpha_L \widetilde{p}_L^1]. \tag{22}$$

By continuity and the fact that expected prices move monotonically with $\overline{v}$, there must be a unique cutoff, which we denote by $\overline{v}^n$, for which we have

$$E[\pi_H \alpha_H \widetilde{p}_H^2 + \pi_L \alpha_L \widetilde{p}_L^2] = E[\pi_H \alpha_H \widetilde{p}_H^1 + \pi_L \alpha_L \widetilde{p}_L^1]. \tag{23}$$

It also follows that $E(\widetilde{p}_H^2) > E(\widetilde{p}_H^1)$ and $E(\widetilde{p}_L^2) < E(\widetilde{p}_L^1)$ hold, so all consumers make sequentially rational choices and we have a unique consumer equilibrium (the notation suppresses the dependence on n). ■

In the consumer equilibrium, the cutoff converges to the cutoff in the continuum economy, $\overline{v}^n \to \overline{v}^*$. Because the limiting cutoff is $\overline{v}^*$, by the law of large numbers, the prices, $\widetilde{p}_H^2$, $\widetilde{p}_L^2$, and $\widetilde{p}_L^1$ converge in probability: $\widetilde{p}_H^2 \to p_H^c$, $\widetilde{p}_L^2 \to p_L^{2*}$, and $\widetilde{p}_L^1 \to R^1$. By (20) and (23), the expectation of $\widetilde{p}_H^1$ converges to p_H^1, $E(\widetilde{p}_H^1) \to p_H^1$. However, significant uncertainty about $\widetilde{p}_H^1$ remains when n is large. The law of large numbers tells us that the fraction of excess demand or excess supply is converging to zero, but $\widetilde{p}_H^1$ depends on whether there is a small amount of excess demand or excess supply. In the former case, $\widetilde{p}_H^1$ is approximately $\overline{v}^*$ (the valuation of the highest rejected bid), and in the later case, $\widetilde{p}_H^1$ is exactly R^1.

Proof of Lemma 1

We show Lemma 1 in two steps. First, we provide arguments that Lemma 1 holds when R^1 is less than p_L^{2*}. Next, we provide arguments for the case where $R^1 > p_L^{2*}$ holds.

If $R^1 \leq p_L^{2*}$ holds, then $CE(R^1, 0)$ is either in Regime 4 or Regime 5. In Regime 4, we have $p_L^1 = p_L^2 = R^1$ and $R^1 < p_H^1 = p_H^2 = p_H^c$. Thus, for R^1 in Regime 4 given $R^2 = 0$, even if R^2 were to be increased from 0 to a positive R^2 with $R^2 < R^1$ (as assumed in Lemma 1), in the resulting $CE(R^1, R^2)$, such an R^2 would not bind in either state, and therefore not affect the CE relative to $CE(R^1, 0)$. Similarly, for R^1 in Regime 5 given $R^2 = 0$, we have $R^1 \leq p_L^c$ and R^1 does not bind in either state. Thus, replacing $R^2 = 0$ with a positive R^2 lower than R^1 does not affect the CE relative to $CE(R^1, 0)$ when $R^1 \leq p_L^{2*}$ holds.

Next, we consider $R^1 > p_L^{2*}$. Note that $R^1 > p_L^{2*}$ implies $CE(R^1, 0)$ is in Regime 1, 2, or 3. We now show that $CE(R^1, 0)$ in Regimes 1, 2, or 3 yields $p_L^2 \geq p_L^{2*}$. Thus, replacing $R^2 = 0$ with a positive R^2 weakly lower than p_L^{2*} implies that in $CE(R^1, R^2)$ consumer behavior and prices are exactly as in $CE(R^1, 0)$.

First consider $CE(R^1, 0)$ with R^1 in Regime 1. Prices at firm 2 are given by

$$p_H^2 = D^{-1}(\frac{1}{\alpha_H}) \text{ and } p_L^2 = D^{-1}(\frac{1}{\alpha_L}),$$

and both $p_H^2 = D^{-1}(\frac{1}{\alpha_H})$ and $p_L^2 = D^{-1}(\frac{1}{\alpha_L})$ are greater than $p_L^{2*} = D^{-1}(\frac{1}{\alpha_H} + \frac{1}{\alpha_L})$. Next, consider $CE(R^1, 0)$ with R^1 in Regime 2. At the lowest $\overline{v}$ consistent with Regime 2, $\overline{v}^*$, we have $\alpha_H D(\overline{v}^*) = 1$, which implies that the lowest p_H^2 in Regime 2 is p_H^c, and the lowest

p_L^2 in regime 2 is p_L^{2*}. Third, consider $CE(R^1, 0)$ with R^1 in Regime 3. The prices at firm 2 are given by $p_H^2 = p_H^c$ and $p_L^2 = p_L^{2*}$. Thus, in $CE(R^1, 0)$ for R^1 in either of Regimes 1-3, p_L^2 is weakly greater than p_L^{2*}. It follows from $R^2 \leq p_L^{2*}$ that R^2 does not bind, and in $CE(R^1, R^2)$ consumer behavior and prices are exactly as in $CE(R^1, 0)$. ■

Proof of Proposition 2

Regime 1 (with $R^2 \geq p_L^{2*}$)

Recall that $CE(R^1, 0)$ is in Regime 1 if we have $\widehat{R}^1 \leq R^1$. For $R^1 < \widehat{R}^1$, $CE(R^1, R^2)$ cannot be in Regime 1. Higher R^2 can only increase the attractiveness of firm 1, so it cannot be the case that all consumers choose firm 2. Thus, we consider $R^1 \geq \widehat{R}^1$.

Consider $R^1 \geq D^{-1}(\frac{1}{\alpha_H})$. In this case, for any R^2 such that $R^2 < R^1$ holds, $CE(R^1, R^2)$ is in Regime 1. To demonstrate this, we need to show

$$\pi_H \alpha_H p_H^2 + \pi_L \alpha_L p_L^2 \leq \pi_H \alpha_H R^1 + \pi_L \alpha_L R^1, \tag{24}$$

with $p_H^2 = max\{R^2, D^{-1}(\frac{1}{\alpha_H})\}$ and $p_L^2 = max\{R^2, D^{-1}(\frac{1}{\alpha_L})\}$. Inequality (24) holds, since by assumption we have $R^1 > R^2$ and $R^1 > D^{-1}(\frac{1}{\alpha_H})$, which also means $R^1 > D^{-1}(\frac{1}{\alpha_L})$.

Note that, for $R^1 \geq \widehat{R}^1$ and $R^2 \leq D^{-1}(\frac{1}{\alpha_L})$, the analysis is identical to $CE(R^1, 0)$, since firm 2's reserve price does not bind and firm 2 gets excess demand in both states. In this case, $CE(R^1, R^2)$ is in Regime 1.

Now consider $R^1 \in [\widehat{R}^1, D^{-1}(\frac{1}{\alpha_H})]$ and $R^2 > D^{-1}(\frac{1}{\alpha_L})$. Given $R^1 \in [\widehat{R}^1, D^{-1}(\frac{1}{\alpha_H})]$, $CE(R^1, R^2)$ is in Regime 1 for $R^2 \in (D^{-1}(\frac{1}{\alpha_L}), R^1)$ if and only if the expected price at firm 1 is higher. The condition is

$$\pi_H \alpha_H D^{-1}(\frac{1}{\alpha_H}) + \pi_L \alpha_L R^2 \leq \pi_H \alpha_H R^1 + \pi_L \alpha_L R^1.$$

Rearranging yields:

$$R^2 \leq R^1 - \frac{\pi_H \alpha_H}{\pi_L \alpha_L}(D^{-1}(\frac{1}{\alpha_H}) - R^1). \tag{25}$$

Thus, for $R^1 \in [\widehat{R}^1, D^{-1}(\frac{1}{\alpha_H})]$ and R^2 satisfying (25), $CE(R^1, R^2)$ is in Regime 1.[14]

Regime 2 (with $R^2 \geq p_L^{2*}$)

For $CE(R^1, R^2)$ in Regime 2, firm 1 has excess supply with R^1 binding in both states, and there is a cutoff $\overline{v}$ such that each consumer with valuation above $\overline{v}$ goes to firm 1 and each

[14] For $R^1 \in [\widehat{R}^1, D^{-1}(\frac{1}{\alpha_H})]$ and R^2 "close enough" to R^1, in the sense of not satisfying (25), $CE(R^1, R^2)$ is not in Regime 1, even though $CE(R^1, 0)$ is in Regime 1.

consumer with valuation below $\overline{v}$ goes to firm 2. For all types with valuation such that they can purchase from any firm in any state, we have the following indifference condition:

$$\pi_H \alpha_H p_H^2 + \pi_L \alpha_L p_L^2 = \pi_H \alpha_H R^1 + \pi_L \alpha_L R^1. \tag{26}$$

Here p_H^2 is the maximum of R^2 and the solution to

$$\alpha_H D(p_H^2) - \alpha_H D(\overline{v}) = 1, \tag{27}$$

while p_L^2 is the maximum of R^2 and the solution to

$$\alpha_L D(p_L^2) - \alpha_L D(\overline{v}) = 1. \tag{28}$$

And finally we have

$$\alpha_H D(\overline{v}) < 1, \tag{29}$$

to ensure that firm 1 has excess supply. Note that R^2 cannot bind in both high and low states. This is because given $R^2 < R^1$, R^2 binding in both states would mean all consumers strictly prefer firm 2.

Case (i) in the statement of Proposition 2: Consider the case in which $R^1 \in [\widehat{R}^1, D^{-1}(\frac{1}{\alpha_H}))$ holds, so $CE(R^1, 0)$ is in Regime 1. If (25) holds, then for any cutoff $\overline{v}$, the left side of (26) is strictly less than the right side, so $CE(R^1, R^2)$ cannot be in Regime 2. However, if (25) does not hold, so we have

$$R^2 > R^1 - \frac{\pi_H \alpha_H}{\pi_L \alpha_L}(D^{-1}(\frac{1}{\alpha_H}) - R^1),$$

then if the cutoff is at the highest valuation, $\overline{v} = b$, prices at firm 2 are $p_H^2 = D^{-1}(\frac{1}{\alpha_H})$ and $p_L^2 = R^2$, so the left side of (26) is strictly greater than the right side. To be in Regime 2, it must also be the case that, if the cutoff is $\overline{v}^*$, the left side of (26) is less than the right side. By continuity, there would be a cutoff between $\overline{v}^*$ and b such that (26) is satisfied. With a cutoff of $\overline{v}^*$, the left side of (26) is $\pi_H \alpha_H p_H^c + \pi_L \alpha_L R^2$ if R^2 binds in state L, and even lower if R^2 does not bind in state L. The

Therefore, if

$$R^2 \leq R^1 - \frac{\pi_H \alpha_H}{\pi_L \alpha_L}(p_H^c - R^1) \tag{30}$$

holds, then the left side of (26) is less than the right side and $CE(R^1, R^2)$ is in Regime 2.

Cases (ii) and (iii) in the statement of Proposition 2: Consider the case in which $R^1 \in [R^{1*}, \widehat{R}^1)$ holds, so $CE(R^1, 0)$ is in Regime 2. Define $\hat{p}_L^2$ to be the corresponding value of p_L^2, solving (26)-(29) when $R^2 = 0$ holds. $CE(R^1, R^2)$ cannot be in Regime 1 for any R^2.

Therefore, if the cutoff is at the highest valuation, $\overline{v} = b$, the left side of (26) is strictly greater than the right side, irregardless of whether or not (25) holds. However, we must still verify that, if the cutoff is $\overline{v}^*$, the left side of (26) is less than the right side. There are two subcases, depending on whether we have $R^1 \geq p_H^c$ or $R^1 < p_H^c$.

If we have $R^1 \in [R^{1*}, \widehat{R}^1)$ and $R^1 \geq p_H^c$, and if the cutoff is $\overline{v}^*$, then the left side of (26) is less than the right side for any R^2. The reason is that we have $p_H^2 = p_H^c \leq R^1$ and $p_L^2 = \max[R^2, p_L^{2*}] \leq R^1$.

If we have $R^1 \in [R^{1*}, \widehat{R}^1)$ and $R^1 < p_H^c$, and if the cutoff is $\overline{v}^*$, then the left side of (26) is $\pi_H \alpha_H p_H^c + \pi_L \alpha_L R^2$ if R^2 binds in state L, and even lower if R^2 does not bind in state L. Therefore, if (30) holds, then the left side of (26) is less than the right side and $CE(R^1, R^2)$ is in Regime 2.

Regime 3 (with $R^2 \geq p_L^{2*}$)

For $CE(R^1, R^2)$ in Regime 3, the threshold remains constant at $\overline{v}^*$, with consumers above $\overline{v}^*$ going to firm 1 and those below $\overline{v}^*$ going to firm 2. Note that it cannot be the case that R^2 binds in both states, since otherwise the expected price at firm 2 is strictly lower, which makes a $\overline{v}^*$ cutoff equilibrium unsustainable. Thus, R^2 can bind only in the low state, if it binds at all, with $p_H^2 = p_H^c$. We also require $p_H^1 < p_H^c$, or else firm 2 would always have the lower price. Since the threshold is at $\overline{v}^*$, the demand at firm 1 is exactly equal to the supply, and $p_H^1 \in [R^1, p_H^c]$ represents the expected price with a distribution over the realizations R^1 and $\overline{v}^*$. $CE(R^1, R^2)$ in Regime 3 is characterized by

$$\pi_H \alpha_H p_H^c + \pi_L \alpha_L p_L^2 = \pi_H \alpha_H p_H^1 + \pi_L \alpha_L R^1, \text{ where} \tag{31}$$

$$p_L^2 = \max[R^2, p_L^{2*}], \tag{32}$$

$$p_H^1 \in [R^1, p_H^c]. \tag{33}$$

Case (i) in the statement of Proposition 2: Consider the case in which $R^1 \in [R^{1*}, D^{-1}(\frac{1}{\alpha_H}))$ holds, so $CE(R^1, 0)$ is in Regime 1 or Regime 2 and the condition, $R^1 < p_H^c$, is satisfied. Notice that the right side of (31) is greater than the left side when we set $p_H^1 = p_H^c$, because $R^1 \geq \max\{R^2, p_L^{2*}\}$ holds. We also require the left side of (31) to be greater than the right side when we set $p_H^1 = R^1$, If we can show that, then $CE(R^1, R^2)$ is in Regime 3, because by continuity some choice of $p_H^1 \in [R^1, p_H^c]$ will cause (31) to hold. When R^2 does not bind in state L, the left side of (31) cannot be greater than the right side when we set $p_H^1 = R^1$, since $CE(R^1, 0)$ is in Regime 1 or Regime 2. Thus, we require R^2 to bind, and to satisfy

$$\pi_H \alpha_H p_H^c + \pi_L \alpha_L R^2 > \pi_H \alpha_H R^1 + \pi_L \alpha_L R^1,$$

or equivalently,

$$R^2 > R^1 - \frac{\pi_H \alpha_H}{\pi_L \alpha_L}(p_H^c - R^1).$$

This is exactly the condition that (30) does not hold.

Case (ii) in the statement of Proposition 2: Finally, consider the case in which $R^1 \in [p_L^{2*}, R^{1*}]$ holds, so $CE(R^1, 0)$ is in Regime 3. We will show that $CE(R^1, R^2)$ is in Regime 3 for any $R^2 < R^1$. The right side of (31) is greater than the left side when we set $p_H^1 = p_H^c$, because $R^1 \geq \max[R^2, p_L^{2*}]$ holds. When we set $p_H^1 = R^1$, since $CE(R^1, 0)$ is in Regime 3, we have

$$\pi_H \alpha_H p_H^c + \pi_L \alpha_L p_L^{2*} > \pi_H \alpha_H R^1 + \pi_L \alpha_L R^1.$$

It follows from $\max[R^2, p_L^{2*}] \geq p_L^{2*}$ that the left side of (31) is greater than the right side when we set $p_H^1 = R^1$ and, therefore, that $CE(R^1, R^2)$ is in Regime 3.■

Proof of Proposition 3

To prove Proposition 3, we will utilize Lemma 2 and Lemma 3 below.

Lemma 2: *Under the conditions of Proposition 3, $R^1 = R^2 = 0$ are subgame perfect equilibrium strategies of the reserve price game.*

Proof: Without loss of generality, let $R^2 = 0$. We have to show that $R^1 = 0$ is a best response, given our characterization of $CE(R^1, 0)$. The proof strategy is to show that firm 1 can earn more profit by setting $R^1 = 0$ and earning

$$\pi_H \frac{\alpha_H D(p_H^c) p_H^c}{2} + \pi_L \frac{\alpha_L D(p_L^c) p_L^c}{2} = \pi_H p_H^c + \pi_L p_L^c,$$

relative to any other $R^1 > 0$. Since any such R^1 belongs to one of the five regimes, with the resulting outcome characterized in Proposition 1, we go regime-by-regime in this proof. The comparison with Regime 1 and Regime 5 is trivial since firm 1's profit in Regime 1 is 0, and in Regime 5 the profit is identical to the profit from $R^1 = 0$. Comparisons with other regimes are below: we start with Regime 4 and work backwards to Regime 2.

Comparing $R^1 = 0$ and R^1 in Regime 4. Consider R^1 in Regime 4, i.e., consider $R^1 \in [p_L^c, p_L^{2*}]$. In $CE(R^1, 0)$, we have $p_L^1 = p_L^2 = R^1$ and $p_H^1 = p_H^2 = p_H^c$. Consumers with valuations greater than p_H^c choose each firm with probability one half, so we have the competitive, market clearing outcome in state H. Consumers with valuations between R^1 and p_H^c choose firm 2 with probability such that the market clearing price at firm 2 is exactly R^1 in state L. Firm 1 has excess supply in state L, so R^1 binds. Let β denote the constant (across valuation) probability with which consumers with valuations between R^1 and p_H^c choose firm 2.

In Regime 4, due to market clearing at firm 2 in the low state, we must have:

$$\frac{1}{2}\alpha_L D(p_H^c) + \alpha_L \beta[D(R^1) - D(p_H^c)] = 1. \tag{34}$$

It is straighforward to verify that β is well defined.[15] So the profit for firm 1 by setting R^1 in Regime 4 is:

$$\pi_H \alpha_H p_H^c D(p_H^c)\frac{1}{2} + \pi_L \alpha_L R^1 D(p_H^c)\frac{1}{2} + \pi_L \alpha_L R^1 (1-\beta)[D(R^1) - D(p_H^c)].$$

Substituting $\alpha_H D(p_H^c) = 2$ and substituting the value of $\alpha_L \beta[D(R^1) - D(p_H^c)]$ from (34) we have that the profit for firm 1 by setting R^1 in Regime 4 is:

$$\pi_H p_H^c + \pi_L \alpha_L R^1 D(p_H^c)\frac{1}{2} + \pi_L \alpha_L R^1[D(R^1) - D(p_H^c)] - \pi_L R^1[1 - \frac{1}{2}\alpha_L D(p_H^c)].$$

Utilizing $\alpha_H D(p_H^c) = 2$, and rearranging and canceling terms yields that firm 1's profit in Regime 4 is:

$$\pi_H p_H^c + \pi_L \alpha_L R^1 D(R^1) - \pi_L R^1. \tag{35}$$

Recall that the profit from setting $R^1 = 0$ is $\pi_H p_H^c + \pi_L p_L^c$. Hence, the profit advantage from setting $R^1 = 0$ relative to an R^1 in Regime 4, denoted by $PA(R^1)$, is given by:

$$PA(R^1) = \pi_L p_L^c - \pi_L \alpha_L R^1 D(R^1) + \pi_L R^1.$$

[15] If $R^1 = p_L^c$, then $\beta = \frac{1}{2}$. For other R^1 in Regime 4, β is given by:

$$\frac{\alpha_L}{\alpha_H} + \alpha_L \beta[D(R^1) - \frac{2}{\alpha_H}] = 1, \text{ or}$$

$$\beta = \frac{(1 - \frac{\alpha_L}{\alpha_H})}{\alpha_L[D(R^1) - \frac{2}{\alpha_H}]} = \frac{(\alpha_H - \alpha_L)}{\alpha_L[\alpha_H D(R^1) - 2]}.$$

Thus β is increasing in R^1 and highest at $R^1 = p_L^{2*}$, where

$$\beta = \frac{(\alpha_H - \alpha_L)}{\alpha_L[\alpha_H(\frac{1}{\alpha_H} + \frac{1}{\alpha_L}) - 2]} = \frac{(\alpha_H - \alpha_L)}{\alpha_L[\frac{\alpha_H}{\alpha_L} - 1]} = 1.$$

From $\alpha_L D(p_L^c) = 2$, it follows that at $R^1 = p_L^c$, $PA(R^1) = 0$ holds. Note that,

$$\frac{1}{\pi_L}\frac{\partial PA(R^1)}{\partial R^1} = [1 - \alpha_L D(R^1) - \alpha_L R^1 D'(R^1)].$$

Dividing both sides by $\alpha_L D(R^1)$ yields:

$$\frac{1}{\pi_L \alpha_L D(R^1)}\frac{\partial PA}{\partial R^1} = \frac{1}{\alpha_L D(R^1)} - 1 - \frac{R^1 D'(R^1)}{D(R^1)}.$$

Thus, we have:

$$\frac{1}{\pi_L \alpha_L D(p_L^c)}\frac{\partial PA(p_L^c)}{\partial R^1} = \frac{1}{\alpha_L D(p_L^c)} - 1 - \frac{p_L^c D'(p_L^c)}{D(p_L^c)}. \tag{36}$$

Since $\alpha_L D(p_L^c) = 2$, and $\pi_L > 0$, we have:

$$\frac{\partial PA(p_L^c)}{\partial R^1} \geq 0 \iff -\frac{p_L^c D'(p_L^c)}{D(p_L^c)} \geq \frac{1}{2}. \tag{37}$$

Thus, for $R^1 = 0$ to yield greater profit than R^1 in Regime 4, it is a necessary condition that the price elasticity of demand at p_L^c, given by $-\frac{p_L^c D'(p_L^c)}{D(p_L^c)}$ and denoted by $E(p_L^c)$, be greater than $\frac{1}{2}$. This is because otherwise, firm 1 can increase R^1 slightly above p_L^c and increase profit relative to $R^1 = 0$. Next, Claim 1 (below) specifies that under our mantained assumption that $E(p)$ is increasing in p, $E(p_L^c) \geq \frac{1}{2}$ is also a sufficient condition for $R^1 = 0$ to yield greater profit than R^1 in Regime 4.

Claim 1: If $E(p_L^c) = -\frac{p_L^c D'(p_L^c)}{D(p_L^c)} \geq \frac{1}{2}$ holds, then the profit from $R^1 = 0$ is greater than the profit from R^1 in Regime 4.
Proof: Recall that $PA(p_L^c)$= 0 holds. Hence, to show Claim 1, we will show that under the assumptions of Claim 1, $\frac{\partial PA(R^1)}{\partial R^1} \geq 0$ holds for all R^1 in Regime 4. Since we have

$$\frac{1}{\pi_L \alpha_L D(R^1)}\frac{\partial PA(R^1)}{\partial R^1} = [\frac{1}{\alpha_L D(R^1)} - 1 - \frac{R^1 D'(R^1)}{D(R^1)}],$$

and $\pi_L \alpha_L D(R^1) > 0$ holds, it will suffice to show

$$[\frac{1}{\alpha_L D(R^1)} - 1 - \frac{R^1 D'(R^1)}{D(R^1)}] \geq 0 \text{ for } R^1 \in [p_L^c, p_L^{2*}]. \tag{38}$$

Recall that $E(p_L^c) \geq \frac{1}{2}$ implies $\frac{\partial PA(p_L^c)}{\partial R^1} \geq 0$. Since $\frac{1}{\alpha_L D(R^1)} > \frac{1}{\alpha_L D(p_L^c)}$ holds for $R^1 > p_L^c$, and since $-\frac{R^1 D'(R^1)}{D(R^1)}$ is greater than $-\frac{p_L^c D'(p_L^c)}{D(p_L^c)}$ due to $E(p)$ increasing with p, comparing the expression in (38) with the right side of (36), it follows that $\frac{\partial PA(p_L^c)}{\partial R^1} \geq 0$ implies $\frac{\partial PA(R^1)}{\partial R^1} > 0$ for all R^1 greater than p_L^c in Regime 4.

Comparing $R^1 = 0$ and R^1 in Regime 3. Consider R^1 in Regime 3, where $R^1 \in (p_L^{2*}, R^{1*})$, with $R^{1*} = \frac{\pi_H \alpha_H p_H^c + \pi_L \alpha_L p_L^{2*}}{\pi_H \alpha_H + \pi_L \alpha_L}$. For R^1 in Regime 3, consumers with value above $\overline{v}^*$ choose firm 1, while those with value in $[p_L^2, \overline{v}^*]$ choose firm 2. The prices at firm 2 are given by $p_H^2 = p_H^c$ and $p_L^2 = p_L^{2*}$. The prices at firm 1 are given by $p_L^1 = R^1$ and, for p_H^1, the solution to the indifference condition for consumers with v greater than $\overline{v}^*$:

$$\pi_H \alpha_H p_H^c + \pi_L \alpha_L p_L^{2*} = \pi_H \alpha_H p_H^1 + \pi_L \alpha_L R^1. \tag{39}$$

The profit of firm 1 in Regime 3 is:

$$\pi_H \alpha_H p_H^1 D(\overline{v}^*) + \pi_L \alpha_L D(\overline{v}^*) R^1.$$

Rearranging (39), we have

$$\frac{[\pi_H \alpha_H p_H^c + \pi_L \alpha_L p_L^{2*}]}{\pi_H \alpha_H} - \frac{\pi_L \alpha_L}{\pi_H \alpha_H} R^1 = p_H^1.$$

And $\alpha_H D(\overline{v}^*) = 1$ holds by definition. Hence profit in Regime 3 can be written as:

$$\pi_H \left(\frac{[\pi_H \alpha_H p_H^c + \pi_L \alpha_L p_L^{2*}]}{\pi_H \alpha_H} - \frac{\pi_L \alpha_L}{\pi_H \alpha_H} R^1 \right) + \pi_L \frac{\alpha_L}{\alpha_H} R^1$$
$$= \pi_H p_H^c + \pi_L \frac{\alpha_L}{\alpha_H} p_L^{2*},$$

which is independent of R^1 for all R^1 in Regime 3. Note that this profit value is precisely the profit value ontained by setting $R^1 = p_L^{2*}$ in Regime 4. To see this, recall that the profit from an R^1 in Regime 4 is:

$$\pi_H p_H^c + \pi_L \alpha_L R^1 D(R^1) - \pi_L R^1.$$

At $R^1 = p_L^{2*}$, this profit is:

$$\pi_H p_H^c + \pi_L \alpha_L p_L^{2*} D(p_L^{2*}) - \pi_L p_L^{2*}.$$

Using $D(p_L^{2*}) = (\frac{1}{\alpha_L} + \frac{1}{\alpha_H})$, and simplifying, we have that the profit from $R^1 = p_L^{2*}$ in Regime 4 is:

$$\pi_H p_H^c + \pi_L \frac{\alpha_L}{\alpha_H} p_L^{2*}.$$

We have already shown that $E(p_L^c) \geq \frac{1}{2}$ is sufficient for the profit from $R^1 = 0$ to be greater than the profit from all $R^1 \in [p_L^c, p_L^{2*}]$. Since the profit from any R^1 in Regime 3 is identical to the profit from $R^1 = p_L^{2*}$, it follows that $E(p_L^c) \geq \frac{1}{2}$ is also sufficient for the profit from

$R^1 = 0$ to be greater than the profit from all R^1 in Regime 3.

Comparing $R^1 = 0$ and R^1 in Regime 2. Recall that in Regime 2, consumers with $v > \overline{v}$ choose firm 1 and all consumers with $v < \overline{v}$ choose firm 2. Furthermore, due to excess supply at firm 1 in both states, $p_H^1 = p_L^1 = R^1$ holds. Firm 1's profit from Regime 2 is given by:

$$\pi_H \alpha_H R^1 D(\overline{v}) + \pi_L \alpha_L R^1 D(\overline{v}). \tag{40}$$

For the $CE(R^1, 0)$ in Regime 2, we have $p_H^2 > R^1 > p_L^2$ and the indifference condition:

$$\pi_H \alpha_H p_H^2 + \pi_L \alpha_L p_L^2 = \pi_H \alpha_H R^1 + \pi_L \alpha_L R^1. \tag{41}$$

Furthermore, market clearing prices at firm 2 are given by

$$\alpha_H D(p_H^2) - \alpha_H D(\overline{v}) \quad = \quad 1 \tag{42}$$

$$\alpha_L D(p_L^2) - \alpha_L D(\overline{v}) \quad = \quad 1. \tag{43}$$

As R^1 decreases in Regime 2, $\overline{v}$, p_H^2, and p_L^2 all fall. We will show that, Under Condition 2 [i.e., $E(v) > 1$ for all $v > p_L^{2*}$], the most profitable R^1 in Regime 2 occurs at the lowest $\overline{v}$ in this regime, $\overline{v}^*$.

Solving (41) for R^1 and substituting this expression into (40), we have an expression for firm 1's profit as a function of $\overline{v}$,

$$D(\overline{v}) \left[\pi_H \alpha_H p_H^2 + \pi_L \alpha_L p_L^2\right] \tag{44}$$

where p_H^2 and p_L^2 are implicitly functions of $\overline{v}$. Differentiating (44) with respect to $\overline{v}$ yields

$$D(\overline{v})[\pi_H \alpha_H \frac{\partial p_H^2}{\partial \overline{v}} + \pi_L \alpha_L \frac{\partial p_L^2}{\partial \overline{v}}] + D'(\overline{v}) \left[\pi_H \alpha_H p_H^2 + \pi_L \alpha_L p_L^2\right]. \tag{45}$$

Differentiating the firm-2 market clearing conditions, (42) and (43), with respect to $\overline{v}$ yields

$$D'(p_H^2) \frac{\partial p_H^2}{\partial \overline{v}} \quad = \quad D'(\overline{v}), \tag{46}$$

$$D'(p_L^2) \frac{\partial p_L^2}{\partial \overline{v}} \quad = \quad D'(\overline{v}). \tag{47}$$

Substituting (46) and (47) into (45), the derivative of profits is given by

$$D(\overline{v})[\pi_H \alpha_H \frac{D'(\overline{v})}{D'(p_H^2)} + \pi_L \alpha_L \frac{D'(\overline{v})}{D'(p_L^2)}] + D'(\overline{v}) \left[\pi_H \alpha_H p_H^2 + \pi_L \alpha_L p_L^2\right].$$

The most profitable deviation into Regime 2 for firm 1 occurs at $R^1 = R^{1*}$ and $\overline{v} = \overline{v}^*$ if the above expression is negative, or equivalently, if we have

$$D(\overline{v})[\frac{\pi_H\alpha_H}{D'(p_H^2)}+\frac{\pi_L\alpha_L}{D'(p_L^2)}]+\left[\pi_H\alpha_H p_H^2+\pi_L\alpha_L p_L^2\right]>0. \tag{48}$$

Since $\overline{v}>p_H^2$ and $\overline{v}>p_L^2$ hold and $D'(\cdot)$ is negative, the left side of (48) is greater than

$$[\frac{\pi_H\alpha_H D(p_H^2)}{D'(p_H^2)}+\frac{\pi_L\alpha_L D(p_L^2)}{D'(p_L^2)}]+\left[\pi_H\alpha_H p_H^2+\pi_L\alpha_L p_L^2\right].$$

This expression is positive, since Condition 2 and $p_L^{2*}<p_L^2<p_H^2$ imply

$$\begin{aligned}\frac{D(p_H^2)}{D'(p_H^2)}+p_H^2 &> 0 \text{ and}\\ \frac{D(p_L^2)}{D'(p_L^2)}+p_L^2 &> 0.\end{aligned}$$

Therefore, (48) holds.

We have shown that the highest profit for firm 1 in Regime 2 occurs at $R^1=R^{1*}$ and $\overline{v}=\overline{v}^*$. Recall that $\overline{v}^*$ satisfies

$$\alpha_H D(\overline{v}^*)=1. \tag{49}$$

As R^1 falls within Regime 2, $\overline{v}$, p_H^2, and p_L^2 all fall. At the threshold satisfying (49), from (42), we have

$$\alpha_H D(p_H^2)=2,$$

so the lowest p_H^2 in Regime 2 is p_H^c. From (43) and (49), we see that the lowest p_L^2 in Regime 2, which we denote by p_L^{2*}, satisfies

$$D(p_L^{2*})=\frac{1}{\alpha_L}+\frac{1}{\alpha_H}. \tag{50}$$

Thus by the indifference condition we have:

$$\pi_H\alpha_H p_H^c+\pi_L\alpha_L p_L^{2*}=(\pi_H\alpha_H+\pi_L\alpha_L)R^{1*}. \tag{51}$$

We have shown that the highest profit in Regime 2 is when R^1 is at its lowest value within Regime 2, R^{1*}, and this maximized profit from Regime 2 is given by

$$\pi_H\alpha_H R^{1*}D(\overline{v}^*)+\pi_L\alpha_L R^{1*}D(\overline{v}^*),$$

where $\alpha_H D(\overline{v}^*) = 1$ holds. Hence this profit is (using 51)

$$\pi_H R^{1*} + \pi_L \frac{\alpha_L}{\alpha_H} R^{1*} = \frac{1}{\alpha_H}(\pi_H \alpha_H + \pi_L \alpha_L) R^{1*} = \pi_H p_H^c + \pi_L \frac{\alpha_L}{\alpha_H} p_L^{2*}.$$

But note that this maximized Regime 2 profit is exactly the profit from any R^1 in Regime 3, and we have already shown that under Condition 1 ($E(p_L^c) \geq \frac{1}{2}$) the Regime 3 profit is strictly lower than the profit from setting $R^1 = 0$.■

Lemma 3. *If demand is elastic at p_L^{2*}, so we have*

$$-\frac{D'(p_L^{2*}) p_L^{2*}}{D(p_L^{2*})} \geq 1,$$

then there cannot be an SPE in pure strategies with R^1 and R^2 both strictly greater than p_L^{2*}.

Proof of Lemma 3: The condition of Lemma 3 means that demand is elastic at p_L^{2*} and (under our maintained assumption) demand is strictly elastic at all prices above p_L^{2*}. We will first show that for any (R^1, R^2) in Regime 2, under the condition of Lemma 3, firm 1 can strictly increase profit by reducing R^1 slightly. Second, we rule out the possibility that a pure strategy SPE (R^1, R^2) is in Regime 3. Finally, we rule out the possibility that $R^1 = R^2 = R$ holds in SPE, with R strictly greater than p_L^{2*}.

Ruling out Regime 2. Recall from the proof of Lemma 2 (see (44)) that in Regime 2, firm 1's profit can be written as:

$$D(\overline{v}) \left[\pi_H \alpha_H p_H^2 + \pi_L \alpha_L p_L^2 \right].$$

Here p_H^2 and p_L^2 are implicitly functions of $\overline{v}$, with p_L^2 equal to the maximum of R^2 and the solution to

$$\alpha_L D(p_L^2) - \alpha_L D(\overline{v}) = 1.$$

Call this solution $\hat{p}_L^2$.

To prove that there cannot be a pure strategy SPE in Regime 2 under the condition of Lemma 3, we will argue that when $(-\frac{D'(v)v}{D(v)}) > 1$ holds for all $v > p_L^{2*}$, the derivative of firm 1's profit (44) with respect to R^1 is strictly negative. Equivalently, we will show that in Regime 2, the derivative of firm 1's profit (44) with respect to $\overline{v}$ is strictly negative. While p_H^2 increases with $\overline{v}$, p_L^2 increases only if R^2 does not bind (otherwise p_L^2 stays equal to R^2). Accordingly there are two cases within Regime 2.

Regime 2, case (i): $\hat{p}_L^2 \geq R^2$ holds.

In this case, R^2 does not bind, hence there is no difference in the analysis by assuming that $R^2 = 0$ holds. In the proof of Lemma 2, for $R^2 = 0$, we have already shown that the derivative firm 1's profit (44) with respect to $\overline{v}$ is strictly negative (see the **Comparing $R^1 = 0$ and R^1 in Regime 2** section of the proof).

Regime 2, case (ii): $\hat{p}_L^2 < R^2$ holds.

Consider again the expression for firm 1's profit as a function of $\overline{v}$,

$$D(\overline{v}) \left[\pi_H \alpha_H p_H^2 + \pi_L \alpha_L p_L^2\right], \tag{52}$$

where p_H^2 and p_L^2 are implicitly functions of $\overline{v}$. Differentiating (52) with respect to $\overline{v}$ yields

$$D(\overline{v})[\pi_H \alpha_H \frac{\partial p_H^2}{\partial \overline{v}} + \pi_L \alpha_L \frac{\partial p_L^2}{\partial \overline{v}}] + D'(\overline{v}) \left[\pi_H \alpha_H p_H^2 + \pi_L \alpha_L p_L^2\right].$$

Since $\hat{p}_L^2 < R^2$ holds, $\frac{\partial p_L^2}{\partial \overline{v}} = 0$ holds, thus, the above expression can be re-written as:

$$D(\overline{v})[\pi_H \alpha_H \frac{\partial p_H^2}{\partial \overline{v}}] + D'(\overline{v}) \left[\pi_H \alpha_H p_H^2 + \pi_L \alpha_L p_L^2\right]. \tag{53}$$

Differentiating the firm-2 market clearing condition (42) with respect to $\overline{v}$ yields

$$D'(p_H^2) \frac{\partial p_H^2}{\partial \overline{v}} = D'(\overline{v}).$$

Substituting this into (53), the derivative of profits is given by

$$D(\overline{v})[\pi_H \alpha_H \frac{D'(\overline{v})}{D'(p_H^2)}] + D'(\overline{v}) \left[\pi_H \alpha_H p_H^2 + \pi_L \alpha_L p_L^2\right].$$

We want to show that the above expression is negative, or equivalently:

$$D(\overline{v})[\frac{\pi_H \alpha_H}{D'(p_H^2)}] + \left[\pi_H \alpha_H p_H^2 + \pi_L \alpha_L p_L^2\right] > 0. \tag{54}$$

Since $\overline{v} > p_H^2$ holds and $D'(\cdot)$ is negative, the left side of (54) is greater than

$$\frac{\pi_H \alpha_H D(p_H^2)}{D'(p_H^2)} + \left[\pi_H \alpha_H p_H^2 + \pi_L \alpha_L p_L^2\right].$$

The sum of the first two terms in this expression is positive, since demand is elastic above p_L^{2*} and we have $p_L^{2*} < p_H^2$, which means

$$\frac{D(p_H^2)}{D'(p_H^2)} + p_H^2 > 0.$$

Therefore, (54) holds.

Ruling out Regime 3.

If (R^1, R^2) are in Regime 3 with $R^2 > p_L^{2*}$ and $R^1 > R^2$, then firm 2's profits are

$$\text{Profit2} = \pi_H p_H^c + \pi_L \alpha_L [D(R^2) - D(\overline{v}^*)]R^2.$$

Differentiating with respect to R^2, we have

$$\begin{aligned} sign(\frac{\partial \text{Profit2}}{\partial R^2}) &= D'(R^2)R^2 + D(R^2) - D(\overline{v}^*) \text{ or} \\ & D(R^2)[\frac{D'(R^2)R^2}{D(R^2)} + 1] - D(\overline{v}^*). \end{aligned}$$

The term in brackets is negative from our elasticity assumption, so firm 2 strictly increases profits by reducing its reserve price. Thus, under our elasticity assumption, there cannot be a pure strategy SPE in Regime 3 with $R^1 > R^2$ and $R^2 > p_L^{2*}$.

Ruling out both firms setting equal reserve prices—$R^1 = R^2 = R$.

In this case, each of the two firms' profit depends on the level of R.[16]
(a) For $R > D^{-1}(\frac{1}{\alpha_H})$, each firm makes the following profit:

$$\pi_H \alpha_H \frac{RD(R)}{2} + \pi_L \alpha_L \frac{RD(R)}{2}.$$

Firm 2's profit if it chooses R^2 slightly lower than R puts us in Regime 1, which means firm 2's profit is:

$$\pi_H \alpha_H R^2 D(R^2) + \pi_L \alpha_L R^2 D(R^2).$$

This latter profit, for R^2 close enough to R, is clearly strictly greater than the profit from setting $R^2 = R$.

[16] The argument assumes that in $CS(R, R)$, each consumer chooses each firm with probability one half, so prices and excess supplies in each state are the same for each firm. If a different consumer equilibrium is selected, there will be a disadvantaged firm with an even greater incentive to lower its reserve price slightly.

(b) For $R \in (p_H^c, D^{-1}(\frac{1}{\alpha_H})]$, each firm makes the following profit:

$$\pi_H \alpha_H \frac{RD(R)}{2} + \pi_L \alpha_L \frac{RD(R)}{2}. \tag{55}$$

Firm 2's profit if it chooses R^2 slightly lower than R is given by the profit from being in Regime 2 (by Proposition 2):

$$\pi_H \alpha_H p_H^2 [D(p_H^2) - D(\overline{v})] + \pi_L \alpha_L R^2 [D(R^2) - D(\overline{v})]. \tag{56}$$

Taking limits as R^2 approaches R from below, p_H^2 converges to R and the deviation profit in (56) converges to

$$(\pi_H \alpha_H + \pi_L \alpha_L)[D(R) - D(\overline{v})]R \tag{57}$$

By market clearing at firm 2 in state H, the limiting $\overline{v}$ satisfies

$$[D(R) - D(\overline{v})] = \frac{1}{\alpha_H}.$$

Therefore, (57) becomes

$$(\pi_H \alpha_H + \pi_L \alpha_L)\frac{R}{\alpha_H}. \tag{58}$$

Since $R > p_H^c$ holds, we have

$$\frac{D(R)}{2} < \frac{D(p_H^c)}{2} = \frac{1}{\alpha_H}.$$

Therefore, firm 2's profit from offering reserve price of exactly R, (55), is strictly less than the limiting profit of deviating to a reserve price slightly below R, (58).

(c) For $R \in (p_L^{2*}, p_H^c)$, each firm makes the following profit from setting $R^1 = R^2 = R$:

$$\pi_H \alpha_H \frac{D(p_H^c) p_H^c}{2} + \pi_L \alpha_L \frac{D(R)R}{2},$$

which, using $\alpha_H D(p_H^c) = 2$, equals

$$\pi_H p_H^c + \pi_L \alpha_L \frac{D(R)}{2} R. \tag{59}$$

But (59) is lower than

$$\pi_H p_H^c + \pi_L \alpha_L R[D(R) - D(\overline{v}^*)]$$

since $D(\overline{v}^*) = \frac{1}{\alpha_H}$ holds by the definition of $\overline{v}^*$, and because we have

$$\frac{D(R)}{2} \leq [D(R) - D(\overline{v}^*)]$$

since $\frac{2}{\alpha_H} \leq D(R)$ holds for $R < p_H^c$ as we have assumed in this case (c).
Thus, firm 2's profit from setting $R^2 = R^1 = R$ is lower than:

$$\pi_H p_H^c + \pi_L \alpha_L R^2 [D(R^2) - D(\overline{v}^*)],$$

which is the profit of firm 2 in Regime 3. If firm 2 slightly lowers R^2 from R (with the margin small enough), then by Proposition 2 (given $R^1 = R < p_H^c$ within case (c)) it will cause a movement into the interior of Regime 3.

As argued above, $\frac{\partial \text{Profit2}}{R^2} < 0$ holds in Regime 3, thus, firm 2 slightly lowering R^2 strictly increases firm 2's profit relative to setting $R^2 = R^1 = R$, with $R \in (p_L^{2*}, p_H^c)$.

Note that the same argument as (c) applies when $R^1 = R^2 = R = p_H^c$ holds but $R \in (p_L^{2*}, R^{1*})$ holds.

(d) None of our arguments above cover the case with $R^1 = R^2 = R = p_H^c$ and $R \geq R^{1*}$, since in this case (by Propositions 1 and 2) we move to Regime 2 if firm 2 undercuts by any amount, but the arguments in case (b) rely on $R < p_H^c$.

If $R^1 = R^2 = p_H^c$ holds, then the consumer equilibrium has $p_L^1 = p_L^2 = p_H^1 = p_H^2 = p_H^c$. Exactly half of the consumers with $v > p_H^c$ go to each firm. If firm 2 marginally reduces its reserve price, we move to the interior of Regime 2 and firm 2's profits are:

$$\pi_H [\alpha_H D(p_H^2) - \alpha_H D(\overline{v})] p_H^2 + \pi_L \alpha_L [D(R^2) - D(\overline{v})] R^2.$$

Market clearing in state H at firm 2 yields

$$\alpha_H D(p_H^2) - \alpha_H D(\overline{v}) = 1,$$

so we can simplify the profit expression to

$$\pi_H p_H^2 + \pi_L \alpha_L [D(R^2) - D(\overline{v})] R^2.$$

We can rewrite the market clearing condition as

$$D(\overline{v}) = D(p_H^2) - \frac{1}{\alpha_H},$$

so we can write profits as

$$\pi_H p_H^2 + \pi_L \alpha_L D(R^2) R^2 - \pi_L \alpha_L D(p_H^2) R^2 + \frac{\pi_L \alpha_L}{\alpha_H} R^2.$$

The indifference condition can be written as

$$\pi_H p_H^2 + \frac{\pi_L \alpha_L R^2}{\alpha_H} = [\frac{\pi_H \alpha_H + \pi_L \alpha_L}{\alpha_H}] p_H^c. \tag{60}$$

The left side of (60) is equal to the first and fourth terms of the profit expression, so by substituting the right side of (60), we can write profits as

$$\pi_L \alpha_L D(R^2) R^2 - \pi_L \alpha_L D(p_H^2) R^2 + [\frac{\pi_H \alpha_H + \pi_L \alpha_L}{\alpha_H}] p_H^c. \tag{61}$$

Dividing (61) by $\pi_L \alpha_L$ and differentiating with respect to R^2 yields that the sign of the profit derivative equals the sign of

$$\frac{\partial D(R^2) R^2}{\partial R^2} - D(p_H^2) - D'(p_H^2) R^2 \frac{\partial p_H^2}{\partial R^2}.$$

The first term in this expression is negative, by the elasticity condition in Lemma 3, the second term is negative, and the third term is negative, because $\frac{\partial p_H^2}{\partial R^2}$ is negative (a reduction in R^2 causes $\overline{v}$ to increase, which causes p_H^2 to increase). Thus, a reduction in R^2 yields an increase in profits for firm 2.$\blacksquare$

Now to complete the proof of Proposition 3, we must argue that there cannot be a pure-strategy SPE with both R^1 and R^2 in $(p_L^c, p_L^{2*}]$. First consider the possibility that $R^2 = R^1 = R$ holds for some $R \in (p_L^c, p_L^{2*}]$. Then in the consumer equilibrium of the resulting consumer subgame, $CS(R^1, R^2)$, consumers choose each firm with probability one half, and prices at each firm are R in state L and p_H^c in state H. But then firm 2 can deviate to any strictly lower reserve price $\hat{R}^2 < R$, leading to a consumer subgame in Regime 4. In the resulting consumer equilibrium (by Proposition 2) of $CS(R^1, \hat{R}^2)$, firm 2 sells more output in state L at the same price R, while in state H it sells the same output at the same price. Thus, $\hat{R}^2 < R$ is a profitable deviation available for firm 2 from $R^2 = R^1 = R$.

Finally, suppose there is a pure-strategy SPE with R^1 and R^2 in $(p_L^c, p_L^{2*}]$, and (without loss of generality) $R^1 > R^2$ holds. Then, we are in Regime 4 (by Proposition 2), and as argued in Lemma 2, firm 1's profit is:

$$\pi_H p_H^c + \pi_L \alpha_L R^1 D(R^1) - \pi_L R^1.$$

It will be strictly better for firm 1 to slightly reduce R^1 if the derivative of firm 1's profit with respect to R^1 is negative. This is true if and only if we have:

$$\alpha_L \frac{\partial D(R^1) R^1}{\partial R^1} < 1.$$

To show this inequality for any $R^1 > p_L^c$ in Regime 4, under the maintained assumption of elasticity increasing in prices, it will suffice to show that for $R^1 = p_L^c$ we have

$$\alpha_L \frac{\partial D(p_L^c) p_L^c}{\partial p_L^c} \leq 1 \text{ or,}$$

$$\alpha_L D(p_L^c) + \alpha_L p_L^c D'(p_L^c) \leq 1.$$

Dividing both sides of the last inequality by $\alpha_L D(p_L^c)$, we have

$$1 + \frac{p_L^c D'(p_L^c)}{D(p_L^c)} \leq \frac{1}{\alpha_L D(p_L^c)},$$

which (given $\alpha_L D(p_L^c) = 2$) can be rewritten as:

$$1 + \frac{p_L^c D'(p_L^c)}{D(p_L^c)} \leq \frac{1}{2}.$$

This holds, since $\frac{p_L^c D'(p_L^c)}{D(p_L^c)} \leq -\frac{1}{2}$ holds by Condition 1. ■

Proof of Proposition 4

Suppose, to the contrary, that both firms choose reserve prices that do not bind in state L. It follows that each reserve price is less than p_L^c and the $CE(R^1, R^2)$ is in Regime 5 with competitive prices at each firm in each state. If firm 1 deviates to a binding reserve price in Regime 4, $R^1 > p_L^c$, then its profit advantage from setting $R^1 = 0$ relative to an R^1 in Regime 4, is the same as it would be with $R^2 = 0$. The reason is that we are supposing that firm 2's reserve price is not binding, so it will not bind if R^1 is increased. Therefore, from the analysis in the proof of Proposition 2 (recalling that $PA(R^1)$ denotes the profit advantage from setting $R^1 = 0$ relative to an R^1 in Regime 4), we conclude

$$\frac{\partial PA(p_L^c)}{\partial R^1} \geq 0 \iff -\frac{p_L^c D'(p_L^c)}{D(p_L^c)} \geq \frac{1}{2}.$$

From (16), we conclude that

$$\frac{\partial PA(p_L^c)}{\partial R^1} < 0$$

holds. In other words, marginally increasing R^1 above p_L^c implies that there is a negative advantage of $R^1 = 0$ relative to an R^1 in Regime 4, so firm 1 increases its profits by raising R^1 above p_L^c. This contradicts the supposition that we can have an equilibrium where neither reserve price binds. In any subgame perfect equilibrium, one of the firms must be choosing a binding reserve price in state L. ■

Proof of Proposition 5

Given the characterization of each consumer subgame, $CS(R^1, R^2)$, provided in Proposition 2, denote the corresponding profits as $\Pi^1(R^1, R^2)$ and $\Pi^2(R^1, R^2)$. Since $R^1 \in (p_L^c, p_L^{2*})$ is a best response to $R^2 = 0$, it suffices to show that $R^2 = 0$ is a best response to R^1. Let us consider a deviation by firm 2 to $\widetilde{R}^2$.

If $\widetilde{R}^2 < R^1$ holds, then the consumer equilibrium, $CS(R^1, \widetilde{R}^2)$, is in Regime 4. The reserve price, $\widetilde{R}^2$, is not binding and the outcome is exactly the same as under $R^2 = 0$. This is not a profitable deviation.

If $\widetilde{R}^2 = R^1$ holds, then in the consumer equilibrium, $CS(R^1, \widetilde{R}^2)$, consumers choose each firm with probability one half. Prices at each firm are R^1 in state L and p_H^c in state H. Since prices are the same as in $CS(R^1, 0)$ but firm 2 is selling less output in state L, this cannot be a profitable deviation.

If $\widetilde{R}^2 > R^1$ holds, then the consumer equilibrium, $CS(R^1, \widetilde{R}^2)$, is either in Regime 1, 2, 3, or 4, where now firm 2 is the firm with the higher reserve price. In all these subcases, it follows from $R^1 < p_L^{2*}$ that firm 1's reserve price is not binding. Therefore, we have

$$\Pi^2(R^1, \widetilde{R}^2) = \Pi^2(0, \widetilde{R}^2) = \Pi^1(\widetilde{R}^2, 0) \leq \Pi^1(R^1, 0), \tag{62}$$

where the inequality above follows from the fact that R^1 is a best response to $R^2 = 0$. However, $CS(R^1, 0)$ is in Regime 4, where in each state the prices are the same at the two firms, but firm 2 sells all its output in both states and firm 1 does not sell all its output in state L. Therefore, we have $\Pi^1(R^1, 0) < \Pi^2(R^1, 0)$. Combined with (62), we have $\Pi^2(R^1, \widetilde{R}^2) < \Pi^2(R^1, 0)$, so the deviation is not profitable.■

Proof of Proposition 6

As argued in Proposition 4, Condition 1 ensures that for some R^1 in Regime 4 with $R^1 > p_L^c$, we have $\Pi^1(R^1, 0) > \Pi^1(p_L^c, 0)$. Now consider Condition 2, and recall that firm 1's profit in Regime 4 is (repeating (35)):

$$\pi_H p_H^c + \pi_L \alpha_L R^1 D(R^1) - \pi_L R^1.$$

The derivative of this expression with respect to R^1 is:

$$\pi_L \alpha_L \frac{\partial R^1 D(R^1)}{\partial R^1} - \pi_L.$$

By continuity and our maintained assumption of elasticity being strictly increasing in prices, Condition 2 implies that demand is elastic at $R^1 = p_L^{2*}$. Therefore, the derivative of Regime

4 profit is negative at some R^1 in Regime 4, with $R^1 < p_L^{2*}$, and this derivative stays negative for all higher R^1 in Regime 4. Thus, for some R^1 in Regime 4 with $R^1 < p_L^{2*}$, we have $\Pi^1(R^1, 0) > \Pi^1(p_L^{2*}, 0)$. And finally, under Condition 2, as argued in Proposition 3, $\Pi^1(p_L^{2*}, 0)$ is weakly greater than $\Pi^1(R^1, 0)$ for all $R^1 > p_L^{2*}$. Since firm 1's profit function in Regime 4 is continuous and $[p_L^c, p_L^{2*}]$ is closed and bounded, given the arguments above, some $R_{max}^1 \in (p_L^c, p_L^{2*})$ is a best response to $R^2 = 0$. Finally, Proposition 5 ensures that the resulting $(R_{max}^1, 0)$ is an SPE.■

Proof of Proposition 7

Proof. For $\varepsilon = 0$, it is shown in Peck (2018, online appendix) that there is an equilibrium in which both firms set a reserve price of zero. Half of the consumers go to each firm and the prices are competitive, equal to p_α^c. To verify that prices are competitive in every equilibrium, suppose not. Then, without loss of generality, firm 1 chooses $R^1 > p_\alpha^c$. Firm 2's best response is to set a non-binding $R^2 < R^1$. To see this, in the consumer subgame, the price is R^1 at both firms, but firm 2 sells all its output; any reserve price less than R^1 does not increase firm 2's profits and a reserve price greater than or equal to R^1 yields lower profits.[17] This is inconsistent with equilibrium, because firm 1 (due to the elasticity condition) is receiving lower profits than it would receive with $R^1 = 0$.

Let $\underline{p}$ be a price at which the price elasticity of demand is less than one half. Since elasticity is decreasing in price, we have $\underline{p} < p_\alpha^c$. Set ε to satisfy

$$\varepsilon = \alpha - \frac{2}{D(\underline{p})}, \tag{63}$$

which implies $(\alpha - \varepsilon)D(\underline{p}) = 2$. When ε is set according to (63), then $\underline{p}$ is the market clearing price in state L, $p_L^c = \underline{p}$. It follows from Proposition 4 that, in any equilibrium, at least one firm sets a reserve price greater than p_L^c. ■

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